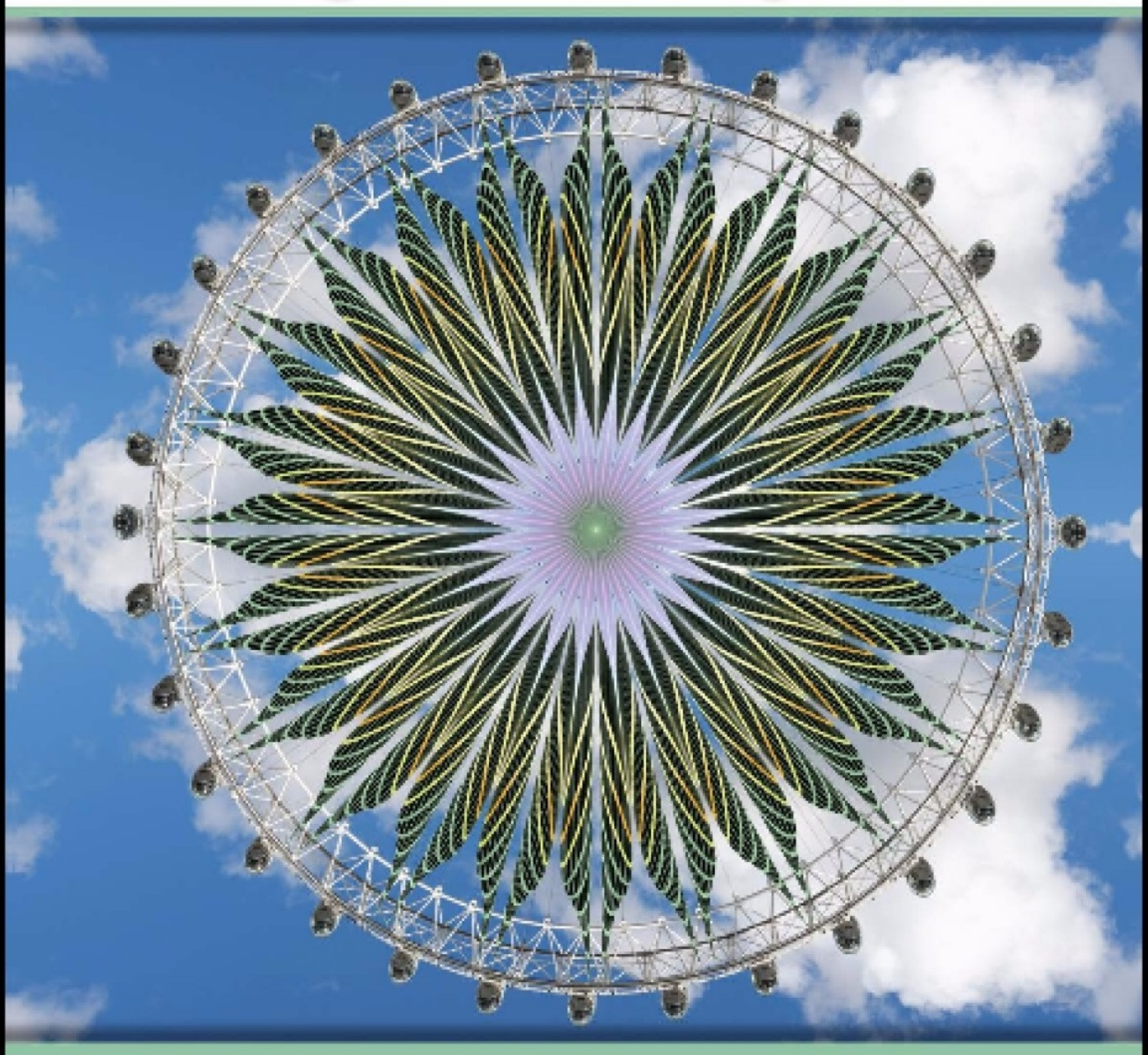
Solutions Manual for Chapters 1-4

Trigonometry 8e



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Chapter 1
The Six Trigonometric Functions

1.1 Angles, Degrees, and Special Triangles

EVEN SOLUTIONS

2. Two angles with a sum of 90° are called complementary angles, and when the sum is 180° they are called supplementary angles.

4. In a right triangle, the longest side opposite the right angle is called the hypotenuse, and the other two sides are called legs.

6. In a 30°–60°–90° triangle, the hypotenuse is always twice the shortest side, and the side opposite the 60° angle is always $\sqrt{3}$ times the shortest side.

8. **a**. ii **b**. iv **c**. i **d**. iii

10. 70° is an acute angle. The complement is 70° – 50° = 20°, and the supplement is 180° – 70° = 110°.

12. 90° is neither an acute or obtuse angle (it is a right angle). The complement is 90° – 90° = 0°, and the supplement is 180° – 90° = 90°.

14. 150° is an obtuse angle. The complement is 90° – 150° = –60°, and the supplement is 180° – 150° = 30°.

16. y is neither an acute or obtuse angle (its measure is unknown). The complement is 90° – y, and the supplement is 180° – y.

18. Adding the angles in triangle BDC:

$$B + 90° + 50° = 180°$$
$$B + 140° = 180°$$
$$B = 40°$$

20. Adding the angles in triangle ADC:

$$\alpha + \alpha + 90° = 180°$$
$$2\alpha + 90° = 180°$$
$$2\alpha = 90°$$
$$\alpha = 45°$$

22. Since $A = 50°$, $\alpha = 180° - 50° - 90° = 40°$. Since $\alpha + \beta = 75°$:

$$40° + \beta = 75°$$
$$\beta = 35°$$

Since $\beta = 35°$, $B = 180° - 35° - 90° = 55°$.

24. No. The posts must be perpendicular to the ground for α and β to be complementary.

26. Since α and β are complementary, $\alpha = 90° - 15° = 75°$.

28. Since 1 revolution = 360°, which corresponds to 24 hours, then in 3 hours the hour earth will turn $\frac{3}{24}(360°) = \frac{1}{8}(360°) = 45°$.

30. Let x represent the measure of each angle. Since the angles must add to 180°:

$$x + x + x = 180°$$
$$3x = 180°$$
$$x = 60°$$

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32. Using the Pythagorean Theorem:

$$2^2 + b^2 = 10^2$$
$$4 + b^2 = 100$$
$$b^2 = 96$$
$$b = \sqrt{96} = 4\sqrt{6}$$

34. Using the Pythagorean Theorem:

$$2^2 + b^2 = 6^2$$
$$4 + b^2 = 36$$
$$b^2 = 32$$
$$b = \sqrt{32} = 4\sqrt{2}$$

36. Using the Pythagorean Theorem:

$$a^2 + 10^2 = 26^2$$
$$a^2 + 100 = 676$$
$$a^2 = 576$$
$$a = 24$$

38. Using the Pythagorean Theorem:

$$4^2 + 4^2 = x^2$$
$$16 + 16 = x^2$$
$$x^2 = 32$$
$$x = \sqrt{32} = 4\sqrt{2}$$

40. Using the Pythagorean Theorem:

$$1^2 + x^2 = 2^2$$
$$1 + x^2 = 4$$
$$x^2 = 3$$
$$x = \sqrt{3}$$

42. Using the Pythagorean Theorem:

$$x^2 + 4^2 = (2x - 1)^2$$
$$x^2 + 16 = 4x^2 - 4x + 1$$
$$0 = 3x^2 - 4x - 15$$
$$3x^2 - 4x - 15 = 0$$
$$(3x + 5)(x - 3) = 0$$
$$x = 3, -\frac{5}{3}$$

Since x represents the side of a triangle, it cannot be a negative number. So the only solution is $x = 3$.

44. First find AC using the Pythagorean Theorem:

$$(AC)^2 + 5^2 = 13^2$$
$$(AC)^2 + 25 = 169$$
$$(AC)^2 = 144$$
$$AC = 12$$

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Thus $DC = AC - AD = 12 - 4 = 8$. Now find BD using the Pythagorean Theorem:

$$8^2 + 5^2 = (BD)^2$$
$$64 + 25 = (BD)^2$$
$$(BD)^2 = 89$$
$$BD = \sqrt{89}$$

46. Let $AB = x$. Since $AC = AB + BC = x + 5$, using the Pythagorean Theorem:

$$12^2 + 5^2 = (x+5)^2$$
$$144 + 25 = (x+5)^2$$
$$(x+5)^2 = 169$$
$$x + 5 = \pm\sqrt{169} = \pm 13$$
$$x = -18, 8$$

Since AB must be positive, $AB = 8$.

48. Let x represent the distance across the pond. Using the Pythagorean Theorem:

$$25^2 + 60^2 = x^2$$
$$625 + 3600 = x^2$$
$$x^2 = 4225$$
$$x = 65$$

The distance across the pond is 65 yards.

50. If the shortest side is 4, the side opposite the 60° angle is $\sqrt{3} \cdot 4 = 4\sqrt{3}$, and the longest side is $2 \cdot 4 = 8$.

52. If the longest side is 5, the shortest side is $\frac{1}{2} \cdot 5 = \frac{5}{2}$, and the side opposite the 60° angle is $\sqrt{3} \cdot \frac{5}{2} = \frac{5\sqrt{3}}{2}$.

54. If the side opposite the 60° angle is 3, the shortest side is $\frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \sqrt{3}$, and the longest side is $2 \cdot \sqrt{3} = 2\sqrt{3}$.

56. Since 20 ft is now opposite the 60° angle, the length of the shortest side is $\frac{20}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{20\sqrt{3}}{3}$. The length of the escalator is the longest side, which is $2 \cdot \frac{20\sqrt{3}}{3} = \frac{40\sqrt{3}}{3} \approx 23.09$ ft.

58. To solve this problem we need to find the widths of the base and sides of the tent. Since 3 ft is the side opposite the 60° angle at the end, the shortest side is $\frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \sqrt{3}$ ft and the longest side is $2\sqrt{3}$ ft. The width of the base and sides are therefore $2\sqrt{3}$ ft. If l represents the length of the tent, the sides and floor of the tent have a total area of $3 \cdot 2\sqrt{3} \cdot l = 6l\sqrt{3}$ ft, and the ends (which are triangles) have a total area of $2 \cdot \frac{1}{2} \cdot 2\sqrt{3} \cdot 3 = 6\sqrt{3}$ ft^2. Thus the total area of material needed is 90:

$$6l\sqrt{3} + 6\sqrt{3} = 90$$
$$6\sqrt{3}(l+1) = 90$$
$$l + 1 = \frac{90}{6\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = 5\sqrt{3}$$
$$l = 5\sqrt{3} - 1 \approx 7.7$$

The length of the tent should be approximately 7.7 feet.

60. Since the shorter sides are each $\frac{1}{2}$, the longer side is $\frac{1}{2}\sqrt{2} = \frac{\sqrt{2}}{2}$.

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62. Since the longest side is $6\sqrt{2}$, the shorter sides are $\frac{6\sqrt{2}}{\sqrt{2}} = 6$.

64. Since the longest side is 12, the shorter sides are $\frac{12}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = 6\sqrt{2}$.

66. Since the height is 1,000 ft, which represents the shorter side, the longer side is $1000\sqrt{2}$ ft. Thus, if the bullet is traveling at 2,828 ft/sec, the total time is $\frac{1000\sqrt{2} \text{ ft}}{2828 \text{ ft/sec}} \approx \frac{1}{2}$ sec.

68. Since $d = 4$ and c is the hypotenuse of a 45°-45°-90° triangle, $c = 4\sqrt{2}$. Since b is the shorter side of a 30°-60°-90° triangle, $b = \frac{4\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{6}}{3}$. Since a is the hypotenuse of a 30°-60°-90° triangle, $a = 2 \cdot \frac{4\sqrt{6}}{3} = \frac{8\sqrt{6}}{3}$.

70. **a**. Since $GD = 5$ cm and ΔGCD is a 45°–45°–90° triangle, then each side has a length of $\frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$ cm.

b. Since $GD = 5$ cm and from part **a** and $BD = \frac{5\sqrt{2}}{2}$ 5 cm, using the Pythagorean Theorem:

$$(GD)^2 + (BD)^2 = (GB)^2$$
$$(5)^2 + \left(\frac{5\sqrt{2}}{2}\right)^2 = (GB)^2$$
$$25 + \frac{25}{2} = (GB)^2$$
$$(GB)^2 = \frac{75}{2}$$
$$GB = \sqrt{\frac{75}{2}} = \frac{5\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{6}}{2} \text{ cm}$$

72. The measure of $\angle GDH$ is 45°, since ΔGHD is a 45°–45°–90° triangle.

74. **a**. Since ΔODB is a right triangle, using the Pythagorean Theorem:

$$(OD)^2 + (DB)^2 = (OB)^2$$
$$1^2 + 2^2 = (OB)^2$$
$$(OB)^2 = 5$$
$$OB = \sqrt{5}$$

b. Since $OB = OE$, $OE = \sqrt{5}$.

c. Since $CE = CO + OE$, $CE = 1 + \sqrt{5}$.

d. Finding the ratio: $\frac{CE}{EF} = \frac{1+\sqrt{5}}{2}$. This is called the golden ratio.

76. Using the Pythagorean Theorem:

$$2^2 + b^2 = 5^2$$
$$4 + b^2 = 25$$
$$b^2 = 21$$
$$b = \sqrt{21}$$

The correct answer is c.

78. Since the leg is 24 feet in a 45°–45°–90° triangle, the length is $24\sqrt{2}$ feet. The correct answer is b.

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ODD SOLUTIONS

1. counterclockwise, clockwise **3.** 180°

5. hypotenuse, sum, legs **7.** equal, $\sqrt{2}$

9. 10° is an acute angle.
The complement of 10° is 80° because $10^\circ + 80^\circ = 90^\circ$.
The supplement of 10° is 170° because $10^\circ + 170^\circ = 180^\circ$.

11. 45° is an acute angle.
The complement of 45° is 45° because $45^\circ + 45^\circ = 90^\circ$.
The supplement of 45° is 135° because $45^\circ + 135^\circ = 180^\circ$.

13. 120° is an obtuse angle.
The complement of 120° is -30° because $120^\circ + (-30)^\circ = 90^\circ$.
The supplement of 120° is 60° because $120^\circ + 60^\circ = 180^\circ$.

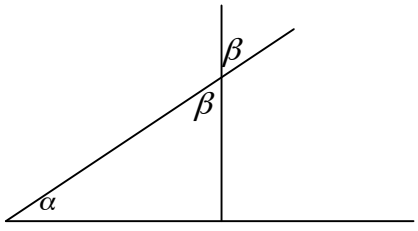
15. We can't tell if x° is acute or obtuse (or neither).
The complement of x° is $90^\circ - x^\circ$ because $x^\circ + (90^\circ - x^\circ) = 90^\circ$.
The supplement of x° is $180^\circ - x^\circ$ because $x^\circ + (180^\circ - x^\circ) = 180^\circ$.

17.
$\alpha = 180^\circ - (\angle A + \angle D)$ The sum of the angles of a triangle is 180°
$= 180^\circ - (30^\circ + 90^\circ)$ Substitute given values
$= 180^\circ - 120^\circ$ Simplify
$= 60^\circ$

19.
$\alpha = 180^\circ - (\angle A + \angle D)$ The sum of the angles of a triangle is 180°
$= 180^\circ - (2\alpha + 90^\circ)$ Substitute the given values
$= 90^\circ - 2\alpha$ Simplify right side
$3\alpha = 90^\circ$ Add 2α to both sides
$\alpha = 30^\circ$ Divide both sides by 3

21.
$\angle A = 180^\circ - (\alpha + \beta + \angle B)$ The sum of the angles of a triangle is 180°
$= 180^\circ - (100^\circ + 30^\circ)$ Substitute given values
$= 180^\circ - 130^\circ$ Simplify
$= 50^\circ$

23. Angles α and β are complementary because
$\alpha + \beta + 90^\circ = 180^\circ$
$\alpha + \beta = 90^\circ$



25.
$\alpha + \beta = 90^\circ$ α and β are complementary
$\alpha = 90^\circ - \beta$ Subtract β from both sides
$= 90^\circ - 52^\circ$ Substitute given value
$= 38^\circ$ Simplify

27. One complete revolution equals 360°.
Therefore, it rotates 360° in 4 seconds and 90° in 1 second.

29. Let α = the degree measure of each base angle
Then $\alpha + \alpha + 40^\circ = 180^\circ$
$2\alpha + 40^\circ = 180^\circ$
$2\alpha = 140^\circ$
$\alpha = 70^\circ$

Each base angle of this isosceles triangle is 70°.

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31. $c^2 = a^2 + b^2$ Pythagorean Theorem

$= 4^2 + 3^2$ Substitute given values

$= 16 + 9$ Simplify

$= 25$

Therefore, $c = \pm 5$. The only solution is $c = 5$, because we cannot use $c = -5$.

33. $a^2 + b^2 = c^2$ Pythagorean Theorem

$b^2 = c^2 - a^2$ Subtract a^2 from both sides

$= 17^2 - 8^2$ Substitute given values

$= 289 - 64$ Simplify

$= 225$

Therefore, $b = \pm 15$. The only solution is $b = 15$, because we cannot use $b = -15$.

35. $a^2 + b^2 = c^2$ Pythagorean Theorem

$a^2 = c^2 - b^2$ Subtract b^2 from both sides

$= 13^2 - 12^2$ Substitute given values

$= 169 - 144$ Simplify

$= 25$

Therefore, $a = \pm 5$. Our only solution is $a = 5$, because we cannot use $a = -5$.

37. $x^2 + 5^2 = \left(5\sqrt{2}\right)^2$ Pythagorean Theorem

$x^2 + 25 = 50$ Simplify

$x^2 = 25$ Subtract 25 from both sides

Therefore, $x = \pm 5$. Our only solution is $x = 5$ because we cannot use $x = -5$.

Note: This must be a $45^\circ - 45^\circ - 90^\circ$ *triangle.*

39. $x^2 = (2)^2 + \left(2\sqrt{3}\right)^2$ Pythagorean Theorem

$= 4 + 12$ Simplify

$= 16$

Therefore $x = \pm 4$. Our only solution is $x = 4$, because we cannot use $x = -4$.

Note: This must be a $30^\circ - 60^\circ - 90^\circ$ *triangle.*

41. $\left(\sqrt{10}\right)^2 = x^2 + (x+2)^2$ Pythagorean Theorem

$10 = x^2 + x^2 + 4x + 4$ Simplify

$10 = 2x^2 + 4x + 4$ Combine like terms

$0 = 2x^2 + 4x - 6$ Subtract 10 from both sides

$0 = x^2 + 2x - 3$ Divide both sides by 2

$0 = (x+3)(x-1)$ Factor

$x + 3 = 0$ or $x - 1 = 0$ Set each factor equal to zero

$x = -3 \qquad x = 1$

Therefore, $x = 1$ because $x = -3$ is not possible.

43. $(BD)^2 = (CD)^2 + (BC)^2$ Pythagorean Theorem

$5^2 = (CD)^2 + (4)^2$ Substitute given values

$25 = (CD)^2 + 16$ Simplify

$9 = (CD)^2$ Subtract 16 from both sides

$CD = 3$ or $CD = -3$ Take square root of both sides

$CD = 3$ Eliminate negative solution

Therefore, $AC = 2 + 3 = 5$ $AC = AD + DC$

$(AB)^2 = (AC)^2 + (BC)^2$ Pythagorean Theorem

$= 5^2 + 4^2$ Substitute given values

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$= 25 + 16$ Simplify

$= \sqrt{41}$

$AB = \sqrt{41}$ or $AB = -\sqrt{41}$ Take square root of both sides

$AB = \sqrt{41}$ Eliminate negative solution

45. $(AB + BC)^2 = (CD)^2 + (AD)^2$ Pythagorean Theorem

$(4 + r)^2 = r^2 + 8^2$ Substitute given values

$16 + 8r + r^2 = r^2 + 64$ Simplify

$16 + 8r = 64$ Subtract r^2 from both sides

$8r = 48$ Subtract 16 from both sides

$r = 6$ Divide both sides by 8

47. This is an isosceles triangle. Therefore, the altitude must bisect the base.

$x^2 = (18)^2 + (13.5)^2$ Pythagorean Theorem

$= 324 + 182.5$ Simplify

$= 506.25$

$x = 22.5$ or $x = -22.5$ Take square root of both sides

$x = 22.5$ ft Eliminate negative solution

49. The shortest side is 1.

The longest side is twice the shortest side. Therefore, it is 2.

The side opposite the 60° angle is $1\sqrt{3}$ or $\sqrt{3}$.

51. The longest side is 8 which is twice the shortest side.

Therefore, the shortest side is 4.

The side opposite the 60° angle is $4\sqrt{3}$.

53. Let t = the shortest side, $2t$ = the longest side, and $t\sqrt{3}$ = the side opposite 60°

Therefore, $t\sqrt{3} = 6$ Side opposite 60° is 6

$t = \dfrac{6}{\sqrt{3}}$ Divide both sides by $\sqrt{3}$

$= \dfrac{6\sqrt{3}}{3} = 2\sqrt{3}$ Rationalize the denominator

Since $t = 2\sqrt{3}$, $2t = 2\left(2\sqrt{3}\right) = 4\sqrt{3}$.

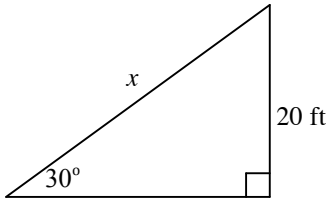
The shortest side is $2\sqrt{3}$ and the longest side is $4\sqrt{3}$.

55. The shortest side is 20 feet.

The longest side is twice the shortest side

Therefore, $x = 2(20)$

$x = 40$ feet



57. The tent is made up of 3 congruent rectangles and 2 congruent triangles.

First we'll find the sides of the $30^\circ - 60^\circ - 90^\circ$ triangle.

The side opposite 60° is 4 ft. Let t = the shortest side.

$t\sqrt{3} = 4$

$t = \dfrac{4}{\sqrt{3}}$

$t = \dfrac{4}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{4\sqrt{3}}{3}$

The shortest side is $\dfrac{4\sqrt{3}}{3}$. The hypotenuse is $2\left(\dfrac{4\sqrt{3}}{3}\right) = \dfrac{8\sqrt{3}}{3}$.

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Also the base of the triangular sides are $2\left(\frac{4\sqrt{3}}{3}\right)=\frac{8\sqrt{3}}{3}$.

Note: This is an equilateral triangle.
Area of rectangles = length · width

Area of rectangles $=6\left(\frac{8\sqrt{3}}{3}\right)$

$=16\sqrt{3}$ ft^2

Area of triangles $=\frac{1}{2}\cdot base\cdot height$

$=\frac{1}{2}\left(\frac{8\sqrt{3}}{3}\right)(4)$

$=\frac{16\sqrt{3}}{3}$ ft^2

Area of tent = 3 rectangles + 2 triangles

$=3\left(16\sqrt{3}\right)+2\left(\frac{16\sqrt{3}}{3}\right)$

$=48\sqrt{3}+\frac{32\sqrt{3}}{3}$

≈ 101.6 ft^2

59. hypotenuse $=\frac{4}{5}\cdot\sqrt{2}$ Hypotenuse is $t\sqrt{2}$

$=\frac{4\sqrt{2}}{5}$ Simplify

61. hypotenuse $=t\sqrt{2}$ t is the shorter side

$8\sqrt{2}=t\sqrt{2}$ Substitute given value

$8=t$ Divide both sides by $\sqrt{2}$

63. hypotenuse $=t\sqrt{2}$ t is the shorter side

$4=t\sqrt{2}$ Substitute given value

$\frac{4}{\sqrt{2}}=t$ Divide both sides by $\sqrt{2}$

$t=2\sqrt{2}$ Rationalize denominator by multiplying numerator and

65. We are looking for the hypotenuse of a $45^\circ-45^\circ-90^\circ$ triangle where the shorter sides are 1000 feet.

hypotenuse $=1000\sqrt{2}$ Hypotenuse is $t\sqrt{2}$

≈ 1414 ft Rounded to the nearest foot

The bullet travels 1414 feet.

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67. First, we will find the leg of the $45^\circ - 45^\circ - 90^\circ$ triangle where the hypotenuse is 3:

$3 = d\sqrt{2}$ Hypotenuse $= d\sqrt{2}$

$\frac{3}{\sqrt{2}} = d$ or $d = \frac{3\sqrt{2}}{2}$ Divide both sides by $\sqrt{2}$ and rationalize the denominator

Next, we will find the shorter side of the $30^\circ - 60^\circ - 90^\circ$ triangle where the side opposite the 60° angle is 3:

$b\sqrt{3} = 3$ $b\sqrt{3}$ = the side opposite 60°

$b = \frac{3}{\sqrt{3}} = \sqrt{3}$ Divide both sides by $\sqrt{3}$ and rationalize the denominator

Last, we will find the hypotenuse of the $30^\circ - 60^\circ - 90^\circ$ triangle:

$a = 2(b) = 2\sqrt{3}$ Hypotenuse = twice the shorter side

69. **a.** hypotenuse $= t\sqrt{2}$ t is the edge of the cube

$= 1\sqrt{2}$ Substitute given value

$= \sqrt{2}$ Simplify

Therefore, $CH = \sqrt{2}$ inches

b. $(CF)^2 = (CH)^2 + (FH)^2$ Pythagorean Theorem

$= \left(\sqrt{2}\right)^2 + (1)^2$ Substitute given values

$= 2 + 1$ Simplify

$= 3$

$CF = \pm\sqrt{3}$ Take easy square root of both sides

$CF = \sqrt{3}$ inches Eliminate the negative solution

71. **a.** hypotenuse $= x\sqrt{2}$ x is the edge of the cube

The length of the diagonal of any face of the cube will be $x\sqrt{2}$.

b. $(CF)^2 = (CH)^2 + (FH)^2$ Pythagorean Theorem

$= \left(x\sqrt{2}\right)^2 + (x)^2$ Substitute given values

$= 2x^2 + x^2$ Simplify

$= 3x^2$

$CF = \sqrt{3x^2}$ or $CF = -\sqrt{3x^2}$

$= x\sqrt{3}$ This is impossible.

The length of any diagonal that passes through the center of the cube will be $x\sqrt{3}$.

75. The complement of 61° is $90^\circ - 61^\circ = 29^\circ$.

The supplement of 61° is $180^\circ - 61^\circ = 119^\circ$. The answer is d.

77. The longest side is 6. The shortest side is one-half the longest side. Therefore it is 3.

The side opposite the 60° angle is $3\sqrt{3}$. The answer is c.

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1.2 The Rectangular Coordinate System

EVEN SOLUTIONS

2. The unit circle has center $(0, 0)$ and a radius of 1.

4. The notation $\theta \in \text{QIII}$ means that θ is in standard position and its terminal side lies in quadrant three.

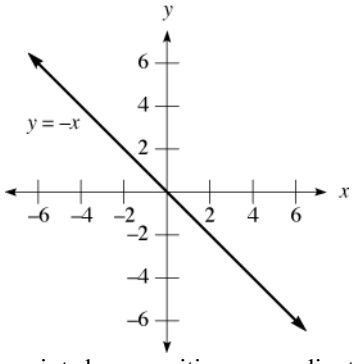
6. Coterminal angles are two angles in standard position having the same terminal side. We can find a coterminal angle by adding or subtracting any multiple of 360°.

8. The formula for a circle with center (h, k) and radius r is $(x-h)^2 + (y-k)^2 = r^2$.

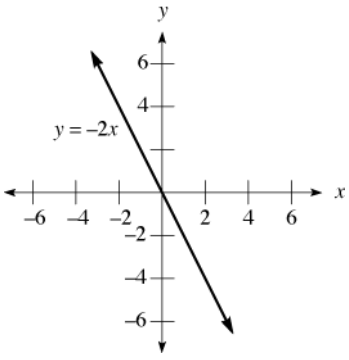
10. The ordered pair $(4,2)$ lies in the first quadrant.

12. The ordered pair $\left(-1,-\sqrt{3}\right)$ lies in the third quadrant.

14. Graphing the line:



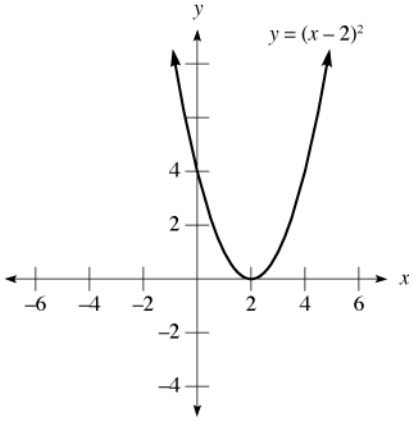
16. Graphing the line:



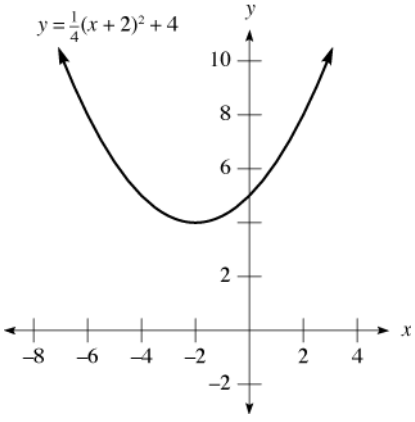
18. All the points have positive y-coordinates in quadrants I and II.

20. The ratio $\frac{x}{y}$ is also negative in quadrant IV, since x is positive and y is negative in that quadrant.

22. Graphing the parabola:



24. Graphing the parabola:



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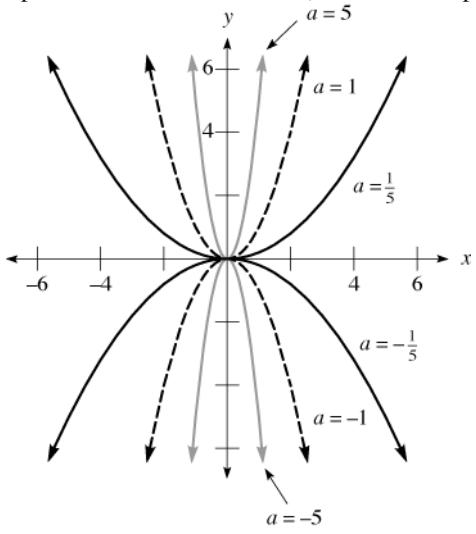
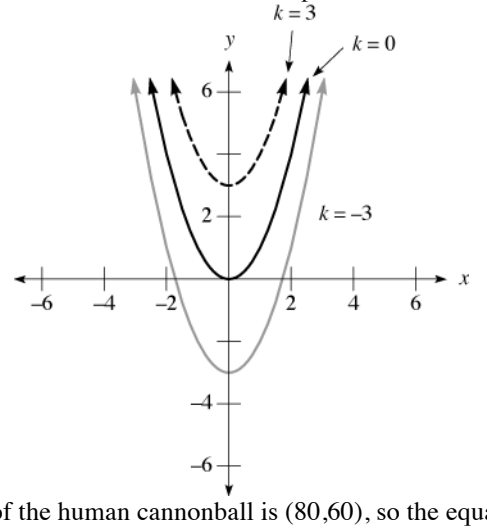
26. The negative value of a reflects the parabola across the x-axis (so that it now points down):



28. When $k < 0$, the parabola is shifted down and when $k > 0$, the parabola is shifted up:



30. The vertex of the parabolic path of the human cannonball is (80,60), so the equation of the path is $y = a(x - 80)^2 + 60$. Substituting the point $(160, 0)$:

$$0 = a(160 - 80)^2 + 60$$
$$-60 = 6400a$$
$$a = -\frac{60}{6400} = -\frac{3}{320}$$

Thus the equation is $y = -\frac{3}{320}(x - 80)^2 + 60$. Substituting $x = 40$ and $x = 140$:

$x = 40$: $y = -\frac{3}{320}(40 - 80)^2 + 60 = 45$ feet

$x = 140$: $y = -\frac{3}{320}(140 - 80)^2 + 60 = 26.25$ feet

These values are verified using a graphing calculator.

32. Using the distance formula: $r = \sqrt{(7-4)^2 + (1-8)^2} = \sqrt{3^2 + (-7)^2} = \sqrt{9+49} = \sqrt{58}$

34. Using the distance formula: $r = \sqrt{(0+8)^2 + (6-0)^2} = \sqrt{8^2 + 6^2} = \sqrt{64+36} = \sqrt{100} = 10$

36. Using the distance formula: $r = \sqrt{(1+5)^2 + (-2-8)^2} = \sqrt{6^2 + (-10)^2} = \sqrt{36+100} = \sqrt{136} = 2\sqrt{34}$

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38. Using the points $(0,0)$ and $(-5,5)$ in the distance formula:

$$r = \sqrt{(-5-0)^2 + (5-0)^2} = \sqrt{(-5)^2 + 5^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$$

40. Using the points $(7,y)$ and $(3,3)$ in the distance formula:

$$\sqrt{(3-7)^2 + (-1-y)^2} = 5$$
$$\sqrt{16 + (-1-y)^2} = 5$$
$$16 + (-1-y)^2 = 25$$
$$(-1-y)^2 = 9$$
$$-1-y = \pm\sqrt{9} = \pm 3$$
$$-1-y = 3 \quad \text{or} \quad -1-y = -3$$
$$-y = 4 \quad \text{or} \quad -y = -2$$
$$y = -4 \quad \text{or} \quad y = 2$$

42. Since the triangle from home plate to first base to second base is a right triangle, use the Pythagorean Theorem. Let x represent the distance from home plate to second base, so:

$$60^2 + 60^2 = x^2$$
$$3600 + 3600 = x^2$$
$$x^2 = 7200$$
$$x = \sqrt{7200} = 60\sqrt{2}$$

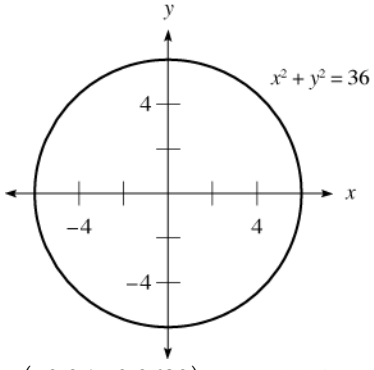
The distance is $60\sqrt{2} \approx 85$ ft.

44. Since the distance from home plate to second base is $60\sqrt{2}$ ft, the distance from home plate to the pitcher's mound is $30\sqrt{2}$ ft, so the coordinates of the pitcher's mound are $(30\sqrt{2}, 0)$. Since the distance from the pitcher's mound to either first base or third base is also $30\sqrt{2}$ ft, the coordinates of first base are $(30\sqrt{2}, -30\sqrt{2})$ and the coordinates of third base are $(30\sqrt{2}, 30\sqrt{2})$.

46. Substituting the point $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$: $\left(-\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2} + \frac{1}{2} = 1$

48. Substituting the point $\left(-\frac{2}{3}, -\frac{\sqrt{5}}{3}\right)$: $\left(\frac{2}{3}\right)^2 + \left(-\frac{\sqrt{5}}{3}\right)^2 = \frac{4}{9} + \frac{5}{9} = 1$

50. The circle with have a center of $(0,0)$ and a radius of 6. Graphing the circle:



52. The two points are $(-0.25, 0.9682)$ and $(-0.25, -0.9682)$. The –0.25 value is exact.

54. The two points are $(0.7454, 0.6667)$ and $(0.7454, -0.6667)$.

56. The two points are $(0.8, 0.6)$ and $(0.8, -0.6)$. Note that these values are exact.

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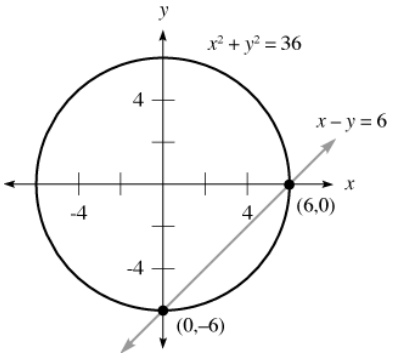
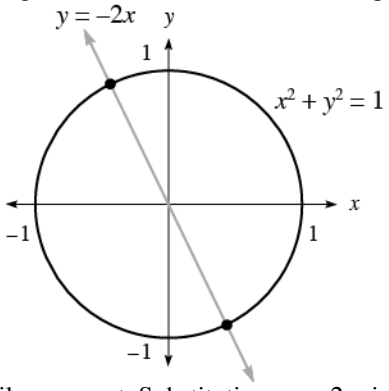
58. Note that graphing the line and circle together results in two intersection points:



The intersection points are $(0,-6)$ and $(6,0)$.

60. Note that graphing the line and circle together results in two intersection points:



The points, however, may not be readily apparent. Substituting $y = -2x$ into the equation for the circle:

$$x^2 + (-2x)^2 = 1$$
$$x^2 + 4x^2 = 1$$
$$5x^2 = 1$$
$$x^2 = \frac{1}{5}$$
$$x = \pm\frac{1}{\sqrt{5}} = \pm\frac{\sqrt{5}}{5}$$

Since $y = -2x$, the two intersection points are $\left(\frac{\sqrt{5}}{5}, -\frac{2\sqrt{5}}{5}\right)$ and $\left(-\frac{\sqrt{5}}{5}, \frac{2\sqrt{5}}{5}\right)$.

62. The complement is $90° - 90° = 0°$.

64. The complement is $90° - 30° = 60°$.

66. The supplement is $180° - 30° = 150°$.

68. The supplement is $180° - 135° = 45°$.

70. The angle between 0° and 360° which is coterminal with –45° is 315°.

72. The angle between 0° and 360° which is coterminal with –300° is 60°.

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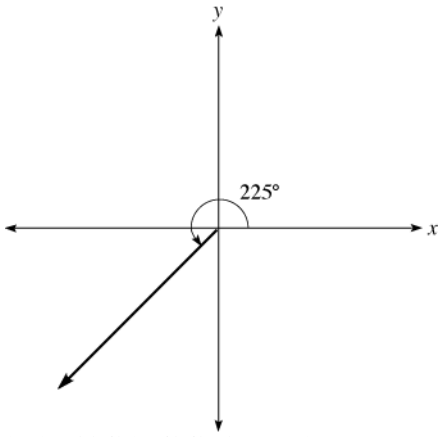
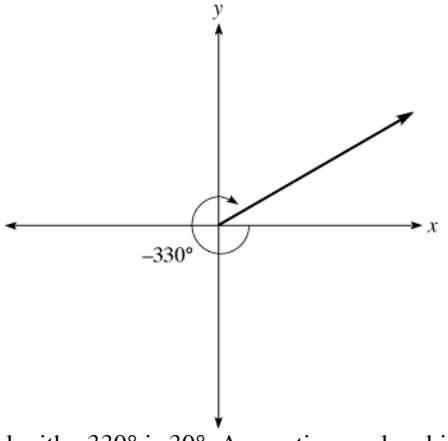
74. Drawing 225° in standard position:



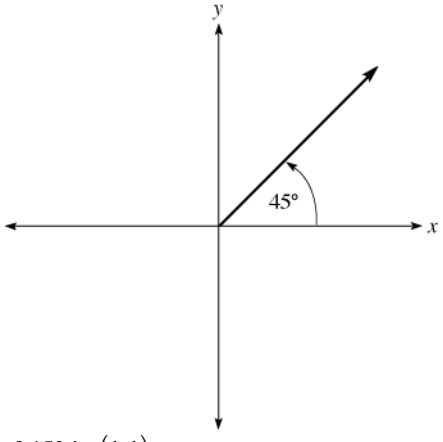
A positive angle which is coterminal with 225° is 585°. A negative angle which is coterminal with 225° is –135°.

76. Drawing –330° in standard position:



A positive angle which is coterminal with –330° is 30°. A negative angle which is coterminal with –330° is –690°.

78. Drawing 45° in standard position:



a. A point on the terminal side of 45° is $(1,1)$.

b. The distance to that point is $r = \sqrt{(1-0)^2 + (1-0)^2} = \sqrt{1+1} = \sqrt{2}$.

c. Another angle which is coterminal with 45° is –315°.

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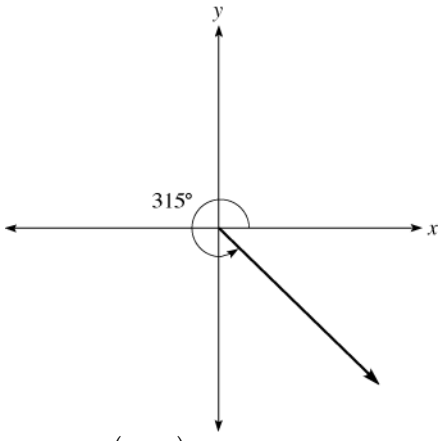
80. Drawing 315° in standard position:

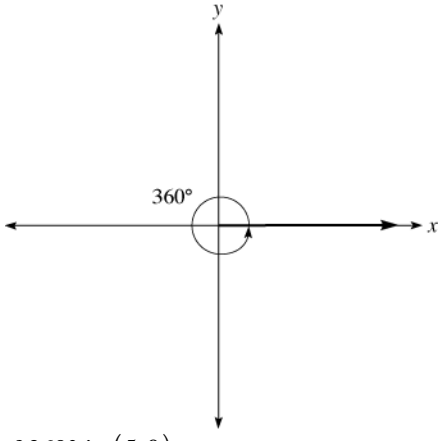


a. A point on the terminal side of 315° is $(2,-2)$.

b. The distance to that point is $r = \sqrt{(2-0)^2 + (-2-0)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$.

c. Another angle which is coterminal with 315° is –45°.

82. Drawing 360° in standard position:

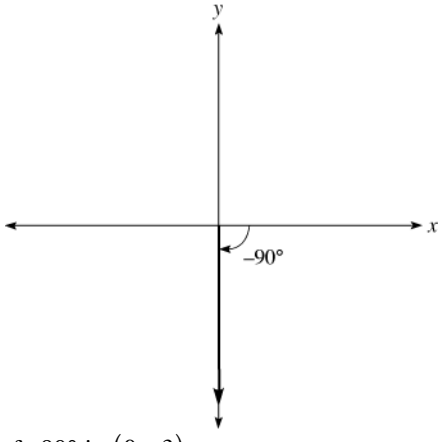


a. A point on the terminal side of 360° is $(5,0)$.

b. The distance to that point is 5.

c. Another angle which is coterminal with 360° is 0°.

84. Drawing –90° in standard position:



a. A point on the terminal side of –90° is $(0,-3)$.

b. The distance to that point is 3.

c. Another angle which is coterminal with –90° is 270°.

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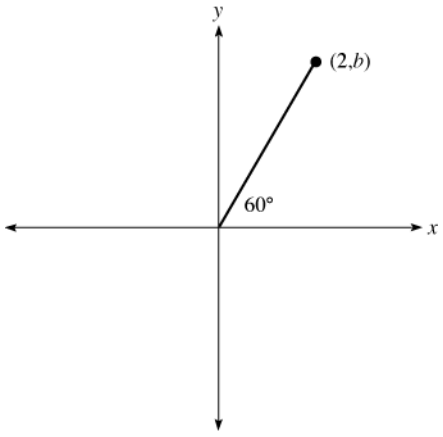
86. For any integer k, $-60° + 360°k$ will be coterminal with $-60°$. Other answers are possible.

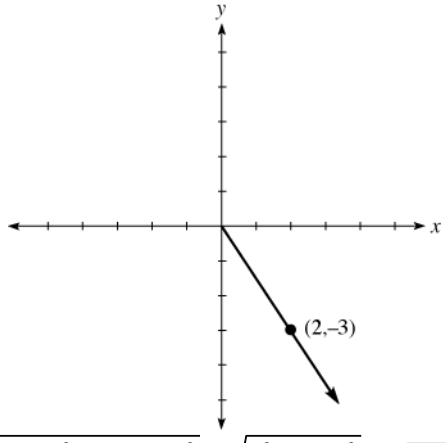
88. For any integer k, $180° + 360°k$ will be coterminal with $180°$. Other answers are possible.

90. Drawing 60° in standard position and labeling the point $(2,b)$:



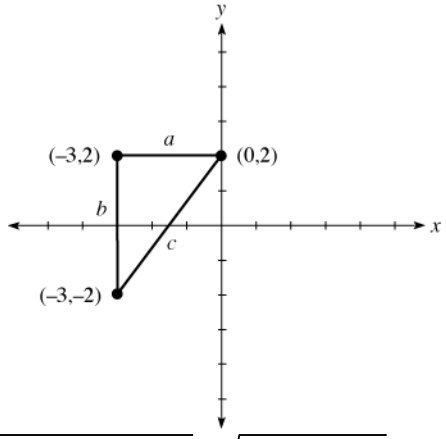
Since 2 is the shorter side of the triangle (opposite the 30° angle), the value of b must be $b = 2\sqrt{3}$.

92. Drawing an angle whose terminal side contains the point $(2,-3)$:



Using the distance formula: $r = \sqrt{(2-0)^2 + (-3-0)^2} = \sqrt{2^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13}$

94. Plotting the points:



Note that $a = 3$, $b = 4$, and $c = \sqrt{(-3-0)^2 + (-2-2)^2} = \sqrt{(-3)^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$. Since $a^2 + b^2 = c^2$, by the Pythagorean Theorem these points form a right triangle.

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96. Pascal's triangle appears as:

$$\begin{array}{ccccccccccc} & & & & & 1 & & & & & \\ & & & & 1 & & 1 & & & & \\ & & & 1 & & 2 & & 1 & & & \\ & & 1 & & 3 & & 3 & & 1 & & \\ & 1 & & 4 & & 6 & & 4 & & 1 & \\ 1 & & 5 & & 10 & & 10 & & 5 & & 1 \\ 1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1 \end{array}$$

These numbers are the coefficients of the expansion of $(a+b)^n$, where n represents the line number of the triangle. The first few expansions are:

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$
$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$
$$(a+b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$$

This pattern continues for all natural numbers n.

98. Using the distance formula: $d = \sqrt{(4+2)^2 + (5-8)^2} = \sqrt{6^2 + (-3)^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$

The correct answer is b.

100. A coterminal angle is 160° – 360° = –200°. The correct answer is d.

ODD SOLUTIONS

1. quadrants, I, IV, counterclockwise

3. $(0,0)$, 1

5. quadrantal

7. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

9. Since x is positive and y is negative, the point $(2,-4)$ must lie in quadrant IV.

11. Since x is negative and y is positive, the point $(-\sqrt{3},1)$ must lie in quadrant II.

13. If we let $x = 0$, the equation $y = x$ becomes: $y = 0$. This gives us the point $(0,0)$.

If we let $x = 2$, the equation $y = x$ becomes: $y = 2$. This gives us the point $(2,2)$.

Graphing the points (0, 0) and (2, 2) and then drawing a line through them, we have the graph of $y = x$.

15. If we let $x = 0$, the equation $y = \frac{1}{2}x$ becomes: $y = \frac{1}{2}(0) = 0$

This gives us (0, 0) as one solution to $y = \frac{1}{2}x$.

If we let $x = 4$, the equation $y = \frac{1}{2}x$ becomes: $y = \frac{1}{2}(4) = 2$

This gives us (4, 2) as a second solution to $y = \frac{1}{2}x$.

Graphing the points (0, 0) and (4, 2) and then drawing a line through them, we have the graph of $y = \frac{1}{2}x$.

17. Quadrants II and III lie to the left of the y-axis. Therefore, all points in these two quadrants have negative x-coordinates.

19. In quadrant III, x and y are always negative. Therefore the ratio $\frac{x}{y}$ will always be positive.

21. The vertex of this parabola is at $(0,-4)$.

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If we let $x = -2$, the equation $y = x^2 - 4$ becomes $y = (-2)^2 - 4$

$= 4 - 4 = 0$

This gives us $(-2,0)$ as a point on the curve.

If we let $x = -1$, the equation $y = x^2 - 4$ becomes $y = (-1)^2 - 4$

$= 1 - 4 = -3$

This gives us $(-1,-3)$ as a point on the curve.

Using the symmetry of a parabola, the points $(2,0)$ and $(1,-3)$ will also be points on the curve.

Graphing the points $(-2,0), (-1,-3), (0,-4), (1,-3)$, and $(2,0)$, and then drawing a smooth curve through them, we have the graph of the parabola $y = x^2 - 4$.

23. The vertex of this parabola is $(-2,4)$.

If we let $x = -1$, the equation $y = (x+2)^2 + 4$ becomes

$$y = (-1+2)^2 + 4$$
$$= 1^2 + 4 = 5$$

This gives us $(-1,5)$ as a point on the curve.

If we let $x = 0$, the equation $y = (x+2)^2 + 4$ becomes

$$y = (0+2)^2 + 4$$
$$= 2^2 + 4 = 8$$

This gives us (0, 8) as a point on the curve.

Using the symmetry of a parabola, the points $(-3,5)$ and $(-4,8)$ will also be points on the curve.

Graphing the points $(0,8), (-1,5), (-2,4), (-3,5)$ and $(-4,8)$ and then drawing a smooth curve through them, we have the graph of the parabola $y = (x+2)^2 + 4$.

29. The cannonball is on the ground ($y = 0$) when $x = 0$ and when $x = 160$. The x-coordinate of the vertex (the maximum) will be at $\frac{1}{2}(160)$ or 80, and the y-coordinate will be 60.

We can now sketch the parabola through the points (0, 0), (80, 60), and (160, 0).

The equation will be in the form $y = a(x-80)^2 + 60$. We will use the point (160, 0) to find a.

Let $x = 160$ and $y = 0$:

$$y = a(x-80)^2 + 60$$
$$0 = a(160-80)^2 + 60$$
$$-60 = a(80)^2$$
$$-60 = 6400a$$
$$a = -\frac{3}{320}$$

Therefore, the equation is $y = -\frac{3}{320}(x-80)^2 + 60$.

31.

$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$= \sqrt{(3-6)^2 + (7-3)^2}$ Substitute given values

$= \sqrt{(-3)^2 + 4^2}$ Simplify

$= \sqrt{9+16}$

$= \sqrt{25} = 5$

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33. $r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$= \sqrt{(0-5)^2 + (12-0)^2}$ Substitute given values

$= \sqrt{(-5)^2 + 12^2}$ Simplify

$= \sqrt{25 + 144}$

$= \sqrt{169} = 13$

35. $r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$= \sqrt{[-1-(-10)]^2 + (-2-5)^2}$ Substitute given values

$= \sqrt{9^2 + (-7)^2} = \sqrt{81+49} = \sqrt{130}$ Simplify

37. $r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$= \sqrt{(3-0)^2 + (-4-0)^2}$ Substitute given values

$= \sqrt{(3)^2 + (-4)^2}$ Simplify

$= \sqrt{9+16} = \sqrt{25} = 5$

39. $r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$\sqrt{13} = \sqrt{(x-1)^2 + (2-5)^2}$ Substitute given values

$\sqrt{13} = \sqrt{(x-1)^2 + (-3)^2}$ Simplify

$\sqrt{13} = \sqrt{(x-1)^2 + 9}$

$13 = (x-1)^2 + 9$ Square both sides

$4 = (x-1)^2$ Subtract 9 from both sides

$\pm 2 = x - 1$ Use the square root method to solve

$x - 1 = 2$ or $x - 1 = -2$

$x = 3$ or $x = -1$

41. First, we convert 1.2 miles to feet: 1.2 mi = 1.2(5,280) ft = 6,336 ft

$c^2 = a^2 + b^2$ Pythagorean Theorem

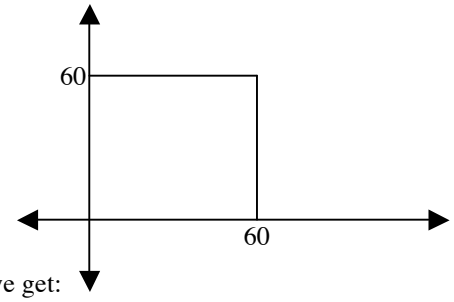
$= (2{,}640)^2 + (6{,}336)^2$ Substitute given values

$= 6{,}969{,}600 + 40{,}144{,}896$ Simplify

$= 47{,}114{,}496$

$c = 6{,}864$ ft (or 1.3 mi) Take square root of both sides

43. The x-axis goes from home plate to first base, a distance of 60 feet. Therefore, home plate is $(0, 0)$ and first base is $(60, 0)$.
The y-axis goes from home plate to third base.
Therefore, third base is $(0, 60)$. Second base will be at $(60, 60)$.



45. We substitute $x = 0$ and $y = -1$ into the equation of the unit circle and we get:

$x^2 + y^2 = 0^2 + (-1)^2$

$= 0 + 1 = 1$ It checks.

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47. We substitute $x = \frac{1}{2}$ and $y = \frac{\sqrt{3}}{2}$ into the equation of the unit circle and we get:

$$x^2 + y^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \left(\frac{1}{4}\right) + \left(\frac{3}{4}\right) = 1 \qquad \text{It checks.}$$

49. The center of this circle is $(0, 0)$ and the radius is 5.

57. From the graph of $x^2 + y^2 = 25$, we can see that $(0, 5)$ and $(5, 0)$ are the points at which the line $x + y = 5$ will intersect the circle.

59. Solving this system of equations by substitution,, we get:

$x^2 + y^2 = 1$ $\qquad$ $x^2 + x^2 = 1$

$y = x$ $\qquad$ $2x^2 = 1$

$x^2 = \frac{1}{2}$

$x = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$

If $x = \frac{\sqrt{2}}{2}, y = \frac{\sqrt{2}}{2}$. $\qquad$ If $x = -\frac{\sqrt{2}}{2}, y = -\frac{\sqrt{2}}{2}$

The solution is $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ and $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$.

61. The complement of a 45° angle is $90^\circ - 45^\circ = 45^\circ$.

63. The complement of a 60° angle is $90^\circ - 60^\circ = 30^\circ$.

65. The supplement of a 120° angle is $180^\circ - 120^\circ = 60^\circ$.

67. The supplement of a 90° angle is $180^\circ - 90^\circ = 90^\circ$.

69. An angle coterminal with an angle of -135° is $-135^\circ + 360^\circ = 225^\circ$.

71. An angle coterminal with an angle of -210° is $-210^\circ + 360^\circ = 150^\circ$.

73. This angle in standard position lies in quadrant IV.
One revolution in a positive direction gives us: $300^\circ + 360^\circ = 660^\circ$
One revolution in a negative direction gives us: $300^\circ - 360^\circ = -60^\circ$

75. This angle in standard position lies in quadrant III.
One revolution in a positive direction gives us: $-150^\circ + 360^\circ = 210^\circ$
One revolution in a negative direction gives us: $-150^\circ - 360^\circ = -510^\circ$

77. **a.** If we draw 135° in standard position, we see that the terminal side is along the line $y = -x$. Since the terminal side lies in the second quadrant, x is negative and y is positive. Some of points on the terminal side are $(-1,1), (-3,3)$, and $\left(-\sqrt{2}, \sqrt{2}\right)$.

b. To find the distance from $(0, 0)$ to $(-3,3)$, we use the distance formula:

$$r = \sqrt{(-3-0)^2 + (3-0)^2}$$

$$= \sqrt{(-3)^2 + (3)^2}$$

$$= \sqrt{9+9}$$

$$= \sqrt{18} = 3\sqrt{2}$$

c. One revolution in a negative direction gives us: $135^\circ - 360^\circ = -225^\circ$

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79. **a.** If we draw 225° in standard position, we see that the terminal side is along the line $y = x$. Since the terminal side lies in the third quadrant, x and y are both negative. Some of points on the terminal side are $(-1,-1), (-3,-3)$, and $\left(-\sqrt{2},-\sqrt{2}\right)$.

b. To find the distance from $(0, 0)$ to $(-3,-3)$, we use the distance formula:

$$r = \sqrt{(-3-0)^2 + (-3-0)^2}$$
$$= \sqrt{(-3)^2 + (-3)^2}$$
$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

c. One revolution in a negative direction gives us: $225^\circ - 360^\circ = -135^\circ$

81. **a.** If we draw 90° in standard position, we see that the terminal side is the positive y-axis. Some of the points on the terminal side are $(0,1), (0,2)$, and $(0,3)$.

b. The distance between (0, 0) and (0, 3) is 3 units.

c. One revolution in a negative direction gives us: $90^\circ - 360^\circ = -270^\circ$

83. **a.** If we draw -45° in standard position, we see that the terminal side is along the line $y = -x$. Since the terminal side lies in the fourth quadrant, x is positive and y is negative. Some of points on the terminal side are $(1,-1), (3,-3)$, and $\left(\sqrt{2},-\sqrt{2}\right)$.

b. To find the distance from $(0, 0)$ to $(3,-3)$, we use the distance formula:

$$r = \sqrt{(3-0)^2 + (-3-0)^2}$$
$$= \sqrt{(3)^2 + (-3)^2}$$
$$= \sqrt{9+9}$$
$$= \sqrt{18} = 3\sqrt{2}$$

c. One revolution in a positive direction gives us: $-45^\circ + 360^\circ = 315^\circ$

85. For any integer k, $30^\circ + 360^\circ k$ will be coterminal with 30°.

87. For any integer k, $-135^\circ + 360^\circ k$ will be coterminal with -135°.

89. The side opposite 30° is 1.
The side opposite 60° is $1\sqrt{3}$ or $\sqrt{3}$.
Therefore, the point is $\left(\sqrt{3},1\right)$.

91.
$$r = \sqrt{(3-0)^2 + (-2-0)^2}$$
$$= \sqrt{(3)^2 + (-2)^2}$$
$$= \sqrt{9+4} = \sqrt{13}$$

93. We will find the lengths of the three sides:
From (0, 0) to (5, 0) is 5 units.
From (5, 0) to (5, 12) is 12 units.
From (0. 0) to (5, 12), we use the distance formula:

$$r = \sqrt{(5-0)^2 + (12-0)^2}$$
$$= \sqrt{5^2 + 12^2}$$
$$= \sqrt{25+144}$$
$$= \sqrt{169} = 13$$

If this is a right triangle, then $c^2 = a^2 + b^2$.
We check: $13^2 = 5^2 + 12^2$
$169 = 25 + 144$
$169 = 169$ It checks.

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97. If a point (x,y) lies on the unit circle, then $x^2 + y^2 = 1$. Testing the given points, we find that (a) is true: $\left(\frac{3}{4}\right)^2 + \left(\frac{\sqrt{7}}{4}\right)^2 = \frac{9}{16} + \frac{7}{16} = \frac{16}{16} = 1$

99. To draw 140° in standard position, place the vertex at the origin and draw the terminal side 140° counterclockwise from the positive x-axis. The answer is a.

1.3 Definition 1: Trigonometric Functions

EVEN SOLUTIONS

2. In quadrant I, all of the six trigonometric functions are positive. In each of the other three quadrants, only two of the six functions are positive.

4. The tangent and cotangent functions do not depend on the value of r.

6. Begin by finding the distance r from the origin to $(-4,-3)$:

$$r = \sqrt{(-4-0)^2 + (-3-0)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

Now applying the definitions for the six trigonometric functions using the values $x = -4$, $y = -3$, and $r = 5$:

$$\sin\theta = \frac{y}{r} = -\frac{3}{5} \qquad \csc\theta = \frac{r}{y} = -\frac{5}{3}$$

$$\cos\theta = \frac{x}{r} = -\frac{4}{5} \qquad \sec\theta = \frac{r}{x} = -\frac{5}{4}$$

$$\tan\theta = \frac{y}{x} = \frac{-3}{-4} = \frac{3}{4} \qquad \cot\theta = \frac{x}{y} = \frac{-4}{-3} = \frac{4}{3}$$

8. Begin by finding $r = \sqrt{(12)^2 + (-5)^2} = \sqrt{144+25} = \sqrt{169} = 13$. Now applying the definitions for the six trigonometric functions using the values $x = 12$, $y = -5$, and $r = 13$:

$$\sin\theta = \frac{y}{r} = -\frac{5}{13} \qquad \csc\theta = \frac{r}{y} = -\frac{13}{5}$$

$$\cos\theta = \frac{x}{r} = \frac{12}{13} \qquad \sec\theta = \frac{r}{x} = \frac{13}{12}$$

$$\tan\theta = \frac{y}{x} = -\frac{5}{12} \qquad \cot\theta = \frac{x}{y} = -\frac{12}{5}$$

10. Begin by finding $r = \sqrt{6^2 + 6^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$. Now applying the definitions for the six trigonometric functions using the values $x = 6$, $y = 6$, and $r = 6\sqrt{2}$:

$$\sin\theta = \frac{6}{6\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \csc\theta = \frac{r}{y} = \frac{6\sqrt{2}}{6} = \sqrt{2}$$

$$\cos\theta = \frac{6}{6\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \sec\theta = \frac{r}{x} = \frac{6\sqrt{2}}{6} = \sqrt{2}$$

$$\tan\theta = \frac{y}{x} = \frac{6}{6} = 1 \qquad \cot\theta = \frac{x}{y} = \frac{6}{6} = 1$$

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12. Begin by finding $r = \sqrt{(-2)^2 + \left(\sqrt{5}\right)^2} = \sqrt{4+5} = \sqrt{9} = 3$. Now applying the definitions for the six trigonometric functions using the values $x = -2$, $y = \sqrt{5}$, and $r = 3$:

$$\sin\theta = \frac{y}{r} = \frac{\sqrt{5}}{3} \qquad \csc\theta = \frac{r}{y} = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

$$\cos\theta = \frac{x}{r} = -\frac{2}{3} \qquad \sec\theta = \frac{r}{x} = -\frac{3}{2}$$

$$\tan\theta = \frac{y}{x} = -\frac{\sqrt{5}}{2} \qquad \cot\theta = \frac{x}{y} = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

14. Begin by finding $r = \sqrt{4^2 + 0^2} = \sqrt{16} = 4$. Now applying the definitions for the six trigonometric functions using the values $x = 4$, $y = 0$, and $r = 4$:

$$\sin\theta = \frac{y}{r} = \frac{0}{4} = 0 \qquad \csc\theta = \frac{r}{y} = \frac{4}{0}, \text{ which is undefined}$$

$$\cos\theta = \frac{x}{r} = \frac{4}{4} = 1 \qquad \sec\theta = \frac{r}{x} = \frac{4}{4} = 1$$

$$\tan\theta = \frac{y}{x} = \frac{0}{4} = 0 \qquad \cot\theta = \frac{x}{y} = \frac{4}{0}, \text{ which is undefined}$$

16. Begin by finding $r = \sqrt{m^2 + n^2}$. Now applying the definitions for the six trigonometric functions using the values $x = m$, $y = n$, and $r = \sqrt{m^2 + n^2}$:

$$\sin\theta = \frac{y}{r} = \frac{n}{\sqrt{m^2 + n^2}} \qquad \csc\theta = \frac{r}{y} = \frac{\sqrt{m^2 + n^2}}{n}$$

$$\cos\theta = \frac{x}{r} = \frac{m}{\sqrt{m^2 + n^2}} \qquad \sec\theta = \frac{r}{x} = \frac{\sqrt{m^2 + n^2}}{m}$$

$$\tan\theta = \frac{y}{x} = \frac{n}{m} \qquad \cot\theta = \frac{x}{y} = \frac{m}{n}$$

18. Begin by finding $r = \sqrt{(-6)^2 + (-2)^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$. Now applying the definitions for $\sin\theta$, $\cos\theta$, and $\tan\theta$ using $x = -6$, $y = -2$, and $r = 2\sqrt{10}$:

$$\sin\theta = \frac{y}{r} = \frac{-2}{2\sqrt{10}} = -\frac{\sqrt{10}}{10} \qquad \cos\theta = \frac{-6}{2\sqrt{10}} = -\frac{3\sqrt{10}}{10} \qquad \tan\theta = \frac{y}{x} = \frac{-2}{-6} = \frac{1}{3}$$

20. Begin by finding $r = \sqrt{(3)^2 + (-8)^2} = \sqrt{9+64} = \sqrt{73}$. Now applying the definitions for $\sin\theta$, $\cos\theta$, and $\tan\theta$ using $x = 3$, $y = -8$, and $r = \sqrt{73}$:

$$\sin\theta = \frac{y}{r} = -\frac{8}{\sqrt{73}} = -\frac{8\sqrt{73}}{73} \qquad \cos\theta = \frac{x}{r} = \frac{3}{\sqrt{73}} = \frac{3\sqrt{73}}{73} \qquad \tan\theta = \frac{y}{x} = -\frac{8}{3}$$

22. Begin by finding r: $r = \sqrt{(3.63)^2 + (6.25)^2} = \sqrt{13.1769 + 39.0625} = \sqrt{52.2394} \approx 7.2277$
Now applying the definitions for $\sin\theta$ and $\cos\theta$ using $x = 3.63$, $y = 6.25$, and $r = 7.2277$:

$$\sin\theta = \frac{y}{r} = \frac{6.25}{7.2277} \approx 0.865 \qquad \cos\theta = \frac{x}{r} = \frac{3.63}{7.2277} \approx 0.502$$

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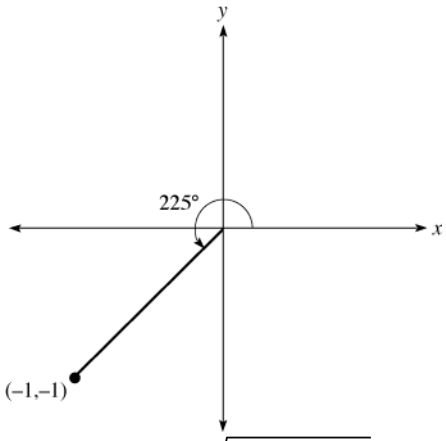
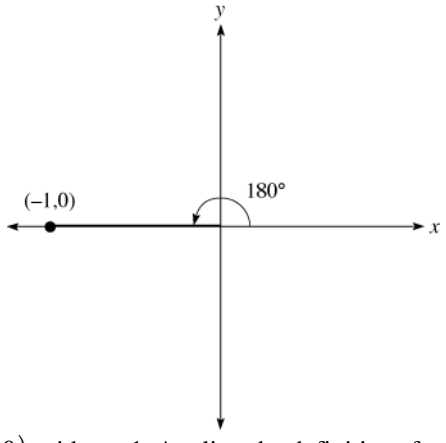
24. Drawing 225° in standard position:



A point on the terminal side is $(-1,-1)$. Now find $r = \sqrt{(-1)^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$. Appling the definitions for sin 225°, cos 225°, and tan 225°:

$$\sin 225° = \frac{y}{r} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} \qquad \cos 225° = \frac{x}{r} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} \qquad \tan 225° = \frac{y}{x} = \frac{-1}{-1} = 1$$

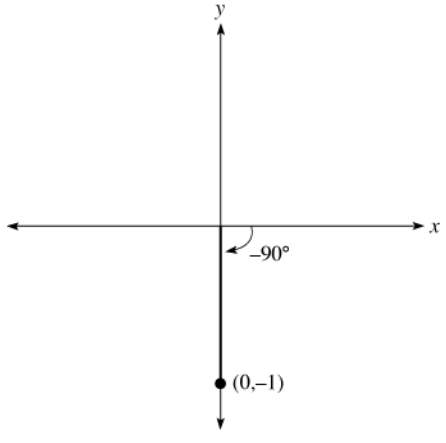
26. Drawing 180° in standard position:



A point on the terminal side is $(-1,0)$ with $r = 1$. Appling the definitions for sin 180°, cos 180°, and tan 180°:

$$\sin 180° = \frac{y}{r} = \frac{0}{1} = 0 \qquad \cos 180° = \frac{x}{r} = \frac{-1}{1} = -1 \qquad \tan 180° = \frac{y}{x} = \frac{0}{-1} = 0$$

28. Drawing –90° in standard position:



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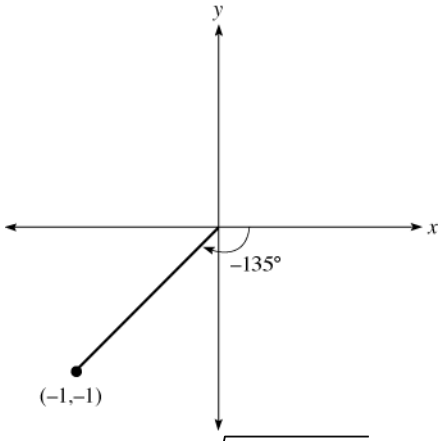
A point on the terminal side is $(0,-1)$ with $r = 1$. Appling the definitions for sin (–90°), cos (–90°), and tan (–90°):

$$\sin(-90°) = \frac{y}{r} = \frac{-1}{1} = -1 \quad \cos(-90°) = \frac{x}{r} = \frac{0}{1} = 0 \quad \tan(-90°) = \frac{y}{x} = \frac{-1}{0}, \text{ which is undefined}$$

30. Drawing –135° in standard position:



A point on the terminal side is $(-1,-1)$. Now find $r = \sqrt{(-1)^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$. Appling the definitions for sin (–135°), cos (–135°), and tan (–135°):

$$\sin(-135°) = \frac{y}{r} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} \quad \cos(-135°) = \frac{x}{r} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} \quad \tan(-135°) = \frac{y}{x} = \frac{-1}{-1} = 1$$

32. This statement is true.

34. This statement is false.

36. Since $r \geq x$, $\frac{r}{x} \geq 1$. Thus $\sec\theta \geq 1$, so there is no angle θ such that $\sec\theta = \frac{1}{2}$.

38. Since $y \leq r$, $\frac{y}{r} \leq 1$. Thus $\sin\theta \leq 1$ for any angle θ.

40. The value of $\cos\theta$ tends toward 0 as θ increases from 0° to 90°.

42. As θ increases from 0° to 90°, the value of $\sec\theta$ tends towards 1.

44. θ could terminate in quadrant II or quadrant IV.

46. θ could terminate in quadrant III or quadrant IV.

48. θ could terminate in quadrant II or quadrant IV.

50. θ could terminate in quadrant I or quadrant II.

52. If $\sin\theta$ is positive, $y > 0$, and if $\cos\theta$ is negative, $x < 0$, so the terminal side of θ must lie in quadrant II.

54. $\csc\theta$ and $\cot\theta$ are both positive in quadrant I, and $\csc\theta$ and $\cot\theta$ are both negative in quadrant IV. Thus $\csc\theta$ and $\cot\theta$ have the same sign in quadrants I and IV.

56. If $\cos\theta = \frac{24}{25}$, we can choose $x = 24$ and $r = 25$. To find y, use $x^2 + y^2 = r^2$:

$$24^2 + y^2 = 25^2$$
$$576 + y^2 = 625$$
$$y^2 = 49$$
$$y = \pm 7$$

Since θ terminates in quadrant IV, $y < 0$ and thus $y = -7$. Using $x = 24$, $y = -7$, and $r = 25$ in the definitions:

$$\sin\theta = \frac{y}{r} = -\frac{7}{25} \quad \sec\theta = \frac{r}{x} = \frac{25}{24} \quad \csc\theta = \frac{r}{y} = -\frac{25}{7}$$
$$\tan\theta = \frac{y}{x} = -\frac{7}{24} \quad \cot\theta = \frac{x}{y} = -\frac{24}{7}$$

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58. If $\sin\theta = -\frac{20}{29}$, we can choose $y = -20$ and $r = 29$. To find y, use $x^2 + y^2 = r^2$:

$$x^2 + (-20)^2 = 29^2$$
$$x^2 + 400 = 841$$
$$x^2 = 441$$
$$x = \pm 21$$

Since θ terminates in quadrant III, $x < 0$ and thus $x = -21$. Using $x = -21$, $y = -20$, and $r = 29$ in the definitions:

$$\cos\theta = \frac{x}{r} = -\frac{21}{29} \qquad \csc\theta = \frac{r}{y} = -\frac{29}{20} \qquad \sec\theta = \frac{r}{x} = -\frac{29}{21}$$

$$\tan\theta = \frac{y}{x} = \frac{20}{21} \qquad \cot\theta = \frac{x}{y} = \frac{21}{20}$$

60. If $\sin\theta = \frac{\sqrt{2}}{2}$, we can choose $y = \sqrt{2}$ and $r = 2$. To find x, use $x^2 + y^2 = r^2$:

$$x^2 + \left(\sqrt{2}\right)^2 = 2^2$$
$$x^2 + 2 = 4$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}$$

Since θ terminates in quadrant II, $x < 0$ and thus $x = -\sqrt{2}$. Using $x = -\sqrt{2}$, $y = \sqrt{2}$, and $r = 2$ in the definitions:

$$\sin\theta = \frac{y}{r} = \frac{\sqrt{2}}{2} \qquad \csc\theta = \frac{r}{y} = \frac{2}{\sqrt{2}} = \sqrt{2} \qquad \sec\theta = \frac{r}{x} = \frac{2}{-\sqrt{2}} = -\sqrt{2}$$

$$\tan\theta = \frac{y}{x} = \frac{\sqrt{2}}{-\sqrt{2}} = -1 \qquad \cot\theta = \frac{x}{y} = \frac{-\sqrt{2}}{\sqrt{2}} = -1$$

62. If $\tan\theta = -3$ and θ terminates in quadrant IV, $x > 0$ and $y < 0$, so choose $y = -3$ and $x = 1$. Finding r using $x^2 + y^2 = r^2$:

$$1^2 + (-3)^2 = r^2$$
$$1 + 9 = r^2$$
$$r^2 = 10$$
$$r = \sqrt{10}$$

Using $x = 1$, $y = -3$, and $r = \sqrt{10}$ in the definitions:

$$\sin\theta = \frac{y}{r} = \frac{-3}{\sqrt{10}} = -\frac{3\sqrt{10}}{10} \qquad \csc\theta = \frac{r}{y} = -\frac{\sqrt{10}}{3} \qquad \cot\theta = \frac{x}{y} = -\frac{1}{3}$$

$$\cos\theta = \frac{x}{r} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10} \qquad \sec\theta = \frac{r}{x} = \sqrt{10}$$

64. If $\sec\theta = \frac{13}{5}$, we can choose $x = 5$ and $r = 13$. To find y, use $x^2 + y^2 = r^2$:

$$5^2 + y^2 = 13^2$$
$$5^2 + y^2 = 169$$
$$y^2 = 144$$
$$y = \pm 12$$

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Since $\sin\theta < 0$, $y < 0$ and thus $y = -12$. Using $x = 5$, $y = -12$, and $r = 13$ in the definitions:

$$\sin\theta = \frac{y}{r} = -\frac{12}{13} \qquad \tan\theta = \frac{y}{x} = -\frac{12}{5} \qquad \csc\theta = \frac{r}{y} = -\frac{13}{12}$$

$$\cos\theta = \frac{x}{r} = \frac{5}{13} \qquad \cot\theta = \frac{x}{y} = -\frac{5}{12}$$

66. If $\cot\theta = -\frac{1}{4}$ and $\sin\theta > 0$, $y > 0$, so choose $y = 4$ and $x = -1$. Finding r using $x^2 + y^2 = r^2$:

$$(-1)^2 + 4^2 = r^2$$
$$1 + 16 = r^2$$
$$r^2 = 17$$
$$r = \sqrt{17}$$

Using $x = -1$, $y = 4$, and $r = \sqrt{17}$ in the definitions:

$$\sin\theta = \frac{y}{r} = \frac{4}{\sqrt{17}} = \frac{4\sqrt{17}}{17} \qquad \csc\theta = \frac{r}{y} = \frac{\sqrt{17}}{4} \qquad \tan\theta = \frac{x}{y} = -4$$

$$\cos\theta = \frac{x}{r} = -\frac{1}{\sqrt{17}} = -\frac{\sqrt{17}}{17} \qquad \sec\theta = \frac{r}{x} = -\sqrt{17}$$

68. If $\cot\theta = \frac{m}{n}$, we can choose $x = m$ and $y = n$, so $r = \sqrt{m^2 + n^2}$. Using the definitions:

$$\sin\theta = \frac{y}{r} = \frac{n}{\sqrt{m^2 + n^2}} \qquad \csc\theta = \frac{r}{y} = \frac{\sqrt{m^2 + n^2}}{n} \qquad \tan\theta = \frac{y}{x} = \frac{n}{m}$$

$$\cos\theta = \frac{x}{r} = \frac{m}{\sqrt{m^2 + n^2}} \qquad \sec\theta = \frac{r}{x} = \frac{\sqrt{m^2 + n^2}}{m}$$

70. **a**. If $\tan\theta = 1$, the angel is $\theta = 45°$.

b. If $\tan\theta = -1$, the angel is $\theta = 135°$.

72. A point on $y = \frac{1}{2}x$ in quadrant III is $(-2,-1)$, so choose $x = -2$ and $y = -1$. Thus $r = \sqrt{(-2)^2 + (-1)^2} = \sqrt{4+1} = \sqrt{5}$.

Using the definitions:

$$\sin\theta = \frac{y}{r} = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5} \qquad \cos\theta = \frac{x}{r} = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

74. A point on $y = -3x$ in quadrant IV is $(1,-3)$, so choose $x = 1$ and $y = -3$. Thus $r = \sqrt{1^2 + (-3)^2} = \sqrt{1+9} = \sqrt{10}$.

Using the definitions:

$$\sin\theta = \frac{y}{r} = -\frac{3}{\sqrt{10}} = -\frac{3\sqrt{10}}{10} \qquad \tan\theta = \frac{y}{x} = \frac{-3}{1} = -3$$

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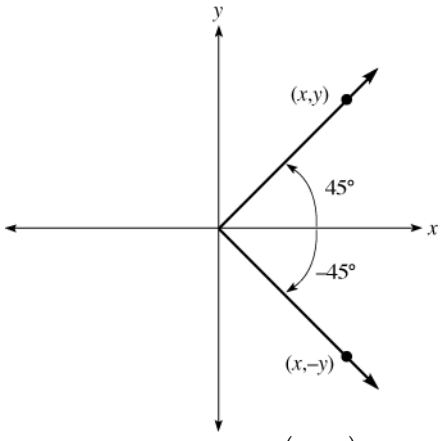
76. Drawing 45° and –45° in standard position:



Let (x,y) represent a point on the terminal side of 45°. Then $(x,-y)$ will represent a point on the terminal side of –45°. Therefore: $\sin(-45°) = \frac{-y}{r} = -\frac{y}{r} = -\sin 45°$

78. If $\sin\theta = -\frac{3}{5}$, we can choose $y = -6$ and $r = 10$. To find x, use $x^2 + y^2 = r^2$:

$$x^2 + (-6)^2 = 10^2$$
$$x^2 + 36 = 100$$
$$x^2 = 64$$
$$x = \pm 8$$

Note that both values of x are possible, since the points $(8,-6)$ and $(-8,-6)$ both satisfy the condition $\sin\theta = -\frac{3}{5}$.

80. The value of $\cos\theta$ tends toward 0 as θ increases from 0° to 90°. The correct answer is a.

82. If $\sin\theta = \frac{4}{5}$, we can choose $y = 4$ and $r = 5$. To find x, use $x^2 + y^2 = r^2$:

$$x^2 + 4^2 = 5^2$$
$$x^2 + 16 = 25$$
$$x^2 = 9$$
$$x = \pm 3$$

Since θ terminates in quadrant II, $x < 0$ and thus $x = -3$. Thus $\tan\theta = \frac{y}{x} = -\frac{4}{3}$. The correct answer is a.

ODD SOLUTIONS

1. terminal, distance, origin

3. tangent and secant, cotangent and cosecant

5. $(x,y) = (3,4)$

$x = 3$ and $y = 4$

$r = \sqrt{3^2 + 4^2}$

$= \sqrt{9+16}$

$= \sqrt{25} = 5$

$\sin\theta = \frac{y}{r} = \frac{4}{5}$ $\quad \cos\theta = \frac{x}{r} = \frac{3}{5}$ $\quad \tan\theta = \frac{y}{x} = \frac{4}{3}$

$\cot\theta = \frac{x}{y} = \frac{3}{4}$ $\quad \sec\theta = \frac{r}{x} = \frac{5}{3}$ $\quad \csc\theta = \frac{r}{y} = \frac{5}{4}$

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7. $(x,y)=(-5,12)$

$x=-5$ and $y=12$

$r=\sqrt{(-5)^2+12^2}$

$=\sqrt{25+144}$

$=\sqrt{169}=13$

$\sin\theta=\frac{y}{r}=\frac{12}{13}$ $\quad \cos\theta=\frac{x}{r}=\frac{-5}{13}=-\frac{5}{13}$ $\quad \tan\theta=\frac{y}{x}=\frac{12}{-5}=-\frac{12}{5}$

$\cot\theta=\frac{x}{y}=\frac{-5}{12}=-\frac{5}{12}$ $\quad \sec\theta=\frac{r}{x}=\frac{13}{-5}=-\frac{13}{5}$ $\quad \csc\theta=\frac{r}{y}=\frac{13}{12}$

9. $(x,y)=(-1,-2)$

$x=-1$ and $y=-2$

$r=\sqrt{(-1)^2+(-2)^2}$

$=\sqrt{1+4}\ =\sqrt{5}$

$\sin\theta=\frac{y}{r}=\frac{-2}{\sqrt{5}}=-\frac{2\sqrt{5}}{5}$ $\quad \cos\theta=\frac{x}{r}=\frac{-1}{\sqrt{5}}=-\frac{5\sqrt{5}}{5}$ $\quad \tan\theta=\frac{y}{x}=\frac{-2}{-1}=2$

$\cot\theta=\frac{x}{y}=\frac{-1}{-2}=\frac{1}{2}$ $\quad \sec\theta=\frac{r}{x}=\frac{\sqrt{5}}{-1}=-\sqrt{5}$ $\quad \csc\theta=\frac{r}{y}=\frac{\sqrt{5}}{-2}=-\frac{\sqrt{5}}{2}$

11. $(x,y)=(\sqrt{3},-1)$

$x=\sqrt{3}$ and $y=-1$

$r=\sqrt{(\sqrt{3})^2+(-1)^2}$

$=\sqrt{3+1}$

$=\sqrt{4}\ =2$

$\sin\theta=\frac{y}{r}=\frac{-1}{2}=-\frac{1}{2}$ $\quad \cos\theta=\frac{x}{r}=\frac{\sqrt{3}}{2}$ $\quad \tan\theta=\frac{y}{x}=\frac{-1}{\sqrt{3}}=-\frac{\sqrt{3}}{3}$

$\cot\theta=\frac{x}{y}=-\frac{\sqrt{3}}{1}=-\sqrt{3}$ $\quad \sec\theta=\frac{r}{x}=\frac{2}{\sqrt{3}}=\frac{2\sqrt{3}}{3}$ $\quad \csc\theta=\frac{r}{y}=\frac{2}{-1}=-2$

13. $(x,y)=(0,-5)$

$x=0$ and $y=-5$

$r=\sqrt{0^2+(-5)^2}$

$=\sqrt{25}\ =5$

$\sin\theta=\frac{y}{r}=\frac{-5}{5}=-1$ $\quad \cos\theta=\frac{x}{r}=\frac{0}{5}=0$ $\quad \tan\theta=\frac{y}{x}=\frac{-5}{0}$ is undefined

$\cot\theta=\frac{x}{y}=\frac{0}{-5}=0$ $\quad \sec\theta=\frac{r}{x}=\frac{5}{0}$ is undefined $\quad \csc\theta=\frac{r}{y}=\frac{5}{-5}=-1$

15. $(x,y)=(-9a,-12a)$

$x=-9a$ and $y=-12a$

$r=\sqrt{(-9a)^2+(-12a)^2}$

$=\sqrt{81a^2+144a^2}=\sqrt{225a^2}=15a$

$\sin\theta=\frac{y}{r}=\frac{-12a}{15a}=-\frac{4}{5}$ $\quad \cos\theta=\frac{x}{r}=\frac{-9a}{15a}=-\frac{3}{5}$ $\quad \tan\theta=\frac{y}{x}=\frac{-12a}{-9a}=\frac{4}{3}$

$\cot\theta=\frac{x}{y}=\frac{-9a}{-12a}=\frac{3}{4}$ $\quad \sec\theta=\frac{r}{x}=\frac{15a}{-9a}=-\frac{5}{3}$ $\quad \csc\theta=\frac{r}{y}=\frac{15a}{-12a}=-\frac{5}{4}$

17. $(4, 3)$ lies on the terminal side of θ. Therefore, $x=4, y=3,$ and $r=\sqrt{4^2+3^2}=\sqrt{16+9}=\sqrt{25}=5$.

$\sin\theta=\frac{y}{r}=\frac{3}{5}$ $\quad \cos\theta=\frac{x}{r}=\frac{4}{5}$ $\quad \tan\theta=\frac{y}{x}=\frac{3}{4}$

19. $(-7,6)$ lies on the terminal side of θ. Therefore, $x=-7, y=6,$ and $r=\sqrt{(-7)^2+6^2}=\sqrt{49+36}=\sqrt{85}$.

$\sin\theta=\frac{y}{r}=\frac{6}{\sqrt{85}}=\frac{6\sqrt{85}}{85}$ $\quad \cos\theta=\frac{x}{r}=\frac{-7}{\sqrt{85}}=-\frac{7\sqrt{85}}{85}$ $\quad \tan\theta=\frac{y}{x}=\frac{6}{-7}=-\frac{6}{7}$

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21. $(x,y)=(9.36,7.02)$ $\qquad \sin\theta=\dfrac{y}{r}=\dfrac{7.02}{11.7}=0.6$

$x=9.36$ and $y=7.02$ $\qquad \cos\theta=\dfrac{x}{r}=\dfrac{9.36}{11.7}=0.8$

$$r=\sqrt{(9.36)^2+(7.02)}$$
$$=\sqrt{87.6096+49.2804}$$
$$=\sqrt{136.89}=11.7$$

23. The terminal side of 135° in standard position lies on the line $y=-x$. A point on this line in quadrant II is $(-1,1)$.

$x=-1, y=1$ and $r=\sqrt{(-1)^2+1^2}=\sqrt{1+1}=\sqrt{2}$.

$\sin\theta=\dfrac{y}{r}=\dfrac{1}{\sqrt{2}}=\dfrac{\sqrt{2}}{2}$ $\qquad \cos\theta=\dfrac{x}{r}=\dfrac{-1}{\sqrt{2}}=-\dfrac{\sqrt{2}}{2}$ $\qquad \tan\theta=\dfrac{y}{x}=\dfrac{1}{-1}=-1$

25. A point on the terminal side of an angle of 90° is $(0,1)$.

$x=0, y=1$ and $r=\sqrt{0^2+1^2}=\sqrt{1}=1$.

$\sin\theta=\dfrac{y}{r}=\dfrac{1}{1}=1$ $\qquad \cos\theta=\dfrac{x}{r}=\dfrac{0}{1}=0$ $\qquad \tan\theta=\dfrac{y}{x}=\dfrac{1}{0}$ (undefined)

27. The terminal side of -45° in standard position lies on the line $y=-x$. A point on this line in quadrant IV is $(1,-1)$.

$x=1$, $y=-1$, and $r=\sqrt{1^2+(-1)^2}=\sqrt{1+1}=\sqrt{2}$.

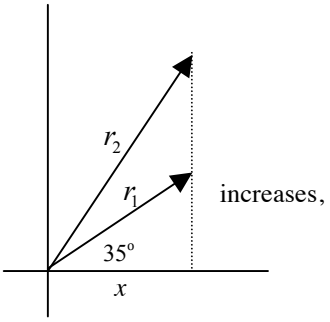
$\sin\theta=\dfrac{y}{r}=\dfrac{-1}{\sqrt{2}}=-\dfrac{\sqrt{2}}{2}$ $\qquad \cos\theta=\dfrac{x}{r}=\dfrac{1}{\sqrt{2}}=\dfrac{\sqrt{2}}{2}$ $\qquad \tan\theta=\dfrac{y}{x}=\dfrac{-1}{1}=-1$

29. A point on the terminal side of an angle of 0° is (1, 0).

$x=1$, $y=0$, and $r=\sqrt{1^2+0^2}=\sqrt{1}=1$.

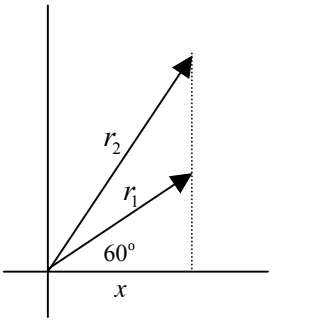
$\sin\theta=\dfrac{y}{r}=\dfrac{0}{1}=0$ $\qquad \cos\theta=\dfrac{x}{r}=\dfrac{1}{1}=1$ $\qquad \tan\theta=\dfrac{y}{x}=\dfrac{0}{1}=0$

31. We have chosen P and Q on the terminal side of 35° and 45°, so that the x-coordinate is the same for both. Because 35° is less than 45°, we can see that $r_1<r_2$.



We also know that $\cos 35^\circ=\dfrac{x}{r_1}$ and $\cos 45^\circ=\dfrac{x}{r_2}$. As the denominator of a fraction increases, its value decreases. Therefore, $\cos 35^\circ>\cos 45^\circ$ and the statement is false.

33. We have chosen P and Q on the terminal side of 60° and 75°, so that the x-coordinate is the same for both. Because 60° is less than 75°, we can see that $r_1<r_2$.



We also know that $\sec 60^\circ=\dfrac{r_1}{x}$ and $\sec 75^\circ=\dfrac{r_2}{x}$.

Since $r_1<r_2$, $\sec 60^\circ<\sec 75^\circ$ and the statement is true.

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35. $\sin\theta = \frac{y}{r}$ where $x^2 + y^2 = r^2$

Assume $\sin\theta = 2$ where $y = 2$ and $r = 1$.

Then $x = \pm\sqrt{r^2 - y^2}$

$= \pm\sqrt{1^2 - 2^2}$

$= \pm\sqrt{1-4} = \pm\sqrt{-3}$ which is not a real number

Therefore, it is impossible to have any θ such that $\sin\theta = 2$.

37. Since $x^2 + y^2 = r^2$, we know that $|r| \geq |y|$. Then it is also true that $\left|\frac{r}{y}\right| \geq \left|\frac{y}{y}\right|$.

Therefore, we can conclude that $|\csc\theta| \geq 1$.

39. A point on the terminal side of $\theta = 0^\circ$ is $(1,0)$. Therefore, $x = 1$, $y = 0$, and $r = 1$.

$$\sin 0^\circ = \frac{y}{r} = \frac{0}{1} = 0$$

A point on the terminal side of $\theta = 90^\circ$ is $(0,1)$. Therefore, $x = 0$, $y = 1$, and $r = 1$.

$$\sin 90^\circ = \frac{y}{r} = \frac{1}{1} = 1$$

As θ increases from 0° to 90°, the value of $\sin\theta$ increases to the number 1.

41. As explained in Example 4 of the textbook, since $\tan\theta$ can be interpreted as the slope of the terminal side of θ, $\tan\theta$ will become larger and larger. Therefore, as θ nears 90°, the slope of the terminal side approaches infinity.

43. From Table 1 in the textbook, $\sin\theta$ will be positive in quadrant I or quadrant II.

45. From Table 1 in the textbook, $\cos\theta$ will be negative in quadrant II or quadrant III.

47. From Table 1 in the textbook, $\tan\theta$ will be positive in quadrant I or quadrant III.

49. From Table 1 in the textbook, $\csc\theta$ will be negative in quadrant III or quadrant IV.

51. The sine function is negative in quadrants III and IV.
If tangent function is positive in quadrants I and III.
Therefore, $\sin\theta$ is negative and $\tan\theta$ is positive in QIII.

53. The sine function is positive in quadrants I and II.
If tangent function is positive in quadrants I and III.
Therefore, both functions are positive in quadrant I.
The sine function is negative in quadrants III and IV.
If tangent function is negative in quadrants II and IV.
Therefore, both functions are negative in quadrant IV.

55. $\sin\theta = \frac{y}{r} = \frac{12}{13}$ and θ terminates in QI

$y = 12$ and $r = 13$

$x^2 + y^2 = r^2$

$x^2 + (12)^2 = (13)^2$

$x^2 + 144 = 169$

$x^2 = 25$

$x = \pm 5$

Since θ terminates in QI, x must equal 5.

$\sin\theta = \frac{12}{13}$

$\cos\theta = \frac{x}{r} = \frac{5}{13}$

$\tan\theta = \frac{y}{x} = \frac{12}{5}$

$\sec\theta = \frac{r}{x} = \frac{13}{5}$

$\sec\theta = \frac{r}{x} = \frac{13}{5}$

$\csc\theta = \frac{r}{y} = \frac{13}{12}$

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57. θ terminates in QII. Therefore, x is negative and y is positive.

$\cos\theta = \dfrac{x}{r} = \dfrac{-20}{29}$

Therefore, $x = -20$ and $r = 29$

$x^2 + y^2 = r^2$

$(-20)^2 + (y)^2 = (29)^2$

$400 + y^2 = 841$

$y^2 = 441$

$y = \pm 21$

Therefore, $y = 21$ because θ is in QII.

$\sin\theta = \dfrac{y}{r} = \dfrac{21}{29}$

$\cos\theta = -\dfrac{20}{29}$

$\tan\theta = \dfrac{y}{x} = \dfrac{21}{-20} = -\dfrac{21}{20}$

$\cot\theta = \dfrac{x}{y} = \dfrac{-20}{21} = -\dfrac{20}{21}$

$\sec\theta = \dfrac{r}{x} = \dfrac{29}{-20} = -\dfrac{29}{20}$

$\csc\theta = \dfrac{r}{y} = \dfrac{29}{21}$

59. θ terminates in QIV. Therefore x is positive and y is negative.

$\cos\theta = \dfrac{x}{r} = \dfrac{\sqrt{3}}{2}$

Therefore $x = \sqrt{3}$ and $r = 2$

$x^2 + y^2 = r^2$

$\left(\sqrt{3}\right)^2 + (y)^2 = (2)^2$

$3 + y^2 = 4$

$y^2 = 1$

Therefore, $y = -1$ because θ is in QIV.

$\sin\theta = \dfrac{y}{r} = -\dfrac{1}{2}$

$\cos\theta = \dfrac{\sqrt{3}}{2}$

$\tan\theta = \dfrac{y}{x} = -\dfrac{1}{\sqrt{3}} = -\dfrac{\sqrt{3}}{3}$

$\cot\theta = \dfrac{x}{y} = -\dfrac{\sqrt{3}}{1} = -\sqrt{3}$

$\sec\theta = \dfrac{r}{x} = \dfrac{2}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$

$\csc\theta = \dfrac{r}{y} = -\dfrac{2}{1} = -2$

61. θ terminates in QIII. Therefore, both x and y are positive.

$\tan\theta = \dfrac{y}{x} = \dfrac{3}{4} = \dfrac{-3}{-4}$

Therefore $x = -4$ and $y = -3$

$r = \sqrt{x^2 + y^2}$

$= \sqrt{(-4)^2 + (-3)^2}$

$= \sqrt{16 + 9}$

$= \sqrt{25} = 5$

$\sin\theta = \dfrac{y}{r} = \dfrac{-3}{5} = -\dfrac{3}{5}$

$\cos\theta = \dfrac{x}{r} = \dfrac{-4}{5} = -\dfrac{4}{5}$

$\tan\theta = \dfrac{3}{4}$

$\cot\theta = \dfrac{x}{y} = \dfrac{-4}{-3} = \dfrac{4}{3}$

$\sec\theta = \dfrac{r}{x} = \dfrac{5}{-4} = -\dfrac{5}{4}$

$\csc\theta = \dfrac{r}{y} = \dfrac{5}{-3} = -\dfrac{5}{3}$

63. The cosecant function is positive in QI and QII. The cosine function is negative in QII and QIII. Therefore, θ is in QII, where x is negative and y is positive.

$\csc\theta = \dfrac{r}{y} = \dfrac{13}{5}$

Therefore $y = 5$ and $r = 13$

$x^2 + y^2 = r^2$

$x^2 + 5^2 = 13^2$

$x^2 + 25 = 169$

$\sin\theta = \dfrac{y}{r} = \dfrac{5}{13}$

$\cos\theta = \dfrac{x}{r} = \dfrac{-12}{13} = -\dfrac{12}{13}$

$\tan\theta = \dfrac{y}{x} = \dfrac{5}{-12} = -\dfrac{5}{12}$

$\cot\theta = \dfrac{x}{y} = \dfrac{-12}{5} = -\dfrac{12}{5}$

$\sec\theta = \dfrac{r}{x} = \dfrac{13}{-12} = -\dfrac{13}{12}$

$\csc\theta = \dfrac{13}{5}$

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$x^2 = 144$

$x = \pm 12$

Therefore, $x = -12$.

65. $\cot\theta$ is positive in QI and QIII. $\cos\theta$ is positive in QI and QIV.
Therefore, θ is in QI and both x and y are positive.

$\cot\theta = \dfrac{x}{y} = \dfrac{1}{2}$

Therefore, $x = 1$ and $y = 2$.

$x^2 + y^2 = r^2$ $\qquad \sin\theta = \dfrac{y}{r} = \dfrac{2}{\sqrt{5}} = \dfrac{2\sqrt{5}}{5}$ $\qquad \cot\theta = \dfrac{1}{2}$

$1^2 + 2^2 = r^2$ $\qquad \cos\theta = \dfrac{x}{r} = \dfrac{1}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$ $\qquad \sec\theta = \dfrac{r}{x} = \dfrac{\sqrt{5}}{1} = \sqrt{5}$

$1 + 4 = r^2$ $\qquad \tan\theta = \dfrac{y}{x} = \dfrac{2}{1} = 2$ $\qquad \csc\theta = \dfrac{r}{y} = \dfrac{\sqrt{5}}{2}$

$r^2 = 5$

$r = \sqrt{5}$ because r is always positive.

67. $\tan\theta = \dfrac{a}{b}$ where a and b are both positive

$\tan\theta = \dfrac{y}{x} = \dfrac{a}{b}$ $\qquad \sin\theta = \dfrac{y}{r} = \dfrac{a}{\sqrt{a^2 + b^2}}$ $\qquad \cot\theta = \dfrac{x}{y} = \dfrac{b}{a}$

Therefore, $x = b$ and $y = a$. $\qquad \cos\theta = \dfrac{x}{r} = \dfrac{b}{\sqrt{a^2 + b^2}}$ $\qquad \sec\theta = \dfrac{r}{x} = \dfrac{\sqrt{a^2 + b^2}}{b}$

$r = \sqrt{a^2 + b^2}$ $\qquad \tan\theta = \dfrac{a}{b}$ $\qquad \csc\theta = \dfrac{r}{y} = \dfrac{\sqrt{a^2 + b^2}}{a}$

69. **a.** If $\sin\theta = -1$, then $y = -1$ and $r = 1$. Using $x^2 + y^2 = r^2$, we find that $(-1)^2 + y^2 = 1^2$ and $x = 0$.

Therefore, the point on the unit circle is $(0,-1)$. If we refer to figure 22 in section 1.2, we see that $\theta = 270^\circ$.

b. If $\cos\theta = -1$, then $x = -1$ and $r = 1$. Using $x^2 + y^2 = r^2$, we find that $x^2 + (-1)^2 = 1^2$ and $y = 0$.

Therefore, the point on the unit circle is $(-1,0)$. If we refer to figure 22 in section 1.2, we see that $\theta = 180^\circ$.

71. The terminal side of θ lies in QI. The point $(1,2)$ lies on $y = 2x$ in QI.

$x = 1$ and $y = 2$

$r = \sqrt{x^2 + y^2}$ $\qquad \sin\theta = \dfrac{y}{r} = \dfrac{2}{\sqrt{5}} = \dfrac{2\sqrt{5}}{5}$

$= \sqrt{1^2 + 2^2}$ $\qquad \cos\theta = \dfrac{x}{r} = \dfrac{1}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$

$= \sqrt{1 + 4} = \sqrt{5}$

73. The terminal side of θ lies in QII. The point $(-1,3)$ lies on $y = -3x$ in QII.

$x = -1$ and $y = 3$

$r = \sqrt{x^2 + y^2}$ $\qquad \sin\theta = \dfrac{y}{r} = \dfrac{3}{\sqrt{10}} = \dfrac{3\sqrt{10}}{10}$

$= \sqrt{(-1)^2 + 3^2} = \sqrt{1 + 9} = \sqrt{10}$ $\qquad \tan\theta = \dfrac{y}{x} = \dfrac{3}{-1} = -3$

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75. The point $(1,1)$ lies on the terminal side of 45° in QI.

Therefore, $x=1$, $y=1$, and $r=\sqrt{x^2+y^2}=\sqrt{1^2+1^2}=\sqrt{2}$

$$\cos 45^\circ=\frac{x}{r}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$$

The point $(1,-1)$ lies on the terminal side of -45° in QIV.

Therefore, $x=1$, $y=-1$, and $r=\sqrt{1^2+(-1)^2}=\sqrt{1+1}=\sqrt{2}$

$$\cos(-45^\circ)=\frac{x}{r}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$$

77. $\cos\theta=\frac{x}{r}=\frac{5}{13}$ $x=5$ and $r=13$

$$\begin{aligned} x^2+y^2&=r^2\\ 5^2+y^2&=13^2\\ y^2+25&=169\\ y^2&=144\\ y&=\pm 12 \end{aligned}$$

79. If $x=1$ and $y=-3$, then $r=\sqrt{1^2+(-3)^2}=\sqrt{1+9}=\sqrt{10}$. Therefore,

$\cos\theta=\frac{x}{r}=\frac{1}{\sqrt{10}}=\frac{\sqrt{10}}{10}$. The answer is d.

81. The cosine function is negative in quadrants I and IV. The answer is c.

1.4 Introduction to Identities

EVEN SOLUTIONS

2. The notation $\cos^2\theta$ is a shorthand for $(\cos\theta)^2$.

4. **a**. iii (cosine) **b**. i (secant) **c**. ii (cosecant)

6. The reciprocal is $\frac{1}{5}$.

8. The reciprocal is $-\frac{13}{12}$.

10. The reciprocal is $-\frac{\sqrt{3}}{2}$.

12. The reciprocal is $\frac{r}{y}$.

14. Using the reciprocal identity: $\sec\theta=\frac{1}{\cos\theta}=\frac{1}{\sqrt{2}/2}=\frac{2}{\sqrt{2}}=\sqrt{2}$

16. Using the reciprocal identity: $\sin\theta=\frac{1}{\csc\theta}=\frac{1}{-13/5}=-\frac{5}{13}$

18. Using the reciprocal identity: $\tan\theta=\frac{1}{\cot\theta}=\frac{1}{-1/m}=-m$

20. Using the ratio identity: $\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{2\sqrt{13}/13}{3\sqrt{13}/13}=\frac{2}{3}$

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22. Using the ratio identity: $\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{-4/5}{3/5} = -\dfrac{4}{3}$

24. Finding $\cos^2\theta$: $\cos^2\theta = (\cos\theta)^2 = \left(\dfrac{\sqrt{3}}{2}\right)^2 = \dfrac{3}{4}$

26. Finding $\sec^3\theta$: $\sec^3\theta = (\sec\theta)^3 = \left(-\dfrac{1}{3}\right)^3 = -\dfrac{1}{27}$

28. Using the ratio identity: $\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{-5/13}{-12/13} = \dfrac{5}{12}$

30. Using the reciprocal identity: $\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{-12/13} = -\dfrac{13}{12}$

32. First find $\sin^2\theta$: $\sin^2\theta = 1 - \cos^2\theta = 1 - \left(\dfrac{12}{13}\right)^2 = 1 - \dfrac{144}{169} = \dfrac{25}{169}$

Since θ terminates in quadrant I, $\sin\theta > 0$, and thus $\sin\theta = \sqrt{\dfrac{25}{169}} = \dfrac{5}{13}$.

34. First find $\cos^2\theta$: $\cos^2\theta = 1 - \sin^2\theta = 1 - \left(-\dfrac{1}{4}\right)^2 = 1 - \dfrac{1}{16} = \dfrac{15}{16}$

Since θ terminates in quadrant III, $\cos\theta < 0$, and thus $\cos\theta = -\sqrt{\dfrac{15}{16}} = -\dfrac{\sqrt{15}}{4}$.

36. First find $\cos^2\theta$: $\cos^2\theta = 1 - \sin^2\theta = 1 - \left(-\dfrac{5}{13}\right)^2 = 1 - \dfrac{25}{169} = \dfrac{144}{169}$

Since θ terminates in quadrant IV, $\cos\theta > 0$, and thus $\cos\theta = \sqrt{\dfrac{144}{169}} = \dfrac{12}{13}$.

38. First find $\sin^2\theta$: $\sin^2\theta = 1 - \cos^2\theta = 1 - \left(-\dfrac{\sqrt{2}}{2}\right)^2 = 1 - \dfrac{1}{2} = \dfrac{1}{2}$

Since θ terminates in quadrant II, $\sin\theta > 0$, and thus $\sin\theta = \sqrt{\dfrac{1}{2}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$.

40. First find $\sin^2\theta$: $\sin^2\theta = 1 - \cos^2\theta = 1 - \left(-\dfrac{2}{3}\right)^2 = 1 - \dfrac{4}{9} = \dfrac{5}{9}$

Since θ terminates in quadrant II, $\sin\theta > 0$, and thus $\sin\theta = \sqrt{\dfrac{5}{9}} = \dfrac{\sqrt{5}}{3}$. Now using the ratio identity:

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{-2/3}{\sqrt{5}/3} = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

42. First find $\csc^2\theta$: $\csc^2\theta = 1 + \cot^2\theta = 1 + \left(\dfrac{24}{7}\right)^2 = 1 + \dfrac{576}{49} = \dfrac{625}{49}$

Since θ terminates in quadrant III, $\csc\theta < 0$, and thus $\csc\theta = -\sqrt{\dfrac{625}{49}} = -\dfrac{25}{7}$.

44. First find $\sec^2\theta$: $\sec^2\theta = 1 + \tan^2\theta = 1 + \left(\dfrac{20}{21}\right)^2 = 1 + \dfrac{400}{441} = \dfrac{841}{441}$

Since $\cos\theta < 0$, $\sec\theta < 0$, and thus $\sec\theta = -\sqrt{\dfrac{841}{441}} = -\dfrac{29}{21}$.

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46. First find $\cos^2\theta$: $\cos^2\theta = 1-\sin^2\theta = 1-\left(\frac{4}{5}\right)^2 = 1-\frac{16}{25} = \frac{9}{25}$

Since θ terminates in quadrant I, $\cos\theta > 0$, and thus $\cos\theta = \sqrt{\frac{9}{25}} = \frac{3}{5}$. Now use the ratio identities to find the remaining four trigonometric ratios:

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{3/5} = \frac{5}{3} \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{1}{4/5} = \frac{5}{4}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{4/5}{3/5} = \frac{4}{3} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{3/5}{4/5} = \frac{3}{4}$$

48. First find $\sin^2\theta$: $\sin^2\theta = 1-\cos^2\theta = 1-\left(-\frac{1}{5}\right)^2 = 1-\frac{1}{25} = \frac{24}{25}$

Since $\cos\theta < 0$ and θ does not terminate in quadrant II, it must terminate in quadrant III and thus $\sin\theta < 0$.

Therefore $\sin\theta = -\sqrt{\frac{24}{25}} = -\frac{2\sqrt{6}}{5}$. Now use the ratio identities to find the remaining four trigonometric ratios:

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{-1/5} = -5 \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{1}{-2\sqrt{6}/5} = -\frac{5}{2\sqrt{6}} = -\frac{5\sqrt{6}}{12}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-2\sqrt{6}/5}{-1/5} = 2\sqrt{6} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{-1/5}{-2\sqrt{6}/5} = \frac{1}{2\sqrt{6}} = \frac{\sqrt{6}}{12}$$

50. Since $\sec\theta = -3$, $\cos\theta = \frac{1}{\sec\theta} = -\frac{1}{3}$. Now find $\sin^2\theta$: $\sin^2\theta = 1-\cos^2\theta = 1-\left(-\frac{1}{3}\right)^2 = 1-\frac{1}{9} = \frac{8}{9}$

Since $\sin\theta > 0$, $\sin\theta = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$. Now use the ratio identities to find the remaining three trigonometric ratios:

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{2\sqrt{2}/3} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2\sqrt{2}/3}{-1/3} = -2\sqrt{2}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{-1/3}{2\sqrt{2}/3} = \frac{-1}{2\sqrt{2}} = -\frac{\sqrt{2}}{4}$$

52. First find $\cos^2\theta$: $\cos^2\theta = 1-\sin^2\theta = 1-\left(\frac{4\sqrt{17}}{17}\right)^2 = 1-\frac{16}{17} = \frac{1}{17}$

Since θ terminates in quadrant II, $\cos\theta < 0$, and thus $\cos\theta = -\sqrt{\frac{1}{17}} = -\frac{1}{\sqrt{17}} = -\frac{\sqrt{17}}{17}$. Now use the ratio identities to find the remaining four trigonometric ratios:

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{-\sqrt{17}/17} = -\sqrt{17} \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{1}{4\sqrt{17}/17} = \frac{\sqrt{17}}{4}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{4\sqrt{17}/17}{-\sqrt{17}/17} = -4 \qquad \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{-\sqrt{17}/17}{4\sqrt{17}/17} = -\frac{1}{4}$$

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54. Since $\sec\theta = b$, $\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{1}{b}$. Now find $\sin^2\theta$: $\sin^2\theta = 1 - \cos^2\theta = 1 - \left(\dfrac{1}{b}\right)^2 = 1 - \dfrac{1}{b^2} = \dfrac{b^2-1}{b^2}$

Since θ terminates in quadrant I, $\sin\theta > 0$, and thus $\sin\theta = \sqrt{\dfrac{b^2-1}{b^2}} = \dfrac{\sqrt{b^2-1}}{b}$. Now use the ratio identities to find the remaining three trigonometric ratios:

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\sqrt{b^2-1}\,/\,b} = \frac{b}{\sqrt{b^2-1}} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{b^2-1}\,/\,b}{1/b} = \sqrt{b^2-1}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1/b}{\sqrt{b^2-1}\,/\,b} = \frac{1}{\sqrt{b^2-1}}$$

56. First find $\sin^2\theta$: $\sin^2\theta = 1 - \cos^2\theta = 1 - (0.59)^2 = 1 - 0.3481 = 0.6519$

Since θ terminates in quadrant I, $\sin\theta > 0$, and thus $\sin\theta = \sqrt{0.6519} \approx 0.8074$. Now use the ratio identities to find the remaining four trigonometric ratios:

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{0.59} \approx 1.69 \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{1}{0.8074} \approx 1.24$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{0.8074}{0.59} \approx 1.37 \qquad \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{0.59}{0.8074} \approx 0.73$$

58. Since $\csc\theta = -2.54$, $\sin\theta = \dfrac{1}{-2.54} \approx -0.3937$. Now find $\cos^2\theta$:

$\cos^2\theta = 1 - \sin^2\theta = 1 - (-0.3937)^2 = 1 - 0.1550 = 0.8450$

Since θ terminates in quadrant IV, $\cos\theta > 0$, and thus $\cos\theta = \sqrt{0.8450} \approx 0.9192$. Now use the ratio identities to find the remaining three trigonometric ratios:

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{0.9192} \approx 1.09 \qquad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-0.3937}{0.9192} \approx -0.43 \qquad \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{0.9192}{-0.3937} \approx -2.33$$

60. Using the points $(0, 0)$ and $(1, m)$: slope $= \dfrac{m-0}{1-0} = \dfrac{m}{1} = m$

62. Forming the right triangle to the point $(1, m)$, note that $\tan\theta = \dfrac{y}{x} = \dfrac{m}{1} = m$.

64. Using the ratio identity: $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\sqrt{5}\,/\,3}{2/3} = \dfrac{\sqrt{5}}{2}$. The correct answer is b.

66. First find $\sin^2\theta$: $\sin^2\theta = 1 - \cos^2\theta = 1 - \left(\dfrac{1}{4}\right)^2 = 1 - \dfrac{1}{16} = \dfrac{15}{16}$

Since θ terminates in quadrant IV, $\sin\theta < 0$, and thus $\sin\theta = -\sqrt{\dfrac{15}{16}} = -\dfrac{\sqrt{15}}{4}$. The correct answer is d.

ODD SOLUTIONS

1. true, defined

3. **a.** ii **b.** iii **c.** i

5. $\dfrac{1}{7}$

7. $\dfrac{1}{-\frac{2}{3}} = 1 \cdot -\dfrac{3}{2} = -\dfrac{3}{2}$

9. $\dfrac{1}{-\frac{1}{\sqrt{2}}} = 1 \cdot \dfrac{\sqrt{2}}{-1} = -\sqrt{2}$

11. $\dfrac{1}{x}, x \neq 0$

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13. $\csc\theta = \dfrac{1}{\sin\theta}$

$= \dfrac{1}{\frac{4}{5}}$

$= \dfrac{5}{4}$

15. $\cos\theta = \dfrac{1}{\sec\theta}$

$= \dfrac{1}{-2}$

$= -\dfrac{1}{2}$

17. $\cot\theta = \dfrac{1}{\tan\theta}$

$= \dfrac{1}{a}, a \neq 0$

19. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{\frac{2\sqrt{5}}{5}}{\frac{\sqrt{5}}{5}}$

$= 2$

21. $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{-12/13}{-5/13} = \dfrac{12}{5}$

23. $\sin^2\theta = (\sin\theta)^2$

$= \left(\dfrac{\sqrt{2}}{2}\right)^2 = \dfrac{2}{4} = \dfrac{1}{2}$

25. $\tan^3\theta = (\tan\theta)^3$

$= (2)^3 = 8$

27. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{-\frac{12}{13}}{-\frac{5}{13}} = \dfrac{12}{5}$

29. $\sec\theta = \dfrac{1}{\cos\theta}$

$= \dfrac{1}{-\frac{5}{13}}$

$= -\dfrac{13}{5}$

31. $\sin\theta = \pm\sqrt{1-\cos^2\theta}$

$= \pm\sqrt{1-\left(\dfrac{3}{5}\right)^2}$

$= \pm\sqrt{1-\dfrac{9}{25}}$

$= \pm\sqrt{\dfrac{16}{25}}$

$= \pm\dfrac{4}{5}$

Since θ terminates in QI, $\sin\theta$ is positive.

Therefore, $\sin\theta = \dfrac{4}{5}$

33. $\cos\theta = \pm\sqrt{1-\sin^2\theta}$

$= \pm\sqrt{1-\left(\dfrac{\sqrt{3}}{2}\right)^2}$

$= \pm\sqrt{1-\dfrac{3}{4}}$

$= \pm\sqrt{\dfrac{1}{4}} = \pm\dfrac{1}{2}$

Since θ terminates in QII, $\cos\theta$ is negative.

Therefore, $\cos\theta = -\dfrac{1}{2}$

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35. $\cos\theta = \pm\sqrt{1-\sin^2\theta}$

$= \pm\sqrt{1-\left(-\frac{4}{5}\right)^2}$

$= \pm\sqrt{1-\frac{16}{25}}$ Since θ terminates in QIII, $\cos\theta$ is negative.

$= \pm\sqrt{\frac{9}{25}} = \pm\frac{3}{5}$ Therefore, $\cos\theta = -\frac{3}{5}$

37. $\sin\theta = \pm\sqrt{1-\cos^2\theta}$

$= \pm\sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}$

$= \pm\sqrt{1-\frac{3}{4}}$

$= \pm\sqrt{\frac{1}{4}}$

$= \pm\frac{1}{2}$

Since θ terminates in QI, $\sin\theta$ is positive.

Therefore, $\sin\theta = \frac{1}{2}$

39. $\cos\theta = \pm\sqrt{1-\sin^2\theta}$ Since θ terminates in QI, $\cos\theta$ is positive.

$= \pm\sqrt{1-\left(\frac{1}{3}\right)^2}$ $\cos\theta = \frac{2\sqrt{2}}{3}$

$= \pm\sqrt{1-\frac{1}{9}}$ $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \pm\sqrt{\frac{8}{9}}$ $= \frac{\frac{1}{3}}{\frac{2\sqrt{2}}{3}}$

$= \pm\frac{2\sqrt{2}}{3}$ $= \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$

41. $\sec^2\theta = \tan^2\theta + 1$

$\sec\theta = \pm\sqrt{\tan^2\theta+1}$

$= \pm\sqrt{\left(\frac{8}{15}\right)^2+1}$

$= \sqrt{\frac{64}{225}+1}$

$= \pm\sqrt{\frac{289}{225}} = \pm\frac{17}{15}$

Since θ terminates in QIII, $\sec\theta$ is negative. Therefore, $\sec\theta = -\frac{17}{15}$

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43. $\csc\theta = \pm\sqrt{1+\cot^2\theta}$ Pythagorean identity

$= \pm\sqrt{1+\left(-\frac{21}{20}\right)^2}$ Substitute given value

$= \pm\sqrt{1+\frac{441}{400}}$ Simplify

$= \pm\sqrt{\frac{841}{400}}$

$= \pm\frac{29}{20}$

Since $\cot\theta$ is negative and $\sin\theta$ is positive, θ must lie in QII.

Therefore, $\csc\theta = \frac{29}{20}$.

45. θ terminates in QI. Therefore, all functions are positive.

$\sin\theta = \sqrt{1-\cos^2\theta}$

$= \sqrt{1-\left(\frac{12}{13}\right)^2}$

$= \sqrt{1-\frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{\frac{5}{13}}{\frac{12}{13}} = \frac{5}{12}$

$\cot\theta = \frac{1}{\tan\theta}$

$= \frac{1}{\frac{5}{12}} = \frac{12}{5}$

$\sec\theta = \frac{1}{\cos\theta}$

$= \frac{1}{\frac{12}{13}} = \frac{13}{12}$

$\csc\theta = \frac{1}{\sin\theta}$

$= \frac{1}{\frac{5}{13}} = \frac{13}{5}$

All six ratios are:

$\sin\theta = \frac{5}{13}$ $\quad$ $\cot\theta = \frac{12}{5}$

$\cos\theta = \frac{12}{13}$ $\quad$ $\sec\theta = \frac{13}{12}$

$\tan\theta = \frac{5}{12}$ $\quad$ $\csc\theta = \frac{13}{5}$

47. In QIII and QIV, $\sin\theta$ is negative, but θ is not in QIII. Therefore, θ must be in QIV.

$\csc\theta = \frac{1}{\sin\theta}$

$= \frac{1}{-1/2} = -2$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{-1/2}{\sqrt{3}/2} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

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$\cos\theta = \pm\sqrt{1-\sin^2\theta}$

$= \pm\sqrt{1-\left(-\frac{1}{2}\right)^2}$

$= \pm\sqrt{1-\frac{1}{4}}$

$= \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$

$\cot\theta = \frac{1}{\tan\theta}$

$= \frac{1}{-1/\sqrt{3}} = -\sqrt{3}$

$\sec\theta = \frac{1}{\cos\theta}$

$= \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\cos\theta = \frac{\sqrt{3}}{2}$ because $\cos\theta$ is positive in QIV.

All six ratios are:

$\sin\theta = -\frac{1}{2}$ $\quad \tan\theta = -\frac{3\sqrt{3}}{3}$ $\quad \sec\theta = \frac{2\sqrt{3}}{3}$

$\cos\theta = \frac{\sqrt{3}}{2}$ $\quad \cot\theta = -\sqrt{3}$ $\quad \csc\theta = -2$

49. The cosecant is positive and cosine function is negative. Therefore, θ must be in QII, where the sine and cosecant functions are positive and all other functions are negative.

$\sin\theta = \frac{1}{\csc\theta}$

$= \frac{1}{2}$

$\cos\theta = \pm\sqrt{1-\sin^2\theta}$

$= \pm\sqrt{1-\left(\frac{1}{2}\right)^2}$

$= \pm\sqrt{1-\frac{1}{4}} = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

$\cos\theta = -\frac{\sqrt{3}}{2}$ because cosine is negative.

$\cot\theta = \frac{1}{\tan\theta}$

$= \frac{1}{-\frac{1}{\sqrt{3}}} = -\sqrt{3}$

$\sec\theta = \frac{1}{\cos\theta}$

$= \frac{1}{-\sqrt{3}/2} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

All six ratios are:

$\sin\theta = \frac{1}{2}$ $\quad \cot\theta = -\sqrt{3}$

$\cos\theta = -\frac{\sqrt{3}}{2}$ $\quad \sec\theta = -\frac{2\sqrt{3}}{3}$

$\tan\theta = -\frac{\sqrt{3}}{3}$ $\quad \csc\theta = 2$

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51. $\sin\theta = \pm\sqrt{1-\cos^2\theta}$ $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\cot\theta = \dfrac{1}{\tan\theta}$

$= \pm\sqrt{1-\left(\dfrac{2}{\sqrt{13}}\right)^2}$ $= \dfrac{-3/\sqrt{13}}{2/\sqrt{13}} = -\dfrac{3}{2}$ $= \dfrac{1}{-3/2} = -\dfrac{2}{3}$

$= \pm\sqrt{1-\dfrac{4}{13}}$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\csc\theta = \dfrac{1}{\sin\theta}$

$= \pm\sqrt{\dfrac{9}{13}}$ $= \dfrac{1}{2/\sqrt{13}}$ $= \dfrac{1}{-3/\sqrt{13}}$

$= \pm\dfrac{3}{\sqrt{13}} = \pm\dfrac{3\sqrt{13}}{13}$ $= \dfrac{\sqrt{13}}{2}$ $= -\dfrac{\sqrt{13}}{3}$

$\sin\theta = -\dfrac{3\sqrt{13}}{13}$ because $\sin\theta$ is negative in QIV.

All six ratios are:

$\sin\theta = -\dfrac{3\sqrt{13}}{13}$ $\cot\theta = -\dfrac{2}{3}$

$\cos\theta = \dfrac{2\sqrt{13}}{13}$ $\sec\theta = \dfrac{\sqrt{13}}{2}$

$\tan\theta = -\dfrac{3}{2}$ $\csc\theta = -\dfrac{\sqrt{13}}{3}$

53. $\sin\theta = \dfrac{1}{\csc\theta}$ $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\cot\theta = \dfrac{1}{\tan\theta}$

$\sin\theta = \dfrac{1}{a}, a \neq 0$ $= \dfrac{1/a}{\sqrt{a^2-1}/a}$ $= \dfrac{1}{1/\sqrt{a^2-1}}$

$\cos\theta = \pm\sqrt{1-\sin^2\theta}$ $= \dfrac{1}{\sqrt{a^2-1}}$ $= \sqrt{a^2-1}$

$= \pm\sqrt{1-\left(\dfrac{1}{a}\right)^2} = \pm\sqrt{1-\dfrac{1}{a^2}}$ $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{\sqrt{a^2-1}/a} = \dfrac{a}{\sqrt{a^2-1}}$

$= \pm\sqrt{\dfrac{a^2-1}{a^2}}$

$= \pm\dfrac{\sqrt{a^2-1}}{a} = \dfrac{\sqrt{a^2-1}}{a}$ because θ is in QI.

All six ratios are:

$\sin\theta = \dfrac{1}{a}$ $\cot\theta = \sqrt{a^2-1}$

$\cos\theta = \dfrac{\sqrt{a^2-1}}{a}$ $\sec\theta = \dfrac{a}{\sqrt{a^2-1}}$

$\tan\theta = \dfrac{1}{\sqrt{a^2-1}}$ $\csc\theta = a$

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55. All functions are positive in QI.

$\cos\theta = \sqrt{1-\sin^2\theta}$
$= \sqrt{1-(0.23)^2}$
$= \sqrt{0.9471} = 0.9732$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
$= \dfrac{0.23}{0.9732}$
$= 0.2363$

$\sec\theta = \dfrac{1}{\cos\theta}$
$= \dfrac{1}{0.9732}$
$= 1.0275$

$\cot\theta = \dfrac{1}{\tan\theta}$
$= \dfrac{1}{0.2363} = 4.2313$

$\csc\theta = \dfrac{1}{\sin\theta}$
$= \dfrac{1}{0.2363} = 4.3478$

All six ratios are:

$\sin\theta = 0.23$ $\quad \tan\theta = 0.24$ $\quad \sec\theta = 1.03$
$\cos\theta = 0.97$ $\quad \cot\theta = 4.23$ $\quad \csc\theta = 4.35$

57. $\cos\theta = \dfrac{1}{\sec\theta}$
$= \dfrac{1}{-1.24}$
$= -0.81$

$\sin\theta = \pm\sqrt{1-\cos^2\theta}$
$= \pm\sqrt{1-(-0.81)^2}$
$= \pm 0.59$
$= 0.59$ because $\sin\theta$ is positive in QII

$\tan\theta = \sqrt{\sec^2-1}$
$= \sqrt{(-0.81)^2-1}$
$= -0.73$

$\cot\theta = \dfrac{1}{\tan\theta}$
$= \dfrac{1}{-0.73}$
$= -1.36$

$\csc\theta = \dfrac{1}{\sin\theta}$
$= \dfrac{1}{0.59}$
$= 1.69$

All six ratios are:

$\sin\theta = 0.59$ $\quad \cot\theta = -1.36$
$\cos\theta = -0.81$ $\quad \sec\theta = -1.24$
$\tan\theta = -0.73$ $\quad \csc\theta = 1.69$

59. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
$= \dfrac{3-0}{1-0}$
$= \dfrac{3}{1}$
$= 3$

61. The point $(1,3)$ lies on $y = 3x$ in QI.

Therefore, $x = 1$ and $y = 3$.

$\tan\theta = \dfrac{y}{x} = \dfrac{3}{1} = 3$

63. If $\cos\theta = \dfrac{1}{3}$, then $\sec\theta = \dfrac{1}{\cos\theta}$ and $\sec\theta = \dfrac{3}{1} = 3$. The answer is c.

65. If $\cos\theta = \dfrac{1}{4}$, then $\cos^2\theta = \left(\dfrac{1}{4}\right)^2 = \dfrac{1}{16}$ The answer is a.

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1.5 More on Identities

EVEN SOLUTIONS

2. To verify an identity, we can start with the left side of the equation and transform it until it is identical to the right side of the equation. Alternatively, we can start with the right side of the equation and transform it until it is identical to the left side of the equation.

4. Writing in terms of $\sin\theta$: $\csc\theta = \dfrac{1}{\sin\theta}$

6. Writing in terms of $\sin\theta$: $\sec\theta = \dfrac{1}{\cos\theta} = \pm\dfrac{1}{\sqrt{1-\sin^2\theta}} = \pm\dfrac{\sqrt{1-\sin^2\theta}}{1-\sin^2\theta}$

8. Writing in terms of $\cos\theta$: $\sin\theta = \pm\sqrt{1-\cos^2\theta}$

10. Using the result from the previous exercise: $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \pm\dfrac{\sqrt{1-\cos^2\theta}}{\cos\theta}$

12. **a**. Simplifying: $\dfrac{\frac{1}{a}}{\frac{b}{a}} = \dfrac{\frac{1}{a}}{\frac{b}{a}} \bullet \dfrac{a}{a} = \dfrac{1}{b}$ **b**. Simplifying: $\dfrac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}} = \dfrac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}} \bullet \dfrac{\cos\theta}{\cos\theta} = \dfrac{1}{\sin\theta} = \csc\theta$

14. Writing in terms of $\sin\theta$ and $\cos\theta$, then simplifying: $\sec\theta\cot\theta = \dfrac{1}{\cos\theta} \bullet \dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\sin\theta}$

16. Writing in terms of $\sin\theta$ and $\cos\theta$, then simplifying: $\cos\theta\tan\theta\csc\theta = \cos\theta \bullet \dfrac{\sin\theta}{\cos\theta} \bullet \dfrac{1}{\sin\theta} = 1$

18. Writing in terms of $\sin\theta$ and $\cos\theta$, then simplifying: $\dfrac{\cot\theta}{\sec\theta} = \dfrac{\cos\theta/\sin\theta}{1/\cos\theta} = \dfrac{\cos\theta}{\sin\theta} \bullet \dfrac{\cos\theta}{1} = \dfrac{\cos^2\theta}{\sin\theta}$

20. Writing in terms of $\sin\theta$ and $\cos\theta$, then simplifying: $\dfrac{\csc\theta}{\cot\theta} = \dfrac{1/\sin\theta}{\cos\theta/\sin\theta} = \dfrac{1}{\sin\theta} \bullet \dfrac{\sin\theta}{\cos\theta} = \dfrac{1}{\cos\theta}$

22. Writing in terms of $\sin\theta$ and $\cos\theta$, then simplifying: $\dfrac{\cot\theta}{\tan\theta} = \dfrac{\cos\theta/\sin\theta}{\sin\theta/\cos\theta} = \dfrac{\cos\theta}{\sin\theta} \bullet \dfrac{\cos\theta}{\sin\theta} = \dfrac{\cos^2\theta}{\sin^2\theta}$

24. Writing in terms of $\sin\theta$ and $\cos\theta$, then simplifying: $\dfrac{\sec\theta}{\cos\theta} = \dfrac{1/\cos\theta}{\cos\theta} = \dfrac{1}{\cos\theta} \bullet \dfrac{1}{\cos\theta} = \dfrac{1}{\cos^2\theta} = \sec^2\theta$

26. Writing in terms of $\sin\theta$ and $\cos\theta$, then simplifying: $\cot\theta - \csc\theta = \dfrac{\cos\theta}{\sin\theta} - \dfrac{1}{\sin\theta} = \dfrac{\cos\theta - 1}{\sin\theta}$

28. Writing in terms of $\sin\theta$ and $\cos\theta$, then simplifying: $\sin\theta\tan\theta + \sec\theta = \sin\theta \bullet \dfrac{\sin\theta}{\cos\theta} + \dfrac{1}{\cos\theta} = \dfrac{\sin^2\theta + 1}{\cos\theta}$

30. Writing in terms of $\sin\theta$ and $\cos\theta$, then simplifying:

$$\csc\theta - \cot\theta\cos\theta = \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} \bullet \cos\theta = \frac{1}{\sin\theta} - \frac{\cos^2\theta}{\sin\theta} = \frac{1-\cos^2\theta}{\sin\theta} = \frac{\sin^2\theta}{\sin\theta} = \sin\theta$$

32. **a**. Subtracting the fractions: $\dfrac{1}{a} - a = \dfrac{1}{a} - \dfrac{a}{1} \bullet \dfrac{a}{a} = \dfrac{1-a^2}{a}$

b. Subtracting the fractions: $\dfrac{1}{\cos\theta} - \cos\theta = \dfrac{1}{\cos\theta} - \dfrac{\cos\theta}{1} \bullet \dfrac{\cos\theta}{\cos\theta} = \dfrac{1-\cos^2\theta}{\cos\theta} = \dfrac{\sin^2\theta}{\cos\theta}$

34. **a**. Subtracting the fractions: $\dfrac{a}{b} - \dfrac{b}{a} = \dfrac{a}{b} \bullet \dfrac{a}{a} - \dfrac{b}{a} \bullet \dfrac{b}{b} = \dfrac{a^2 - b^2}{ab}$

b. Subtracting the fractions: $\dfrac{\sin\theta}{\cos\theta} - \dfrac{\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{\cos\theta} \bullet \dfrac{\sin\theta}{\sin\theta} - \dfrac{\cos\theta}{\sin\theta} \bullet \dfrac{\cos\theta}{\cos\theta} = \dfrac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta}$

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36. Adding the fractions: $\cos\theta - \dfrac{1}{\sin\theta} = \dfrac{\cos\theta}{1} \cdot \dfrac{\sin\theta}{\sin\theta} - \dfrac{1}{\sin\theta} = \dfrac{\sin\theta\cos\theta - 1}{\sin\theta}$

38. Subtracting the fractions: $\dfrac{1}{\sin\theta} - \dfrac{1}{\cos\theta} = \dfrac{1}{\sin\theta} \cdot \dfrac{\cos\theta}{\cos\theta} - \dfrac{1}{\cos\theta} \cdot \dfrac{\sin\theta}{\sin\theta} = \dfrac{\cos\theta - \sin\theta}{\sin\theta\cos\theta}$

40. Adding the fractions: $\dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta} = \dfrac{\cos\theta}{\sin\theta} \cdot \dfrac{\cos\theta}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta} \cdot \dfrac{\sin\theta}{\sin\theta} = \dfrac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta} = \dfrac{1}{\sin\theta\cos\theta}$

42. **a**. Multiplying: $(a+1)(a-1) = a^2 - 1$

b. Multiplying: $(\sec\theta + 1)(\sec\theta - 1) = \sec^2\theta - 1 = \tan^2\theta$

44. **a**. Multiplying: $(a-1)(a+6) = a^2 + 6a - a - 6 = a^2 + 5a - 6$

b. Multiplying: $(\cos\theta - 1)(\cos\theta + 6) = \cos^2\theta + 6\cos\theta - \cos\theta - 6 = \cos^2\theta + 5\cos\theta - 6$

46. Multiplying the expressions: $(\cos\theta + 2)(\cos\theta - 5) = \cos^2\theta + 2\cos\theta - 5\cos\theta - 10 = \cos^2\theta - 3\cos\theta - 10$

48. Multiplying the expressions: $(3\sin\theta + 4)(5\cos\theta + 2) = 15\sin\theta\cos\theta + 6\sin\theta + 20\cos\theta + 8$

50. Multiplying the expressions: $(1 - \cos\theta)(1 + \cos\theta) = 1 - \cos\theta + \cos\theta - \cos^2\theta = 1 - \cos^2\theta = \sin^2\theta$

52. Multiplying the expressions: $(1 - \csc\theta)(1 + \csc\theta) = 1 - \csc\theta + \csc\theta - \csc^2\theta = 1 - \csc^2\theta = -\cot^2\theta$

54. Multiplying the expressions: $(\cos\theta + \sin\theta)(\cos\theta + \sin\theta) = \cos^2\theta + \sin\theta\cos\theta + \sin\theta\cos\theta + \sin^2\theta = 1 + 2\sin\theta\cos\theta$

56. Multiplying the expressions: $(\cos\theta + 3)^2 = (\cos\theta + 3)(\cos\theta + 3) = \cos^2\theta + 3\cos\theta + 3\cos\theta + 9 = \cos^2\theta + 6\cos\theta + 9$

58. Substituting $x = \tan\theta$: $\sqrt{x^2 + 1} = \sqrt{\tan^2\theta + 1} = \sqrt{\sec^2\theta} = |\sec\theta|$

60. Substituting $x = 4\sin\theta$: $\sqrt{16 - x^2} = \sqrt{16 - (4\sin\theta)^2} = \sqrt{16 - 16\sin^2\theta} = \sqrt{16\left(1 - \sin^2\theta\right)} = \sqrt{16\cos^2\theta} = 4|\cos\theta|$

62. Substituting $x = 8\sec\theta$: $\sqrt{x^2 - 64} = \sqrt{(8\sec\theta)^2 - 64} = \sqrt{64\sec^2\theta - 64} = \sqrt{64\left(\sec^2\theta - 1\right)} = \sqrt{64\tan^2\theta} = 8|\tan\theta|$

64. Substituting $x = 5\tan\theta$: $\sqrt{4x^2 + 16} = \sqrt{4(2\tan\theta)^2 + 16} = \sqrt{16\tan^2\theta + 16} = \sqrt{16\left(\tan^2\theta + 1\right)} = \sqrt{16\sec^2\theta} = 4|\sec\theta|$

66. Working from the left side: $\sin\theta\cot\theta = \sin\theta \cdot \dfrac{\cos\theta}{\sin\theta} = \cos\theta$

68. Working from the left side: $\cos\theta\csc\theta\tan\theta = \cos\theta \cdot \dfrac{1}{\sin\theta} \cdot \dfrac{\sin\theta}{\cos\theta} = \dfrac{\sin\theta\cos\theta}{\sin\theta\cos\theta} = 1$

70. Working from the left side: $\dfrac{\cos\theta}{\sec\theta} = \dfrac{\cos\theta}{1/\cos\theta} = \cos\theta \cdot \cos\theta = \cos^2\theta$

72. Working from the left side: $\dfrac{\sec\theta}{\tan\theta} = \dfrac{1/\cos\theta}{\sin\theta/\cos\theta} = \dfrac{1}{\cos\theta} \cdot \dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\sin\theta} = \csc\theta$

74. Working from the left side: $\dfrac{\sec\theta}{\csc\theta} = \dfrac{1/\cos\theta}{1/\sin\theta} = \dfrac{1}{\cos\theta} \cdot \dfrac{\sin\theta}{1} = \dfrac{\sin\theta}{\cos\theta} = \tan\theta$

76. Working from the left side: $\dfrac{\csc\theta\tan\theta}{\sec\theta} = \dfrac{1/\sin\theta \cdot \sin\theta/\cos\theta}{1/\cos\theta} = \dfrac{1}{\sin\theta} \cdot \dfrac{\sin\theta}{\cos\theta} \cdot \dfrac{\cos\theta}{1} = \dfrac{\sin\theta\cos\theta}{\sin\theta\cos\theta} = 1$

78. Working from the left side:

$$\cos\theta\cot\theta + \sin\theta = \cos\theta \cdot \frac{\cos\theta}{\sin\theta} + \sin\theta = \frac{\cos^2\theta}{\sin\theta} + \frac{\sin\theta}{1} \cdot \frac{\sin\theta}{\sin\theta} = \frac{\cos^2\theta + \sin^2\theta}{\sin\theta} = \frac{1}{\sin\theta} = \csc\theta$$

80. Working from the left side: $\tan^2\theta + 1 = \dfrac{\sin^2\theta}{\cos^2\theta} + 1 \cdot \dfrac{\cos^2\theta}{\cos^2\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta} = \sec^2\theta$

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82. Working from the left side: $\csc\theta\tan\theta-\cos\theta=\dfrac{1}{\sin\theta}\cdot\dfrac{\sin\theta}{\cos\theta}-\cos\theta=\dfrac{1}{\cos\theta}-\dfrac{\cos\theta}{1}\cdot\dfrac{\cos\theta}{\cos\theta}=\dfrac{1-\cos^2\theta}{\cos\theta}=\dfrac{\sin^2\theta}{\cos\theta}$

84. Working from the left side: $(1+\sin\theta)(1-\sin\theta)=1+\sin\theta-\sin\theta-\sin^2\theta=1-\sin^2\theta=\cos^2\theta$

86. Working from the left side: $(\cos\theta+1)(\cos\theta-1)=\cos^2\theta+\cos\theta-\cos\theta-1=\cos^2\theta-1=-\left(1-\cos^2\theta\right)=-\sin^2\theta$

88. Working from the left side: $1-\dfrac{\sin\theta}{\csc\theta}=1-\dfrac{\sin\theta}{1/\sin\theta}=1-\sin\theta\cdot\sin\theta=1-\sin^2\theta=\cos^2\theta$

90. Working from the left side:

$$\begin{aligned}(\sin\theta-\cos\theta)^2-1&=(\sin\theta-\cos\theta)(\sin\theta-\cos\theta)-1\\&=\sin^2\theta-\sin\theta\cos\theta-\sin\theta\cos\theta+\cos^2\theta-1\\&=1-2\sin\theta\cos\theta-1\\&=-2\sin\theta\cos\theta\end{aligned}$$

92. Working from the left side: $\sec\theta(\sin\theta+\cos\theta)=\dfrac{1}{\cos\theta}(\sin\theta+\cos\theta)=\dfrac{\sin\theta}{\cos\theta}+1=\tan\theta+1$

94. Working from the left side: $\cos\theta(\csc\theta+\tan\theta)=\cos\theta\left(\dfrac{1}{\sin\theta}+\dfrac{\sin\theta}{\cos\theta}\right)=\dfrac{\cos\theta}{\sin\theta}+\sin\theta=\cot\theta+\sin\theta$

96. Working from the left side: $\cos\theta(\sec\theta-\cos\theta)=\cos\theta\left(\dfrac{1}{\cos\theta}-\cos\theta\right)=1-\cos^2\theta=\sin^2\theta$

98. Simplifying: $\dfrac{\cot\theta}{\csc\theta}=\dfrac{\cos\theta/\sin\theta}{1/\sin\theta}=\dfrac{\cos\theta}{\sin\theta}\cdot\dfrac{\sin\theta}{1}=\cos\theta$. The correct answer is a.

100. The correct answer is d. The identity is not an equation, so we cannot use equation properties to prove it.

ODD SOLUTIONS

1. sines, cosines

3. $\cos\theta=\pm\sqrt{1-\sin^2\theta}$ Pythagorean identity

5. $\cot\theta=\dfrac{\cos\theta}{\sin\theta}$ Ratio identity

$=\pm\dfrac{\sqrt{1-\sin^2\theta}}{\sin\theta}$ Pythagorean identity

7. $\sec\theta=\dfrac{1}{\cos\theta}$ Reciprocal identity

9. $\csc\theta=\dfrac{1}{\sin\theta}$ Reciprocal identity

$=\pm\dfrac{1}{\sqrt{1-\cos^2\theta}}$ Pythagorean identity

11. **a.** $\dfrac{\frac{1}{a}}{\frac{1}{b}}=\dfrac{1}{a}\div\dfrac{1}{b}=\dfrac{1}{a}\cdot\dfrac{b}{1}=\dfrac{b}{a}$

b. $\dfrac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}}=\dfrac{1}{\cos\theta}\div\dfrac{1}{\sin\theta}=\dfrac{1}{\cos\theta}\cdot\dfrac{\sin\theta}{1}=\dfrac{\sin\theta}{\cos\theta}=\tan\theta$

13. $\csc\theta\cot\theta=\dfrac{1}{\sin\theta}\cdot\dfrac{\cos\theta}{\sin\theta}$ Reciprocal and ratio identities

$=\dfrac{\cos\theta}{\sin^2\theta}$ Multiplication of fractions

15. $\csc\theta\tan\theta=\dfrac{1}{\sin\theta}\cdot\dfrac{\sin\theta}{\cos\theta}$ Reciprocal and ratio identities

$=\dfrac{\sin\theta}{\sin\theta\cos\theta}$ Multiplication of fractions

$=\dfrac{1}{\cos\theta}$ Division

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17. $\dfrac{\sec\theta}{\csc\theta} = \dfrac{1/\cos\theta}{1/\sin\theta}$ Reciprocal identities

$= \dfrac{\sin\theta}{\cos\theta}$ Division of fractions

19. $\dfrac{\sec\theta}{\tan\theta} = \dfrac{\dfrac{1}{\cos\theta}}{\dfrac{\sin\theta}{\cos\theta}}$ Reciprocal and ratio identities

$= \dfrac{\cos\theta}{\sin\theta\cos\theta}$ Division of fractions

$= \dfrac{1}{\sin\theta}$ Division

21. $\dfrac{\tan\theta}{\cot\theta} = \dfrac{\sin\theta/\cos\theta}{\cos\theta/\sin\theta}$ Ratio identities

$= \dfrac{\sin^2\theta}{\cos^2\theta}$ Division of fractions

23. $\dfrac{\sin\theta}{\csc\theta} = \dfrac{\sin\theta}{\dfrac{1}{\sin\theta}}$ Reciprocal identity

$= \dfrac{\sin^2\theta}{1}$ Division of fractions

$= \sin^2\theta$ Simplify

25. $\tan\theta + \sec\theta = \dfrac{\sin\theta}{\cos\theta} + \dfrac{1}{\cos\theta}$ Ratio and reciprocal identities

$= \dfrac{\sin\theta + 1}{\cos\theta}$ Addition of fractions

27. $\sin\theta\cot\theta + \cos\theta = \sin\theta \cdot \dfrac{\cos\theta}{\sin\theta} + \cos\theta$ Ratio identity

$= \dfrac{\sin\theta\cos\theta}{\sin\theta} + \cos\theta$ Multiplication of fractions

$= \cos\theta + \cos\theta$ Division

$= 2\cos\theta$ Addition

29. $\sec\theta - \tan\theta\sin\theta = \dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta} \cdot \sin\theta$ Reciprocal and ratio identities

$= \dfrac{1}{\cos\theta} - \dfrac{\sin^2\theta}{\cos\theta}$ Multiplication of fractions

$= \dfrac{1 - \sin^2\theta}{\cos\theta}$ Subtraction of fractions

$= \dfrac{\cos^2\theta}{\cos\theta}$ Pythagorean identity

$= \cos\theta$ Division

31. **a.** $\dfrac{1}{a} - \dfrac{1}{b} = \dfrac{b}{ab} - \dfrac{a}{ab} = \dfrac{b-a}{ab}$

b. $\dfrac{1}{\sin\theta} - \dfrac{1}{\cos\theta} = \dfrac{\cos\theta}{\sin\theta\cos\theta} - \dfrac{\sin\theta}{\sin\theta\cos\theta} = \dfrac{\cos\theta - \sin\theta}{\sin\theta\cos\theta}$

33. **a.** $\dfrac{b}{a} + \dfrac{1}{b} = \dfrac{b^2}{ab} + \dfrac{a}{ab} = \dfrac{b^2 + a}{ab}$

b. $\dfrac{\cos\theta}{\sin\theta} + \dfrac{1}{\cos\theta} = \dfrac{\cos^2\theta}{\sin\theta\cos\theta} + \dfrac{\sin\theta}{\sin\theta\cos\theta} = \dfrac{\cos^2\theta + \sin\theta}{\sin\theta\cos\theta}$

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35. $\sin\theta + \dfrac{1}{\cos\theta} = \sin\theta \cdot \dfrac{\cos\theta}{\cos\theta} + \dfrac{1}{\cos\theta}$ LCD is $\cos\theta$

$= \dfrac{\sin\theta\cos\theta}{\cos\theta} + \dfrac{1}{\cos\theta}$ Multiplication of fractions

$= \dfrac{\sin\theta\cos\theta + 1}{\cos\theta}$ Addition of fractions

37. $\dfrac{1}{\cos\theta} - \cos\theta = \dfrac{1}{\cos\theta} - \cos\theta \cdot \dfrac{\cos\theta}{\cos\theta}$ LCD is $\cos\theta$

$= \dfrac{1}{\cos\theta} - \dfrac{\cos^2\theta}{\cos\theta}$ Multiplication

$= \dfrac{1-\cos^2\theta}{\cos\theta}$ Subtraction of fractions

$= \dfrac{\sin^2\theta}{\cos\theta}$ Pythagorean identity

39. $\dfrac{\sin\theta}{\cos\theta} + \dfrac{1}{\sin\theta} = \dfrac{\sin\theta}{\cos\theta} \cdot \dfrac{\sin\theta}{\sin\theta} + \dfrac{1}{\sin\theta} \cdot \dfrac{\cos\theta}{\cos\theta}$ LCD is $\sin\theta\cos\theta$

$= \dfrac{\sin^2\theta}{\sin\theta\cos\theta} + \dfrac{\cos\theta}{\sin\theta\cos\theta}$ Multiplication of fractions

$= \dfrac{\sin^2\theta + \cos\theta}{\sin\theta\cos\theta}$ Addition of fractions

41. **a.** $(a-b)^2 = (a-b)(a-b) = a^2 - ab - ab + b^2 = a^2 - 2ab + b^2$

b. $(\cos\theta - \sin\theta)^2 = (\cos\theta - \sin\theta)(\cos\theta - \sin\theta)$ Definition of squaring

$= \cos^2\theta - 2\sin\theta\cos\theta + \sin^2\theta$ Multiplication using FOIL

$= \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta$ Commutative property

$= 1 - 2\sin\theta\cos\theta$ Pythagorean identity

43. **a.** $(a-2)(a-3) = a^2 - 3a - 2a + 6 = a^2 - 5a + 6$

b. $(\sin\theta - 2)(\sin\theta - 3) = \sin\theta^2 - 3\sin\theta - 2\sin\theta + 6 = \sin^2\theta - 5\sin\theta + 6$

45. $(\sin\theta + 4)(\sin\theta + 3) = \sin^2\theta + 3\sin\theta + 4\sin\theta + 12 = \sin^2\theta + 7\sin\theta + 12$

47. $(2\cos\theta + 3)(4\cos\theta - 5) = 8\cos^2\theta - 10\cos\theta + 12\cos\theta - 15 = 8\cos^2\theta + 2\cos\theta - 15$

49. $(1-\sin\theta)(1+\sin\theta) = 1 + \sin\theta - \sin\theta - \sin^2\theta$

$= 1 - \sin^2\theta = \cos^2\theta$ Pythagorean identity

51. $(1-\tan\theta)(1+\tan\theta) = 1 + \tan\theta - \tan\theta - \tan^2\theta = 1 - \tan^2\theta$

53. $(\sin\theta - \cos\theta)^2 = (\sin\theta - \cos\theta)(\sin\theta - \cos\theta)$

$= \sin^2\theta - \sin\theta\cos\theta - \sin\theta\cos\theta + \cos^2\theta$

$= \sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta$

$= (\sin^2\theta + \cos^2\theta) - 2\sin\theta\cos\theta$

$= 1 - 2\sin\theta\cos\theta$ Pythagorean identity

55. $(\sin\theta - 4)^2 = (\sin\theta - 4)(\sin\theta - 4)$

$= \sin^2\theta - 4\sin\theta - 4\sin\theta + 16$

$= \sin^2\theta - 8\sin\theta + 16$

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57. $\sqrt{x^2+4} = \sqrt{(2\tan\theta)^2+4}$ Substitute given value

$= \sqrt{4\tan^2\theta+4}$ Multiply

$= \sqrt{4(\tan^2\theta+1)}$ Factor

$= \sqrt{4\sec^2\theta}$ Pythagorean identity

$= |2\sec\theta| = 2|\sec\theta|$ Simplify

59. $\sqrt{9-x^2} = \sqrt{9-(3\sin\theta)^2}$ Substitute given value

$= \sqrt{9-9\sin^2\theta}$ Multiply

$= \sqrt{9(1-\sin^2\theta)}$ Factor

$= \sqrt{9\cos^2\theta}$ Pythagorean identity

$= |3\cos\theta| = 3|\cos\theta|$ Simplify

61. $\sqrt{x^2-36} = \sqrt{(6\sec\theta)^2-36}$ Substitute given value

$= \sqrt{36\sec^2\theta-36}$ Multiply

$= \sqrt{36(\sec^2\theta-1)}$ Factor

$= \sqrt{36\tan^2\theta}$ Pythagorean identity

$= |6\tan\theta| = 6|\tan\theta|$ Simplify

63. $\sqrt{64-4x^2} = \sqrt{64-4(4\sin\theta)^2}$ Substitute given value

$= \sqrt{64-4(16\sin^2\theta)}$ Multiply

$= \sqrt{64-64\sin^2\theta}$ Multiply

$= \sqrt{64(1-\sin^2\theta)}$ Factor

$= \sqrt{64\cos^2\theta}$ Pythagorean identity

$= |8\cos\theta| = 8|\cos\theta|$ Simplify

65. $\cos\theta\tan\theta = \cos\theta\cdot\dfrac{\sin\theta}{\cos\theta}$ Ratio identity

$= \dfrac{\cos\theta\sin\theta}{\cos\theta}$ Multiplication of fractions

$= \sin\theta$ Division of common factors

67. $\sin\theta\sec\theta\cot\theta = \sin\theta\cdot\dfrac{1}{\cos\theta}\cdot\dfrac{\cos\theta}{\sin\theta}$ Reciprocal and ratio identities

$= \dfrac{\sin\theta\cos\theta}{\cos\theta\sin\theta}$ Multiplication of fractions

$= 1$ Division of common factors

69. $\dfrac{\sin\theta}{\csc\theta} = \dfrac{\sin\theta}{\dfrac{1}{\sin\theta}}$ Reciprocal identity

$= \sin\theta\cdot\dfrac{\sin\theta}{1}$ Division of fractions

$= \sin^2\theta$ Simplify

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71. $\dfrac{\csc\theta}{\cot\theta} = \dfrac{1/\sin\theta}{\cos\theta/\sin\theta}$ Reciprocal and ratio identities

$= \dfrac{\sin\theta}{\sin\theta\cos\theta}$ Division of fractions

$= \dfrac{1}{\cos\theta}$ Division of common factor

$= \sec\theta$ Reciprocal identity

73. $\dfrac{\csc\theta}{\sec\theta} = \dfrac{\dfrac{1}{\sin\theta}}{\dfrac{1}{\cos\theta}}$ Reciprocal identity

$= \dfrac{\cos\theta}{\sin\theta}$ Division of fractions

$= \cot\theta$ Ratio identity

75. $\dfrac{\sec\theta\cot\theta}{\csc\theta} = \dfrac{\dfrac{1}{\cos\theta}\cdot\dfrac{\cos\theta}{\sin\theta}}{\dfrac{1}{\sin\theta}}$ Reciprocal and ratio identities

$= \dfrac{\cos\theta\sin\theta}{\cos\theta\sin\theta}$ Division of fractions

$= 1$ Division of common factors

77. $\sin\theta\tan\theta + \cos\theta = \sin\theta\cdot\dfrac{\sin\theta}{\cos\theta} + \cos\theta$ Ratio identity

$= \dfrac{\sin^2\theta}{\cos\theta} + \cos\theta$ Multiplication of fractions

$= \dfrac{\sin^2\theta}{\cos\theta} + \dfrac{\cos\theta}{1}\cdot\dfrac{\cos\theta}{\cos\theta}$ L.C.D. is $\cos\theta$

$= \dfrac{\sin^2\theta}{\cos\theta} + \dfrac{\cos^2\theta}{\cos\theta}$ Multiplication of fractions

$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta}$ Addition of fractions

$= \dfrac{1}{\cos\theta}$ Pythagorean identity

$= \sec\theta$ Reciprocal identity

79. $\tan\theta + \cot\theta = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$ Ratio and reciprocal identities

$= \dfrac{\sin\theta}{\cos\theta}\cdot\dfrac{\sin\theta}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta}\cdot\dfrac{\cos\theta}{\cos\theta}$ L.C.D. is $\sin\theta\cos\theta$

$= \dfrac{\sin^2\theta}{\sin\theta\cos\theta} + \dfrac{\cos^2\theta}{\sin\theta\cos\theta}$ Multiplication

$= \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$ Addition of fractions

$= \dfrac{1}{\sin\theta\cos\theta}$ Pythagorean identity

$= \dfrac{1}{\sin\theta}\cdot\dfrac{1}{\cos\theta}$ Property of fractions

$= \csc\theta\sec\theta$ Reciprocal identities

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81. $\csc\theta - \sin\theta = \dfrac{1}{\sin\theta} - \sin\theta$ Reciprocal identity

$= \dfrac{1}{\sin\theta} - \dfrac{\sin\theta}{1}\cdot\dfrac{\sin\theta}{\sin\theta}$ L.C.D. is $\sin\theta$

$= \dfrac{1}{\sin\theta} - \dfrac{\sin^2\theta}{\sin\theta}$ Multiplication of fractions

$= \dfrac{1-\sin^2\theta}{\sin\theta}$ Subtraction of fractions

$= \dfrac{\cos^2\theta}{\sin\theta}$ Pythagorean identity

83. $(1-\cos\theta)(1+\cos\theta) = 1-\cos^2\theta$ Multiplication

$= \sin^2\theta$ Pythagorean identity

85. $(\sin\theta+1)(\sin\theta-1) = \sin^2\theta - 1$ Multiplication

$= 1-\cos^2\theta - 1$ Pythagorean identity

$= -\cos^2\theta$ Addition

87. $\dfrac{\cos\theta}{\sec\theta} + \dfrac{\sin\theta}{\csc\theta} = \dfrac{\cos\theta}{\dfrac{1}{\cos\theta}} + \dfrac{\sin\theta}{\dfrac{1}{\sin\theta}}$ Reciprocal identities

$= \cos^2\theta + \sin^2\theta$ Division of fractions

$= 1$ Pythagorean identity

89. $(\cos\theta+\sin\theta)^2 - 1 = \cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta - 1$ Multiplication

$= \cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta - 1$ Commutative property

$= 1 + 2\sin\theta\cos\theta - 1$ Pythagorean identity

$= 2\sin\theta\cos\theta$ Addition

91. $\sin\theta(\sec\theta+\csc\theta) = \sin\theta\cdot\sec\theta + \sin\theta\cdot\csc\theta$ Distributive property

$= \dfrac{\sin\theta}{1}\cdot\dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{1}\cdot\dfrac{1}{\sin\theta}$ Reciprocal identities

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\sin\theta}{\sin\theta}$ Multiplication of fractions

$= \tan\theta + 1$ Ratio identity and division of common factor

93. $\sin\theta(\sec\theta+\cot\theta) = \sin\theta\sec\theta + \sin\theta\cot\theta$ Distributive property

$= \dfrac{\sin\theta}{1}\cdot\dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{1}\cdot\dfrac{\cos\theta}{\sin\theta}$ Reciprocal and ratio identities

$= \dfrac{\sin\theta}{\cos\theta} + \cos\theta$ Multiplication and division of fractions

$= \tan\theta + \cos\theta$ Ratio identity

95. $\sin\theta(\csc\theta-\sin\theta) = \sin\theta\csc\theta - \sin^2\theta$ Distributive property

$= \dfrac{\sin\theta}{1}\cdot\dfrac{1}{\sin\theta} - \sin^2\theta$ Reciprocal identity

$= \dfrac{\sin\theta}{\sin\theta} - \sin^2\theta$ Multiplication of fractions

$= 1 - \sin^2\theta$ Division of common factor

$= \cos^2\theta$ Pythagorean identity

97. $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\sin\theta}{\pm\sqrt{1-\sin^2\theta}} = \pm\dfrac{\sin\theta}{\sqrt{1-\sin^2\theta}}$. The answer is b.

99. $\sqrt{x^2+16} = \sqrt{(4\tan\theta)^2+16} = \sqrt{16\tan^2\theta+16} = \sqrt{16(\tan^2\theta+1)} = \sqrt{16\sec^2\theta} = 4|\sec\theta|$. The answer is c.

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Chapter 1 Test

1. The complement is $90° - 70° = 20°$ and the supplement is $180° - 70° = 110°$.

2. Using the Pythagorean Theorem:

$$3^2 + x^2 = 6^2$$
$$9 + x^2 = 36$$
$$x^2 = 27$$
$$x = \sqrt{27} = 3\sqrt{3}$$

3. Since $h = s$, $h = 5\sqrt{3}$. Since $r = \sqrt{2}\,h$, we have $r = \sqrt{2} \bullet 5\sqrt{3} = 5\sqrt{6}$. Since ΔBDC is a 30°-60°-90° triangle, $h = \sqrt{3}\,y$ and thus $y = \dfrac{h}{\sqrt{3}}$. So $y = \dfrac{5\sqrt{3}}{\sqrt{3}} = 5$. Finally $x = 2y$, so $x = 10$.

4. Since ΔABC is a right triangle, using the Pythagorean Theorem:

$$(AB)^2 + 3^2 = 5^2$$
$$(AB)^2 + 9 = 25$$
$$(AB)^2 = 16$$
$$AB = 4$$

Since ΔDAB is a right triangle, using the Pythagorean Theorem:

$$(AB)^2 + (AD)^2 = (DB)^2$$
$$4^2 + 6^2 = (DB)^2$$
$$(DB)^2 = 16 + 36$$
$$(DB)^2 = 52$$
$$DB = \sqrt{52} = 2\sqrt{13}$$

5. Since the hour hand moves 360° in 12 hours, in 3 hours it must move $\dfrac{360°}{4} = 90°$.

6. If the longest side is 9, the shortest side is $\dfrac{1}{2}(9) = \dfrac{9}{2}$ and the middle side is $\dfrac{9\sqrt{3}}{2}$.

7. Since this is a 45°-45°-90° triangle, the hypotenuse has a length of $15\sqrt{2} \approx 21.2$ feet.

8. Constructing the triangles from the center of the pentagon, there are 5 congruent triangles with a total sum of angles of $5 \bullet 180° = 900°$. However, this sum includes the center angles, which add to 360°, so the sum of the interior angles is $900° - 360° = 540°$. Since there are 5 of these interior angles, the measure of each is $\dfrac{540°}{5} = 108°$.

9. Using the distance formula: $r = \sqrt{(-1-4)^2 + (10+2)^2} = \sqrt{(-5)^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13$

10. Using the distance formula:

$$\sqrt{(x+2)^2 + (1-3)^2} = \sqrt{29}$$
$$\sqrt{(x+2)^2 + 4} = \sqrt{29}$$
$$(x+2)^2 + 4 = 29$$
$$(x+2)^2 = 25$$
$$x + 2 = 5 \quad \text{or} \quad x + 2 = -5$$
$$x = 3 \quad \text{or} \quad x = -7$$

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11. Substituting the point $\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$: $x^2+y^2=\left(\frac{1}{2}\right)^2+\left(-\frac{\sqrt{3}}{2}\right)^2=\frac{1}{4}+\frac{3}{4}=1$

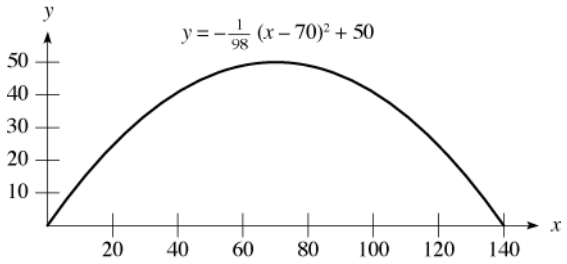
So the point lies on the unit circle.

12. All angles that are coterminal with 150° are $150°+360°k$, for any integer k.

13. The vertex of the parabola is (70,50), so the equation has the form $y=a(x-70)^2+50$. Since the point (140,0) is on this parabola:

$$0=a(140-70)^2+50$$
$$-50=4900a$$
$$a=-\frac{1}{98}$$

Thus the equation is $y=-\frac{1}{98}(x-70)^2+50$. Graphing the parabola:



14. Choose $(-1,0)$ as a point on the terminal side of 180°. Therefore:

$$\sin 180°=\frac{y}{r}=\frac{0}{1}=0 \qquad \cos 180°=\frac{x}{r}=\frac{-1}{1}=-1 \qquad \tan 180°=\frac{y}{x}=\frac{0}{1}=0$$

15. Choose $(1,-1)$ as a point on the terminal side of –45°. First find r: $r=\sqrt{1^2+(-1)^2}=\sqrt{1+1}=\sqrt{2}$

Therefore:

$$\sin(-45°)=\frac{y}{r}=-\frac{1}{\sqrt{2}}=-\frac{\sqrt{2}}{2} \qquad \cos(-45°)=\frac{x}{r}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2} \qquad \tan(-45°)=\frac{y}{x}=\frac{-1}{1}=-1$$

16. Since $\sin\theta<0$, θ could terminate in quadrant III or quadrant IV.

17. Since $\csc\theta>0$, $\sin\theta>0$. Since $\cos\theta<0$, θ must lie in quadrant II.

18. Begin by finding $r=\sqrt{(-3)^2+(-1)^2}=\sqrt{9+1}=\sqrt{10}$. Now applying the definitions for the six trigonometric functions using the values $x=-3$, $y=-1$, and $r=\sqrt{10}$:

$$\sin\theta=\frac{y}{r}=-\frac{1}{\sqrt{10}}=-\frac{\sqrt{10}}{10} \qquad \cos\theta=\frac{x}{r}=-\frac{3}{\sqrt{10}}=-\frac{3\sqrt{10}}{10} \qquad \tan\theta=\frac{y}{x}=\frac{1}{3}$$
$$\csc\theta=\frac{r}{y}=-\sqrt{10}=-\frac{\sqrt{10}}{10} \qquad \sec\theta=\frac{r}{x}=-\frac{\sqrt{10}}{3} \qquad \cot\theta=\frac{x}{y}=3$$

19. Since $y\le r$ we must have $\frac{y}{r}\le 1$. Since $\sin\theta=\frac{y}{r}$, it follows that $\sin\theta\le 1$.

20. If $\sin\theta=\frac{1}{2}$, we can choose $y=1$ and $r=2$. To find x, use $x^2+y^2=r^2$:

$$x^2+1^2=2^2$$
$$x^2+1=4$$
$$x^2=3$$
$$x=\pm\sqrt{3}$$

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Since θ terminates in quadrant II, $x < 0$ and thus $x = -\sqrt{3}$. Using $x = -\sqrt{3}$, $y = 1$, and $r = 2$ in the definitions:

$$\cos\theta = \frac{x}{r} = -\frac{\sqrt{3}}{2} \qquad \sec\theta = \frac{r}{x} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3} \qquad \csc\theta = \frac{r}{y} = 2$$

$$\tan\theta = \frac{y}{x} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3} \qquad \cot\theta = \frac{x}{y} = -\sqrt{3}$$

21. Let $x = 1$ and $y = -2$. Now find $r = \sqrt{1^2 + (-2)^2} = \sqrt{1+4} = \sqrt{5}$. Now applying the definitions for $\sin\theta$ and $\cos\theta$:

$$\sin\theta = \frac{y}{r} = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5} \qquad \cos\theta = \frac{x}{r} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

22. Using the ratio identity: $\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{-4/7} = -\dfrac{7}{4}$

23. Simplifying: $\sin^3\theta = (\sin\theta)^3 = \left(\dfrac{1}{3}\right)^3 = \dfrac{1}{27}$

24. Since $\sec\theta = 3$, $\cos\theta = \dfrac{1}{3}$. We can choose $x = 1$ and $r = 3$. To find y, use $x^2 + y^2 = r^2$:

$$1^2 + y^2 = 3^2$$
$$1 + y^2 = 9$$
$$y^2 = 8$$
$$y = \pm\sqrt{8} = \pm 2\sqrt{2}$$

Since θ terminates in quadrant IV, $y < 0$ and thus $y = -2\sqrt{2}$. Therefore:

$$\sin\theta = \frac{y}{r} = -\frac{2\sqrt{2}}{3} \qquad \tan\theta = \frac{y}{x} = -\frac{2\sqrt{2}}{1} = -2\sqrt{2}$$

25. Multiplying the expressions: $(\cos\theta - \sin\theta)(\cos\theta - \sin\theta) = \cos^2\theta - \sin\theta\cos\theta - \sin\theta\cos\theta + \sin^2\theta = 1 - 2\sin\theta\cos\theta$

26. Subtracting the fractions: $\dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1}{\sin\theta} - \dfrac{\sin\theta}{1} \cdot \dfrac{\sin\theta}{\sin\theta} = \dfrac{1 - \sin^2\theta}{\sin\theta} = \dfrac{\cos^2\theta}{\sin\theta}$

27. Substituting $x = 2\sin\theta$: $\sqrt{4 - x^2} = \sqrt{4 - (2\sin\theta)^2} = \sqrt{4 - 4\sin^2\theta} = \sqrt{4(1 - \sin^2\theta)} = \sqrt{4\cos^2\theta} = 2|\cos\theta|$

28. Working from the left side: $\dfrac{\cot\theta}{\csc\theta} = \dfrac{\cos\theta/\sin\theta}{1/\sin\theta} = \dfrac{\cos\theta}{\sin\theta} \cdot \dfrac{\sin\theta}{1} = \cos\theta$

29. Working from the left side:

$$\begin{aligned}
\cot\theta + \tan\theta &= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} \\
&= \frac{\cos\theta}{\sin\theta} \cdot \frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta} \cdot \frac{\sin\theta}{\sin\theta} \\
&= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta} \\
&= \frac{1}{\sin\theta\cos\theta} \\
&= \frac{1}{\sin\theta} \cdot \frac{1}{\cos\theta} \\
&= \csc\theta\sec\theta
\end{aligned}$$

30. Working from the left side: $(1 - \sin\theta)(1 + \sin\theta) = 1 - \sin\theta + \sin\theta - \sin^2\theta = 1 - \sin^2\theta = \cos^2\theta$

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Chapter 2
Right Triangle Trigonometry

2.1 Definition II: Right Triangle Trigonometry

EVEN SOLUTIONS

2. Using Definition II and Figure 8, we would refer to a as the side opposite A, b as the side adjacent to A, and c as the hypotenuse.

4. **a**. cosine (ii) **b**. cosecant (iii) **c**. cotangent (i)

6. Using the Pythagorean Theorem, first find a:

$$a^2 + 8^2 = 17^2$$
$$a^2 + 64 = 289$$
$$a^2 = 225$$
$$a = 15$$

Using $a = 15$, $b = 8$, and $c = 17$, write the six trigonometric functions of A:

$$\sin A = \frac{a}{c} = \frac{15}{17} \qquad \cos A = \frac{b}{c} = \frac{8}{17} \qquad \tan A = \frac{a}{b} = \frac{15}{8}$$
$$\csc A = \frac{c}{a} = \frac{17}{15} \qquad \sec A = \frac{c}{b} = \frac{17}{8} \qquad \cot A = \frac{b}{a} = \frac{8}{15}$$

8. Using the Pythagorean Theorem, first find c:

$$5^2 + 2^2 = c^2$$
$$25 + 4 = c^2$$
$$c^2 = 29$$
$$c = \sqrt{29}$$

Using $a = 5$, $b = 2$, and $c = \sqrt{29}$, write the six trigonometric functions of A:

$$\sin A = \frac{a}{c} = \frac{5}{\sqrt{29}} = \frac{5\sqrt{29}}{29} \qquad \cos A = \frac{b}{c} = \frac{2}{\sqrt{29}} = \frac{2\sqrt{29}}{29} \qquad \tan A = \frac{a}{b} = \frac{5}{2}$$
$$\csc A = \frac{c}{a} = \frac{\sqrt{29}}{5} \qquad \sec A = \frac{c}{b} = \frac{\sqrt{29}}{2} \qquad \cot A = \frac{b}{a} = \frac{2}{5}$$

10. Using the Pythagorean Theorem, first find c:

$$5^2 + \left(\sqrt{11}\right)^2 = c^2$$
$$25 + 11 = c^2$$
$$c^2 = 36$$
$$c = 6$$

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Using $a = 5$, $b = \sqrt{11}$, and $c = 6$, write the six trigonometric functions of A:

$$\sin A = \frac{a}{c} = \frac{5}{6} \qquad \cos A = \frac{b}{c} = \frac{\sqrt{11}}{6} \qquad \tan A = \frac{a}{b} = \frac{5}{\sqrt{11}} = \frac{5\sqrt{11}}{11}$$

$$\csc A = \frac{c}{a} = \frac{6}{5} \qquad \sec A = \frac{c}{b} = \frac{6}{\sqrt{11}} = \frac{6\sqrt{11}}{11} \qquad \cot A = \frac{b}{a} = \frac{\sqrt{11}}{5}$$

12. Using the Pythagorean Theorem, first find a:

$$a^2 + 3^2 = 4^2$$
$$a^2 + 9 = 16$$
$$a^2 = 7$$
$$a = \sqrt{7}$$

Using $a = \sqrt{7}$, $b = 3$, and $c = 4$, find the three trigonometric functions of A:

$$\sin A = \frac{a}{c} = \frac{\sqrt{7}}{4} \qquad \cos A = \frac{b}{c} = \frac{3}{4} \qquad \tan A = \frac{a}{b} = \frac{\sqrt{7}}{3}$$

Now use the Cofunction Theorem to find the three trigonometric functions of B:

$$\sin B = \cos A = \frac{3}{4} \qquad \cos B = \sin A = \frac{\sqrt{7}}{4} \qquad \tan B = \cot A = \frac{b}{a} = \frac{3}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$$

14. Using the Pythagorean Theorem, first find c:

$$3^2 + 1^2 = c^2$$
$$9 + 1 = c^2$$
$$c^2 = 10$$
$$c = \sqrt{10}$$

Using $a = 3$, $b = 1$, and $c = \sqrt{10}$, find the three trigonometric functions of A:

$$\sin A = \frac{a}{c} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10} \qquad \cos A = \frac{b}{c} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10} \qquad \tan A = \frac{a}{b} = \frac{3}{1} = 3$$

Now use the Cofunction Theorem to find the three trigonometric functions of B:

$$\sin B = \cos A = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10} \qquad \cos B = \sin A = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10} \qquad \tan B = \cot A = \frac{b}{a} = \frac{1}{3}$$

16. Using the Pythagorean Theorem, first find c:

$$1^2 + \left(\sqrt{5}\right)^2 = c^2$$
$$1 + 5 = c^2$$
$$c^2 = 6$$
$$c = \sqrt{6}$$

Using $a = 1$, $b = \sqrt{5}$, and $c = \sqrt{6}$, find the three trigonometric functions of A:

$$\sin A = \frac{a}{c} = \frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6} \qquad \cos A = \frac{b}{c} = \frac{\sqrt{5}}{\sqrt{6}} = \frac{\sqrt{30}}{6} \qquad \tan A = \frac{a}{b} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

Now use the Cofunction Theorem to find the three trigonometric functions of B:

$$\sin B = \cos A = \frac{\sqrt{5}}{\sqrt{6}} = \frac{\sqrt{30}}{6} \qquad \cos B = \sin A = \frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6} \qquad \tan B = \cot A = \frac{b}{a} = \sqrt{5}$$

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18. Using the Pythagorean Theorem, first find c:

$$x^2 + x^2 = c^2$$
$$c^2 = 2x^2$$
$$c = \sqrt{2}\,x$$

Using $a = x$, $b = x$, and $c = \sqrt{2}\,x$, find the three trigonometric functions of A:

$$\sin A = \frac{a}{c} = \frac{x}{\sqrt{2}\,x} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \cos A = \frac{b}{c} = \frac{x}{\sqrt{2}\,x} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \tan A = \frac{a}{b} = \frac{x}{x} = 1$$

Now use the Cofunction Theorem to find the three trigonometric functions of B:

$$\sin B = \cos A = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \cos B = \sin A = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \tan B = \cot A = \frac{b}{a} = \frac{x}{x} = 1$$

20. The coordinates of point B are $B(8, 6)$. Using the Pythagorean Theorem, first find c:

$$6^2 + 8^2 = c^2$$
$$36 + 64 = c^2$$
$$c^2 = 100$$
$$c = 10$$

Using $a = 6$, $b = 8$, and $c = 10$, find the three trigonometric functions of A:

$$\sin A = \frac{a}{c} = \frac{6}{10} = \frac{3}{5} \qquad \cos A = \frac{b}{c} = \frac{8}{10} = \frac{4}{5} \qquad \tan A = \frac{a}{b} = \frac{6}{8} = \frac{3}{4}$$

22. Since $b \le c$, $\frac{c}{b} \ge 1$. Since $\sec\theta = \frac{c}{b} \ge 1$, it is impossible for $\sec\theta = \frac{1}{2}$.

24. Since $b \le c$, $\frac{c}{b} \ge 1$ and can be as large as possible. Since $\sec\theta = \frac{c}{b}$, $\sec\theta$ can be as large as possible.

26. Using the Cofunction Theorem, $\cos 70° = \sin 20°$.

28. Using the Cofunction Theorem, $\cot 22° = \tan 68°$.

30. Using the Cofunction Theorem, $\csc y = \sec(90° - y)$.

32. Using the Cofunction Theorem, $\sin(90° - y) = \cos y$.

34. Complete the table, using the ratio identity $\sec x = \frac{1}{\cos x}$:

x	$\cos x$	$\sec x$
0°	1	1
30°	$\frac{\sqrt{3}}{2}$	$\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{2}{\sqrt{2}} = \sqrt{2}$
60°	$\frac{1}{2}$	2
90°	0	undefined

36. Simplifying the expression: $5\sin^2 60° = 5\left(\frac{\sqrt{3}}{2}\right)^2 = 5 \cdot \frac{3}{4} = \frac{15}{4}$

38. Simplifying the expression: $\cos^3 60° = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

40. Simplifying the expression: $(\sin 60° + \cos 60°)^2 = \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)^2 = \left(\frac{\sqrt{3}+1}{2}\right)^2 = \frac{4 + 2\sqrt{3}}{4} = \frac{2 + \sqrt{3}}{2}$

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42. Simplifying the expression: $(\sin 45° - \cos 45°)^2 = \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)^2 = 0^2 = 0$

44. Simplifying the expression: $\tan^2 45° + \tan^2 60° = 1^2 + \left(\sqrt{3}\right)^2 = 1 + 3 = 4$

46. Simplifying the expression: $6\cos x = 6\cos 30° = 6 \cdot \frac{\sqrt{3}}{2} = 3\sqrt{3}$

48. Simplifying the expression: $-2\sin(90° - y) = -2\sin(90° - 45°) = -2\sin 45° = -2 \cdot \frac{\sqrt{2}}{2} = -\sqrt{2}$

50. Simplifying the expression: $5\sin 2y = 5\sin(2 \cdot 45°) = 5\sin 90° = 5 \cdot 1 = 5$

52. Simplifying the expression: $2\cos(90° - z) = 2\cos(90° - 60°) = 2\cos 30° = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$

54. Finding the exact value: $\csc 30° = \frac{1}{\sin 30°} = \frac{1}{1/2} = 2$

56. Finding the exact value: $\sec 60° = \frac{1}{\cos 60°} = \frac{1}{1/2} = 2$

58. Finding the exact value: $\cot 30° = \frac{\cos 30°}{\sin 30°} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$

60. Finding the exact value: $\csc 45° = \frac{1}{\sin 45°} = \frac{1}{1/\sqrt{2}} = \sqrt{2}$

62. Finding the exact value: $\sec 90° = \frac{1}{\cos 90°} = \frac{1}{0}$, which is undefined

64. Finding the exact value: $\cot 0° = \frac{\cos 0°}{\sin 0°} = \frac{1}{0}$, which is undefined

66. First find a using the Pythagorean Theorem:

$$3.68^2 + b^2 = 5.93^2$$
$$b^2 = 5.93^2 - 3.68^2$$
$$b^2 = 21.6225$$
$$b = 4.65$$

Now find $\sin A$ and $\cos A$:

$$\sin A = \frac{a}{c} = \frac{3.68}{5.93} \approx 0.62 \qquad \cos A = \frac{b}{c} = \frac{4.65}{5.93} \approx 0.78$$

Using the Cofunction Theorem:

$$\sin B = \cos A \approx 0.78 \qquad \cos B = \sin A \approx 0.62$$

68. First find c using the Pythagorean Theorem:

$$13.64^2 + 4.77^2 = c^2$$
$$c^2 = 208.8025$$
$$c = 14.45$$

Now find $\sin A$ and $\cos A$:

$$\sin A = \frac{a}{c} = \frac{13.64}{14.45} \approx 0.94 \qquad \cos A = \frac{b}{c} = \frac{4.77}{14.45} \approx 0.33$$

Using the Cofunction Theorem:

$$\sin B = \cos A \approx 0.33 \qquad \cos B = \sin A \approx 0.94$$

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70. Since $CG = CD = 3$, using the Pythagorean Theorem:

$$(CG)^2 + (CD)^2 = (DG)^2$$
$$3^2 + 3^2 = (DG)^2$$
$$9 + 9 = (DG)^2$$
$$(DG)^2 = 18$$
$$DG = \sqrt{18} = 3\sqrt{2}$$

Now use the Pythagorean Theorem with ΔDGE :

$$(DG)^2 + (GE)^2 = (DE)^2$$
$$\left(3\sqrt{2}\right)^2 + 3^2 = (DE)^2$$
$$18 + 9 = (DE)^2$$
$$(DE)^2 = 27$$
$$DE = \sqrt{27} = 3\sqrt{3}$$

Now, let θ represent the angle formed by diagonals *DE* and *DG*. Therefore:

$$\sin\theta = \frac{GE}{DE} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \qquad \cos\theta = \frac{DG}{DE} = \frac{3\sqrt{2}}{3\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

72. Let $CG = CD = x$, using the Pythagorean Theorem:

$$(CG)^2 + (CD)^2 = (DG)^2$$
$$x^2 + x^2 = (DG)^2$$
$$(DG)^2 = 2x^2$$
$$DG = \sqrt{2x^2} = \sqrt{2}\,x$$

Now use the Pythagorean Theorem with ΔDGE :

$$(DG)^2 + (GE)^2 = (DE)^2$$
$$\left(\sqrt{2}\,x\right)^2 + x^2 = (DE)^2$$
$$2x^2 + x^2 = (DE)^2$$
$$(DE)^2 = 3x^2$$
$$DE = \sqrt{3x^2} = \sqrt{3}\,x$$

Now, let θ represent the angle formed by diagonals *DE* and *DG*. Therefore:

$$\sin\theta = \frac{GE}{DE} = \frac{x}{\sqrt{3}\,x} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \qquad \cos\theta = \frac{DG}{DE} = \frac{\sqrt{2}\,x}{\sqrt{3}\,x} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

74. Using the distance formula:

$$\sqrt{(x-1)^2 + (2-5)^2} = \left(\sqrt{13}\right)^2$$
$$(x-1)^2 + 9 = 13$$
$$(x-1)^2 = 4$$
$$x - 1 = -2, 2$$
$$x = -1, 3$$

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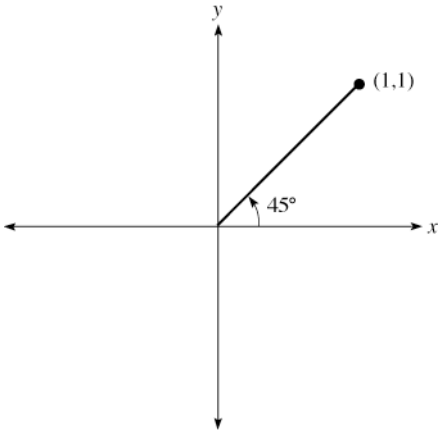
76. A point on the terminal side is $(1,1)$. Drawing the angle in standard position:



78. A coterminal angle to –210° is 150°.

80. Since sin 35° = cos (90° – 35°) = cos 55°, the correct answer is d.

82. Simplifying the expression: $4\cos^2 30° + 2\sin 30° = 4\left(\frac{\sqrt{3}}{2}\right)^2 + 2\left(\frac{1}{2}\right) = 4\cdot\frac{3}{4} + 1 = 3 + 1 = 4$. The correct answer is c.

ODD SOLUTIONS

1. triangle measure

3. complement

5.
$a = \sqrt{c^2 - b^2}$ Pythagorean Theorem
$= \sqrt{(5)^2 - (3)^2}$ Substitute known values
$= \sqrt{25 - 9}$ Simplify
$= \sqrt{16} = 4$

$\sin A = \frac{a}{c} = \frac{4}{5}$ $\quad \cot A = \frac{b}{a} = \frac{3}{4}$

$\cos A = \frac{b}{c} = \frac{3}{5}$ $\quad \sec A - \frac{c}{b} = \frac{5}{3}$

$\tan A = \frac{a}{b} = \frac{4}{3}$ $\quad \csc A = \frac{c}{a} = \frac{5}{4}$

7.
$c = \sqrt{a^2 + b^2}$ Pythagorean Theorem
$= \sqrt{(2)^2 + (1)^2}$ Substitute known values
$= \sqrt{4 + 1}$ Simplify
$= \sqrt{5}$

$\sin A = \frac{a}{c} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ $\quad \cot A = \frac{b}{a} = \frac{1}{2}$

$\cos A = \frac{b}{c} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ $\quad \sec A = \frac{c}{b} = \frac{\sqrt{5}}{1}$

$\tan A = \frac{a}{b} = \frac{2}{1} = 2$ $\quad \csc A = \frac{c}{a} = \frac{\sqrt{5}}{2}$

9.
$c = \sqrt{a^2 + b^2}$ Pythagorean Theorem
$= \sqrt{(2)^2 + (\sqrt{5})^2}$ Substitute known values
$= \sqrt{4 + 5}$ Simplify
$= \sqrt{9} = 3$

$\sin A = \frac{a}{c} = \frac{2}{3}$ $\quad \cot A = \frac{b}{a} = \frac{\sqrt{5}}{2}$

$\cos A = \frac{b}{c} = \frac{\sqrt{5}}{3}$ $\quad \sec A = \frac{c}{b} = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$

$\tan A = \frac{a}{b} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ $\quad \csc A = \frac{c}{a} = \frac{3}{2}$

11.
$b = \sqrt{c^2 - a^2}$ Pythagorean Theorem
$= \sqrt{(6)^2 - (5)^2}$ Substitute known values
$= \sqrt{36 - 25}$ Simplify
$= \sqrt{11}$

$\sin A = \frac{a}{c} = \frac{5}{6}$ $\quad \sin B = \frac{b}{c} = \frac{\sqrt{11}}{6}$

$\cos A = \frac{b}{c} = \frac{\sqrt{11}}{6}$ $\quad \cos B = \frac{a}{c} = \frac{5}{6}$

$\tan A = \frac{a}{b} = \frac{5}{\sqrt{11}} = \frac{5\sqrt{11}}{11}$ $\quad \tan B = \frac{b}{a} = \frac{\sqrt{11}}{5}$

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13. $c = \sqrt{a^2 + b^2}$ Pythagorean Theorem

$= \sqrt{(1)^2 + (1)^2}$ Substitute known values

$= \sqrt{1+1}$ Simplify

$= \sqrt{2}$

$\sin A = \frac{a}{c} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\sin B = \frac{b}{c} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos A = \frac{b}{c} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\cos B = \frac{a}{c} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\tan A = \frac{a}{b} = \frac{1}{1} = 1$ $\tan B = \frac{b}{a} = \frac{1}{1} = 1$

15. $b = \sqrt{c^2 - a^2}$ Pythagorean Theorem

$= \sqrt{10^2 - 6^2}$ Substitute known values

$= \sqrt{100 - 36}$ Simplify

$= \sqrt{64} = 8$

$\sin A = \frac{a}{c} = \frac{6}{10} = \frac{3}{5}$ $\sin B = \frac{b}{c} = \frac{8}{10} = \frac{4}{5}$

$\cos A = \frac{b}{c} = \frac{8}{10} = \frac{4}{5}$ $\cos B = \frac{a}{c} = \frac{6}{10} = \frac{3}{5}$

$\tan A = \frac{a}{b} = \frac{6}{8} = \frac{3}{4}$ $\tan B = \frac{b}{a} = \frac{8}{6} = \frac{4}{3}$

17. $a = \sqrt{c^2 - b^2}$ Pythagorean Theorem

$= \sqrt{(2x)^2 - (x)^2}$ Substitute known values

$= \sqrt{4x^2 - x^2}$ Simplify

$= \sqrt{3x^2}$

$= x\sqrt{3}$

$\sin A = \frac{a}{c} = \frac{x\sqrt{3}}{2x} = \frac{\sqrt{3}}{2}$ $\sin B = \frac{b}{c} = \frac{x}{2x} = \frac{1}{2}$

$\cos A = \frac{b}{c} = \frac{x}{2x} = \frac{1}{2}$ $\cos B = \frac{a}{c} = \frac{x\sqrt{3}}{2x} = \frac{\sqrt{3}}{2}$

$\tan A = \frac{a}{b} = \frac{x\sqrt{3}}{x} = \sqrt{3}$ $\tan B = \frac{b}{a} = \frac{x}{x\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

19. The coordinates of B are (4, 3).

$a = 3, \quad b = 4, \quad c = 5$

$\sin A = \frac{a}{c} = \frac{3}{5}$

$\cos A = \frac{b}{c} = \frac{4}{5}$

$\tan A = \frac{a}{b} = \frac{3}{4}$

21. $\cos\theta = \frac{\text{adj side}}{\text{hyp}} = \frac{3}{1}$ For this to be true, the adjacent side would have to be three times larger than the hypotenuse. This is impossible since the hypotenuse is the longest side of a right triangle.

23. $\tan\theta = \frac{\text{opp side}}{\text{adj side}}$ The opposite and adjacent sides can be any number greater than 0. If we choose a very large number for the opposite side and a very small number for the adjacent side, the ratio will approach infinity.

25. $\sin 10^\circ = \cos(90^\circ - 10^\circ) = \cos 80^\circ$

27. $\tan 8^\circ = \cot(90^\circ - 8^\circ) = \cot 82^\circ$

29. $\sin x^\circ = \cos(90^\circ - x^\circ)$

31. $\tan(90^\circ - x^\circ) = \cot x^\circ$

33. $\csc x = \frac{1}{\sin x}$

$\csc 0^\circ = \frac{1}{0}$ undefined

$\csc 30^\circ = \frac{1}{1/2} = 2$

$\csc 45^\circ = \frac{1}{1/\sqrt{2}} = \sqrt{2}$

$\csc 60^\circ = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\csc 90^\circ = \frac{1}{1} = 1$

35. $4 \sin 30^\circ = 4\left(\frac{1}{2}\right) = 2$

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37. $\left(2\cos 30^\circ\right)^2 = \left[2\left(\frac{\sqrt{3}}{2}\right)\right]^2 = \left(\sqrt{3}\right)^2 = 3$

39. $\sin^2 60^\circ + \cos^2 60^\circ = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

$= \frac{3}{4} + \frac{1}{4}$

$= \frac{4}{4}$

$= 1$

41. $\sin^2 45^\circ - 2\sin 45^\circ \cos 45^\circ + \cos^2 45^\circ = \left(\frac{\sqrt{2}}{2}\right)^2 - 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)^2$

$= \frac{2}{4} - 2\left(\frac{2}{4}\right) + \frac{2}{4} = 0$

43. $\left(\tan 45^\circ + \tan 60^\circ\right)^2 = \left(1 + \sqrt{3}\right)^2$

$= \left(1 + \sqrt{3}\right)\left(1 + \sqrt{3}\right)$

$= 1 + 2\sqrt{3} + 3 = 4 + 2\sqrt{3}$

45. $2\sin 30^\circ = 2\left(\frac{1}{2}\right)$

$= 1$

47. $4\cos\left(z - 30^\circ\right) = 4\cos\left(60^\circ - 30^\circ\right)$

$= 4\cos 30^\circ$

$= 4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$

49. $-3\sin 2\left(30^\circ\right) = -3\sin 60^\circ$

$= -3\left(\frac{\sqrt{3}}{2}\right)$

$= -\frac{3\sqrt{3}}{2}$

51. $2\cos\left(3x - 45^\circ\right) = 2\cos\left(3 \cdot 30^\circ - 45^\circ\right)$

$= 2\cos\left(90^\circ - 45^\circ\right)$

$= 2\cos 45^\circ$

$= 2 \cdot \frac{\sqrt{2}}{2} = \sqrt{2}$

53. $\sec 30^\circ = \frac{1}{\cos 30^\circ}$ Reciprocal identity

$= \frac{1}{\sqrt{3}/2}$ Substitute exact value from Table 1

$= \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ Division of fractions

55. $\csc 60^\circ = \frac{1}{\sin 60^\circ}$

$= \frac{1}{\sqrt{3}/2}$

$= \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

57. $\cot 45^\circ = \frac{\cos 45^\circ}{\sin 45^\circ}$ Ratio identity

$= \frac{\sqrt{2}/2}{\sqrt{2}/2}$ Substitute values from Table 1

$= 1$ Simplify

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59. $\sec 45^\circ = \dfrac{1}{\cos 45^\circ}$

$= \dfrac{1}{1/\sqrt{2}}$

$= \sqrt{2}$

61. $\cot 60^\circ = \dfrac{\cos 60^\circ}{\sin 60^\circ}$ Ratio identity

$= \dfrac{1/2}{\sqrt{3}/2}$ Substitute values from Table 1

$= \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$ Simplify

63. $\csc 90^\circ = \dfrac{1}{\sin 90^\circ}$ Reciprocal identity

$= \dfrac{1}{1} = 1$ Substitute values and simplify

65. $a = \sqrt{c^2 - b^2}$

$= \sqrt{(9.62)^2 - (8.88)^2}$

$= \sqrt{13.69}$

$= 3.70$

$\sin A = \dfrac{a}{c} = \dfrac{3.70}{9.62} = 0.38$

$\cos A = \dfrac{b}{c} = \dfrac{8.88}{9.62} = 0.92$

$\sin B = \dfrac{b}{c} = \dfrac{8.88}{9.62} = 0.92$

$\cos B = \dfrac{a}{c} = \dfrac{3.70}{9.62} = 0.38$

67. $c = \sqrt{a^2 + b^2}$

$= \sqrt{(19.44)^2 + (5.67)^2}$

$= \sqrt{410.0625}$

$= 20.25$

$\sin A = \dfrac{a}{c} = \dfrac{19.44}{20.25} = 0.96$

$\cos A = \dfrac{b}{c} = \dfrac{5.67}{20.25} = 0.28$

$\sin B = \dfrac{b}{c} = \dfrac{5.67}{20.25} = 0.28$

$\cos B = \dfrac{a}{c} = \dfrac{19.44}{20.25} = 0.96$

69. $CH = \sqrt{(CD^2) + (DH)^2}$

$= \sqrt{5^2 + 5^2}$

$= \sqrt{25 + 25}$

$= \sqrt{50} = 5\sqrt{2}$

$CF = \sqrt{(CH)^2 + (FH)^2}$

$= \sqrt{(5\sqrt{2})^2 + (5)^2}$

$= \sqrt{50 + 25}$

$= \sqrt{75} = 5\sqrt{3}$

$\sin\theta = \dfrac{FH}{CF}$

$= \dfrac{5}{5\sqrt{3}}$

$= \dfrac{\sqrt{3}}{3}$

$\cos\theta = \dfrac{CH}{CF}$

$= \dfrac{5\sqrt{2}}{5\sqrt{3}}$

$= \dfrac{\sqrt{2}}{\sqrt{3}} or \dfrac{\sqrt{6}}{3}$

71. $CH = \sqrt{(CD^2) + (DH)^2}$

$= \sqrt{x^2 + x^2}$

$= \sqrt{2x^2}$

$= x\sqrt{2}$

$CF = \sqrt{(CH)^2 + (FH)^2}$

$= \sqrt{(x\sqrt{2})^2 + x^2}$

$= \sqrt{2x^2 + x^2}$

$= \sqrt{3x^2} = x\sqrt{3}$

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$$\sin\theta = \frac{FH}{CF} \qquad \cos\theta = \frac{CH}{CF}$$
$$= \frac{x}{x\sqrt{3}} \qquad = \frac{x\sqrt{2}}{x\sqrt{3}}$$
$$= \frac{\sqrt{3}}{3} \qquad = \frac{\sqrt{2}}{\sqrt{3}} \text{ or } \frac{\sqrt{6}}{3}$$

73. $r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$= \sqrt{[3-(-1)]^2 + [-2-(-4)]^2}$ Substitute known values

$= \sqrt{4^2 + 2^2}$ Simplify

$= \sqrt{16+4}$

$= \sqrt{20} = 2\sqrt{5}$

75. The terminal side is the line $y = -x$. Some points in quadrant II on the line $y = -x$ are $(-1,1)$, $(-2,2)$, and $(-3,3)$.

77. $-135^\circ + 360^\circ = 225^\circ$

79. $\sin A = \frac{a}{c} = \frac{16}{20} = \frac{4}{5}$ The answer is c.

81. Statement a is false because $\sin 30^\circ = \frac{1}{2}$.

2.2 Calculators and Trigonometric Functions of an Acute Angle

EVEN SOLUTIONS

2. If $\theta = 7.25^\circ$ in decimal degrees, then the 7 represents the number of degrees, the 2 represents the number of tenths of a degree, and the 5 represents the number of hundredths of a degree.

4. On a calculator, the SIN^{-1}, COS^{-1}, and TAN^{-1} keys allow us to find an angle given the value of a trigonometric function.

6. Adding the angles: $11°41' + 32°16' = 43°57'$

8. Adding the angles: $63°38' + 24°52' = 87°90' = 88°30'$

10. Adding the angles: $77°21' + 26°44' = 104°5'$

12. Subtracting the angles: $90° - 62°25' = 89°60' - 62°25' = 27°35'$

14. Subtracting the angles: $180° - 132°39' = 179°60' - 132°39' = 47°21'$

16. Subtracting the angles: $89°38' - 28°58' = 88°98' - 28°58' = 60°40'$

18. Converting to degrees and minutes: $83.6° = 83° + 0.6° = 83° + 0.6(60') = 83°36'$

20. Converting to degrees and minutes: $78.5° = 78° + 0.5° = 78° + 0.5(60') = 78°30'$

22. Converting to degrees and minutes: $43.85° = 43° + 0.85° = 43° + 0.85(60') = 43°51'$

24. Converting to degrees and minutes: $8.3° = 8° + 0.3° = 8° + 0.3(60') = 8°18'$

26. Converting to decimal degrees: $74°54' = 74° + 54' = 74° + \left(\frac{54}{60}\right)^\circ = 74.9°$

28. Converting to decimal degrees: $21°15' = 21° + 15' = 21° + \left(\frac{15}{60}\right)^\circ = 21.25°$

30. Converting to decimal degrees: $39°10' = 39° + 10' = 39° + \left(\frac{10}{60}\right)^\circ \approx 39.17°$

32. Converting to decimal degrees: $78°37' = 78° + 37' = 78° + \left(\frac{37}{60}\right)^\circ = 78.62°$

34. Calculating the value: $\cos 79.2° \approx 0.1874$

36. Calculating the value: $\sin 4° \approx 0.0698$

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38. Calculating the value: $\tan 41.88° \approx 0.8966$

40. Calculating the value: $\cot 29° = \dfrac{1}{\tan 29°} \approx 1.8040$

42. Calculating the value: $\sec 18.7° = \dfrac{1}{\cos 18.7°} \approx 1.0557$

44. Calculating the value: $\csc 77.77° = \dfrac{1}{\sin 77.77°} \approx 1.0232$

46. Calculating the value: $\sin 75°50' = \sin\left(75\dfrac{5}{6}\right)^\circ \approx 0.9696$

48. Calculating the value: $\tan 45°19' = \tan\left(45\dfrac{19}{60}\right)^\circ \approx 1.0111$

50. Calculating the value: $\cos 6°4' = \cos\left(6\dfrac{1}{15}\right)^\circ \approx 0.9944$

52. Calculating the value: $\csc 48°48' = \csc\left(48\dfrac{48}{60}\right)^\circ = \csc 48.8° = \dfrac{1}{\sin 48.8°} \approx 1.3291$

54. Completing the table:

x	$\csc x$	$\sec x$	$\cot x$
0°	error (undefined)	1	error (undefined)
15°	3.8637	1.0353	3.7321
30°	2	1.1547	1.7321
45°	1.4142	1.4142	1
60°	1.1547	2	0.5774
75°	1.0353	3.8637	0.2679
90°	1	error (undefined)	error (undefined)

56. Finding the angle θ: $\theta = \sin^{-1}(0.7139) \approx 45.6°$

58. Finding the angle θ: $\theta = \cos^{-1}(0.0945) \approx 84.6°$

60. Finding the angle θ: $\theta = \tan^{-1}(6.2703) \approx 80.9°$

62. Since $\sec\theta = 8.0101$, $\cos\theta = \dfrac{1}{8.0101}$, so $\theta = \cos^{-1}\left(\dfrac{1}{8.0101}\right) \approx 82.8°$.

64. Since $\csc\theta = 4.2319$, $\sin\theta = \dfrac{1}{4.2319}$, so $\theta = \sin^{-1}\left(\dfrac{1}{4.2319}\right) \approx 13.7°$.

66. Since $\cot\theta = 7.0234$, $\tan\theta = \dfrac{1}{7.0234}$, so $\theta = \tan^{-1}\left(\dfrac{1}{7.0234}\right) \approx 8.1°$.

68. Finding the angle θ: $\theta = \sin^{-1}(0.9459) \approx 71.0672° = 71° + 0.0672(60') = 71°4'$

70. Finding the angle θ: $\theta = \tan^{-1}(2.4652) \approx 67.9202° = 67° + 0.9202(60') = 67°55'$

72. Since $\sec\theta = 1.9102$, $\cos\theta = \dfrac{1}{1.9102}$.

Finding the angle θ: $\theta = \cos^{-1}\left(\dfrac{1}{1.9102}\right) \approx 58.4323° = 58° + 0.4323(60') = 58°26'$

74. Calculating the values: $\sin 13° \approx 0.2250$ and $\cos 77° \approx 0.2250$

76. Calculating the values: $\sec 6.7° \approx 1.0069$ and $\csc 83.3° \approx 1.0069$

78. Calculating the values: $\tan 35°15' = \tan 35.25° \approx 0.7067$ and $\cot 54°45' = \cot 54.75° \approx 0.7067$

80. Calculating the value: $\cos^2 58° + \sin^2 58° = 1$

82. To calculate B, $B = \sin^{-1}(4.321)$, which results in an error message. Since, for any angle B, $\sin B \le 1$, it is impossible to find an angle B such that $\sin B = 4.321$.

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84. To calculate $\cot 0°$, we would find $\tan 0° = 0$ then find the reciprocal. This results in an error message. Since $\frac{1}{0}$ is an undefined value, $\cot 0°$ is undefined.

86. **a**. Completing the table:

x	3°	2.5°	2°	1.5°	1°	0.5°	0°
$\cot x$	19.1	22.9	28.6	38.2	57.3	114.6	undefined

b. Completing the table:

x	0.6°	0.5°	0.4°	0.3°	0.2°	0.1°	0°
$\cot x$	95.5	114.6	143.2	191.0	286.5	573.0	undefined

88. Using $\alpha = 36.597°$ and $h = 5$ in the shadow angle formula:

$$\tan\theta = (\sin 36.597°)(\tan(5 \cdot 15°)) \approx 2.2250$$

$$\theta = \tan^{-1}(2.2250) \approx 65.8°$$

90. First find the value of r: $r = \sqrt{\left(-\sqrt{3}\right)^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = 2$

Finding the three trigonometric functions using $x = -\sqrt{3}$, $y = 1$, and $r = 2$:

$$\sin\theta = \frac{y}{r} = \frac{1}{2} \qquad \cos\theta = \frac{x}{r} = -\frac{\sqrt{3}}{2} \qquad \tan\theta = \frac{y}{x} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

92. Let $(-1,-1)$ be a point on the terminal side of –135°. First find the value of r: $r = \sqrt{1^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$

Finding the three trigonometric functions using $x = -1$, $y = -1$, and $r = \sqrt{2}$:

$$\sin\theta = \frac{y}{r} = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} \qquad \cos\theta = \frac{x}{r} = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} \qquad \tan\theta = \frac{y}{x} = \frac{-1}{-1} = 1$$

94. Since $\tan\theta = -\frac{3}{4}$ and θ terminates in quadrant II (where $x < 0$ and $y > 0$), choose $x = -4$ and $y = 3$. Finding r:

$$r = \sqrt{(-4)^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$$

Finding the remaining trigonometric functions using $x = -4$, $y = 3$, and $r = 5$:

$$\sin\theta = \frac{y}{r} = \frac{3}{5} \qquad \cos\theta = \frac{x}{r} = -\frac{4}{5} \qquad \cot\theta = \frac{x}{y} = -\frac{4}{3}$$

$$\csc\theta = \frac{r}{y} = \frac{5}{3} \qquad \sec\theta = \frac{r}{x} = -\frac{5}{4}$$

96. Since $\sec\theta > 0$, $x > 0$. Thus for $\tan\theta < 0$, we must have $y < 0$. Thus the terminal side of θ lies in quadrant IV.

98. Converting to decimal degrees: $76°36' = 76° + 36' = 76° + \left(\frac{36}{60}\right)° = 76.6°$. The correct answer is b.

100. Since $\cot\theta = x$, $\tan\theta = \frac{1}{x}$. Then $\theta = \tan^{-1}\left(\frac{1}{x}\right)$. The correct answer is a.

ODD SOLUTIONS

1. minutes, seconds

3. value, angle

5.
$$\begin{array}{r} 37^\circ\,45' \\ +26^\circ\,24' \\ \hline 63^\circ\,69' = 64^\circ\,9' \end{array} \quad \text{since } 60' = 1^\circ$$

7.
$$\begin{array}{r} 51^\circ\,55' \\ +37^\circ\,45' \\ \hline 88^\circ\,100' = 89^\circ\,40' \end{array}$$

9.
$$\begin{array}{r} 61^\circ\,33' \\ +45^\circ\,16' \\ \hline 106^\circ\,49' \end{array}$$

11.
$$\begin{array}{rcl} 90^\circ & = & 89^\circ\,60' \\ -34^\circ\,12' & & -34^\circ\,12' \\ \hline & & 55^\circ\,48' \end{array}$$

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13.
$$\begin{array}{rcll} 180^\circ & = & 179^\circ 60' & \text{Change } 1^\circ \text{ to } 60' \\ -120^\circ 17' & & -120^\circ\ 17' & \\ & & 59^\circ 43' & \end{array}$$

15.
$$\begin{array}{rcl} 76^\circ 24' & = & 75^\circ 84' \\ -22^\circ 34' & & -22^\circ 34' \\ & & 53^\circ 50' \end{array}$$

17.
$$\begin{aligned} 35.4^\circ &= 35^\circ + 0.4(60)' \\ &= 35^\circ + 24' \\ &= 35^\circ 24' \end{aligned}$$

19.
$$\begin{aligned} 16.25^\circ &= 16^\circ + 0.25(60)' \\ &= 16^\circ + 15' \\ &= 16^\circ 15' \end{aligned}$$

21.
$$\begin{aligned} 92.55^\circ &= 92^\circ + 0.55(60)' \\ &= 92^\circ + 33' \\ &= 92^\circ 33' \end{aligned}$$

23.
$$\begin{aligned} 19.9^\circ &= 19^\circ + 0.9(60)' \\ &= 19^\circ + 54' \\ &= 19^\circ 54' \end{aligned}$$

25.
$$\begin{aligned} 45^\circ 12' &= 45 + \frac{12}{60} \\ &= 45.2^\circ \end{aligned}$$

27.
$$\begin{aligned} 62^\circ 36' &= 62 + \frac{36}{60} \\ &= 62.6^\circ \end{aligned}$$

29.
$$\begin{aligned} 17^\circ 20' &= 17 + \frac{20}{60} \\ &= 17.33^\circ \end{aligned}$$

31.
$$\begin{aligned} 48^\circ 27' &= 48 + \frac{27}{60} \\ &= 48.45^\circ \end{aligned}$$

33. Scientific Calculator: 27.2 [sin]

Graphing Calculator: [sin] [(] 27.2 [)] [*ENTER*]

Answer to 4 places: 0.4571

35. Scientific Calculator: 18 [cos]

Graphing Calculator: [cos] [(] 18 [)] [*ENTER*]

Answer to 4 places: 0.9511

37. Scientific Calculator: 87.32 [tan]

Graphing Calculator: [tan] [(] 87.32 [)] [*ENTER*]

Answer to 4 places: 21.3634

39.
$$\cot 31^\circ = \frac{1}{\tan 31^\circ}$$

Scientific Calculator: 31 [tan] [$1/x$]

Graphing Calculator: [tan] [(] 31 [)] [x^{-1}] [*ENTER*]

Answer: 1.6643

41.
$$\sec 48.2^\circ = \frac{1}{\cos 48.2^\circ}$$

Scientific Calculator: 48.2 [cos] [$1/x$]

Graphing Calculator: [cos] [(] 48.2 [)] [x^{-1}]

Answer: 1.5003

43.
$$\csc 14.15^\circ = \frac{1}{\sin 14.15^\circ}$$

Scientific Calculator: 14.15 [sin] [$1/x$]

Graphing Calculator: [sin] [(] 14.15 [)] [x^{-1}] [*ENTER*]

Answer: 4.0906

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45. $24^\circ 30' = 24 + \frac{30}{60} = 24.5^\circ$

Scientific Calculator: 24.5 [cos]

Graphing Calculator: [cos] [(] 24.5 [)] [*ENTER*]

Answer: 0.9100

47. $42^\circ 15' = 42 + \frac{15}{60}$

$= 42.25^\circ$

Scientific Calculator: 42.25 [tan]

Graphing Calculator: [tan] [(] 42.25 [)] [*ENTER*]

Answer: 0.9083

49. $56^\circ 40' = 56 + \frac{40}{60} = 56.67^\circ$

Scientific Calculator: 56.67 [sin]

Graphing Calculator: [sin] [(] 56.67 [)] [*ENTER*]

Answer: 0.8355

51. $45^\circ 54' = 45 + \frac{54}{60}$

$= 45.9^\circ$

$\sec 45.9^\circ = \frac{1}{\cos 45.9^\circ}$

Scientific Calculator: 45.9 [cos] [$1/x$]

Graphing Calculator: [cos] [(] 45.9 [)] [x^{-1}] [*ENTER*]

Answer: 1.4370

53. Use your calculator to find the values of the sine, cosine, and tangent of each angle:

x	$\sin x$	$\cos x$	$\tan x$
0°	0	1	0
15°	0.2588	0.9659	0.2679
30°	0.5	0.8660	0.5774
45°	0.7071	0.7071	1
60°	0.8660	0.5	1.7321
75°	0.9659	0.2588	3.7321
90°	1	0	Error (undefined)

55. Scientific Calculator: 0.9770 [*inv*] [cos]

Graphing Calculator: [2*nd*] [cos] [(] 0.9770 [)] [*ENTER*]

Answer: 12.3°

57. Scientific Calculator: 0.6873 [*inv*] [tan]

Graphing Calculator: [2*nd*] [tan] [(] 0.6873 [)] [*ENTER*]

Answer: 34.5°

59. Scientific Calculator: 0.9813 [*inv*] [sin]

Graphing Calculator: [2*nd*] [sin] [(] 0.9813 [)] [*ENTER*]

Answer: 78.9°

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61. $\sec\theta = 1.0191$

$\dfrac{1}{\cos\theta} = 1.0191$

$\cos\theta = \dfrac{1}{1.0191}$

Scientific Calculator: $1 \div 1.0191 = inv \cos$

Graphing Calculator: $2nd \cos (1 \div 1.0191) ENTER$

Answer: 11.1°

63. $\csc\theta = 1.8214$

$\dfrac{1}{\sin\theta} = 1.8214$

$\sin\theta = \dfrac{1}{1.8214}$

Scientific Calculator: $1 \div 1.8214 = inv \sin$

Graphing Calculator: $2nd \sin (1 \div 1.8214) ENTER$

Answer: 33.3°

65. $\cot\theta = 0.6873$

$\dfrac{1}{\tan\theta} = 0.6873$

$\tan\theta = \dfrac{1}{0.6873}$

Scientific Calculator: $1 \div 0.6873 = inv \tan$

Graphing Calculator: $2nd \tan (1 \div 0.6873) ENTER$

Answer: 55.5°

67. Scientific Calculator: 0.4112 $inv \cos$

Answer in decimal degrees is 65.719°

Convert the decimal part to minutes: $0.719 \times 60 =$

Graphing Calculator: $2nd \cos (0.4112) 2nd APPS$ ▫ *DMS* $ENTER$

Answer: $\theta = 65^\circ 43'$

69. $\cot\theta = 5.5764$

$\dfrac{1}{\tan\theta} = 5.5764$

$\tan\theta = \dfrac{1}{5.5764}$

Scientific Calculator: $1 \div 5.5764 = inv \tan$

Answer in decimal degrees is 10.1666°

Convert the decimal part to minutes: $0.1666 \times 60 =$

Graphing Calculator: $2nd \tan (5.5764) 2nd APPS$ ▫ *DMS* $ENTER$

Answer: $\theta = 10^\circ 10'$

71. $\csc\theta = 7.0683$

$\dfrac{1}{\sin\theta} = 7.0683$

$\sin\theta = \dfrac{1}{7.0683}$

Scientific Calculator: $1 \div 7.0683 = inv \sin$

Answer in decimal degrees is 8.1333°

Convert the decimal part to minutes: $0.133 \times 60 =$

Graphing Calculator: $2nd \sin (1 \div 7.0683) 2nd APPS$ ▫ *DMS* $ENTER$

Answer: $\theta = 8^\circ 8'$

73. Scientific Calculator: $23 \sin$ and $67 \cos$

Graphing Calculator: $\sin (23) ENTER$ $\cos (67) ENTER$

Both answers should be 0.3907.

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75. To calculate $\sec 34.5^\circ = \dfrac{1}{\cos 34.5^\circ}$:

Scientific Calculator: 34.5 [cos] [1/x]

Graphing Calculaltor: [cos] [(] 34.5 [)] [x^{-1}] [ENTER]

To calculate $\csc 55.5^\circ = \dfrac{1}{\sin 55.5^\circ}$:

Scientific Calculator: 55.5 [sin] [1/x]

Graphing Calculaltor: [sin] [(] 55.5 [)] [x^{-1}] [ENTER]

Both answers should be 1.2134.

77. Scientific Calculator: 4.5 [tan]

Graphing Calculaltor: [tan] [(] 4.5 [)] [ENTER]

To calculate $\cot 85.5^\circ = \dfrac{1}{\tan 85.5^\circ}$:

Scientific Calculator: 85.5 [tan] [1/x]

Graphing Calculaltor: [tan] [(] 85.5 [)] [x^{-1}] [ENTER]

Both answers should be 0.0787.

79. Scientific Calculator: 37 [cos] [x^2] [+] 37 [sin] [x^2] [=]

Graphing Calculator: [cos] [(] 37 [)] [x^2] [+] [sin] [(] 37 [)] [x^2] [ENTER]

Answer should be 1.

81. Scientific Calculator: 1.234 [inv] [sin]

Graphing Calculator: [2nd] [sin] 1.234 [ENTER]

You should get an error message. The sine of an angle can never be greater than 1.

83. Scientific Calculator: 90 [tan]

Graphing Calculaltor: [tan] [(] 90 [)] [ENTER]

You should get an error message. The tangent of 90° is undefined.

87. $\tan\theta = \sin\alpha \tan\left(h \cdot 15^\circ\right)$ where $\alpha = 35.282^\circ$ and $h = 2$

$$\tan\theta = \sin\left(35.282^\circ\right)\tan\left(2 \cdot 15^\circ\right)$$
$$= .333478$$
$$\theta = \tan^{-1}\left(.333478\right)$$
$$\theta = 18.4^\circ$$

89. $(x, y) = (3, -2)$

$x = 3$ and $y = -2$

$r = \sqrt{3^2 + (-2)^2}$

$= \sqrt{9+4} = \sqrt{13}$

$\sin\theta = \dfrac{y}{r} = \dfrac{-2}{\sqrt{13}} = -\dfrac{2\sqrt{13}}{13}$

$\cos\theta = \dfrac{x}{r} = \dfrac{3}{\sqrt{13}} = \dfrac{3\sqrt{13}}{13}$

$\tan\theta = \dfrac{y}{x} = \dfrac{-2}{3} = -\dfrac{2}{3}$

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91. A point on the terminal side of an angle of 90° in standard position is (0, 1), where $x = 0$, $y = 1$, and $r = 1$.

$$\sin 90^\circ = \frac{y}{r} = \frac{1}{1} = 1$$

$$\cos 90^\circ = \frac{x}{r} = \frac{0}{1} = 0$$

$$\tan 90^\circ = \frac{y}{x} = \frac{1}{0} \text{ is undefined}$$

93. $\cos\theta = -\frac{5}{13}$ and θ is in QIII. In QIII, both x and y are negative.

$$\cos\theta = \frac{x}{r} = \frac{-5}{13}$$

$x = -5$ and $r = 13$

$x^2 + y^2 = r^2$

$(-5)^2 + y^2 = 13^2$

$25 + y^2 = 169$

$y^2 = 144$

$y = \pm 12$

$y = -12$ because θ is in QIII

$$\sin\theta = \frac{y}{r} = \frac{-12}{13} = -\frac{12}{13}$$

$$\tan\theta = \frac{y}{x} = \frac{-12}{-5} = \frac{12}{5}$$

$$\cot\theta = \frac{x}{y} = \frac{-5}{-12} = \frac{5}{12}$$

$$\sec\theta = \frac{r}{x} = \frac{13}{-5} = -\frac{13}{5}$$

$$\csc\theta = \frac{r}{y} = \frac{13}{-12} = -\frac{13}{12}$$

95. The $\sin\theta$ is positive in QI and QII.
The $\cos\theta$ is negative in QII and QIII.
Therefore, θ must lie in QII.

97. $67^\circ 22' = 66^\circ 82'$ Change 1° to $60'$

$$\begin{array}{r} -34^\circ 30' \\ \hline \end{array} \qquad \begin{array}{r} -34^\circ 30' \\ \hline 32^\circ 52' \end{array}$$

The answer is d.

2.3 Solving Right Triangles

EVEN SOLUTIONS

2. If the sides of a triangle are accurate to three significant digits, then angles should be measured to the nearest tenth of a degree, or the nearest ten minutes.

4. In general, round answers so that the number of significant digits in your answer matches the number of significant digits in the least significant number given in the problem.

6. **a**. three
b. three
c. five
d. three

8. **a**. five
b. five
c. five
d. seven

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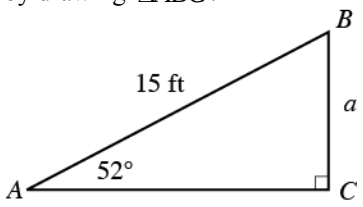
10. Begin by drawing ΔABC:

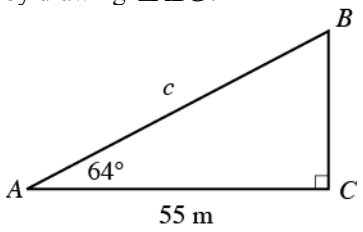


Therefore:

$$\sin 52° = \frac{a}{15}$$

$$a = 15 \sin 52° \approx 12 \text{ ft}$$

12. Begin by drawing ΔABC:



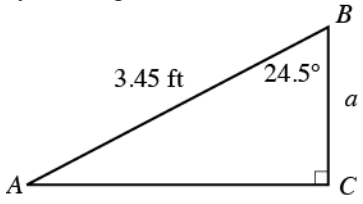
Therefore:

$$\cos 64° = \frac{55}{c}$$

$$c \cos 64° = 55$$

$$c = \frac{55}{\cos 64°} \approx 125 \text{ m} \approx 130 \text{ m}$$

14. Begin by drawing ΔABC:

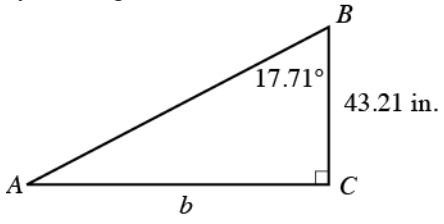


Therefore:

$$\cos 24.5° = \frac{a}{3.45}$$

$$a = 3.45 \cos 24.5° \approx 3.14 \text{ ft}$$

16. Begin by drawing ΔABC:



Therefore:

$$\tan 17.71° = \frac{b}{43.21}$$

$$b = 43.21 \tan 17.71° \approx 13.80 \text{ in.}$$

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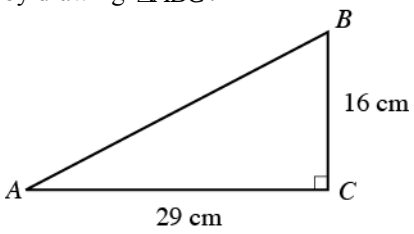
18. Begin by drawing ΔABC:

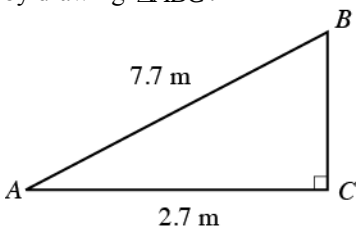


Therefore:

$$\tan A = \frac{16}{29}$$

$$A = \tan^{-1}\left(\frac{16}{29}\right) \approx 29°$$

20. Begin by drawing ΔABC:

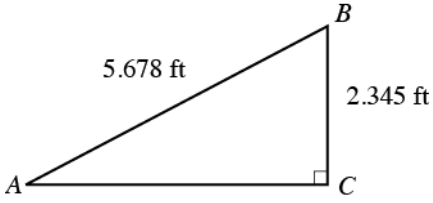


Therefore:

$$\cos A = \frac{2.7}{7.7}$$

$$A = \cos^{-1}\left(\frac{2.7}{7.7}\right) \approx 69°$$

22. Begin by drawing ΔABC:

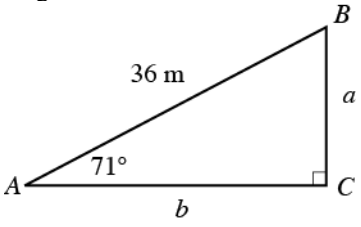


Therefore:

$$\sin A = \frac{2.345}{5.678}$$

$$A = \sin^{-1}\left(\frac{2.345}{5.678}\right) \approx 24.39°$$

24. Begin by drawing ΔABC and label missing information:



Note that $B = 90° - 71° = 19°$. Therefore:

$$\sin 71° = \frac{a}{36} \qquad \cos 71° = \frac{b}{36}$$

$$a = 36 \sin 71° \approx 34 \text{ m} \qquad b = 36 \cos 71° \approx 12 \text{ m}$$

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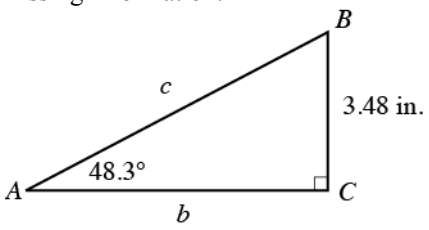
26. Begin by drawing ΔABC and label missing information:



Note that $B = 90° - 48.3° = 41.7°$. Therefore:

$$\sin 48.3° = \frac{3.48}{c}$$

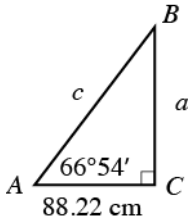
$$c \sin 48.3° = 3.48$$

$$c = \frac{3.48}{\sin 48.3°} \approx 4.66 \text{ in.}$$

$$\cos 48.3° = \frac{b}{4.66}$$

$$b = 4.66 \cos 48.3° \approx 3.10 \text{ in.}$$

28. Begin by drawing ΔABC and label missing information:



Note that $B = 90° - 66°54' = 89°60' - 66°54' = 23°6'$. So:

$$\cos 66°54' = \frac{88.22}{c}$$

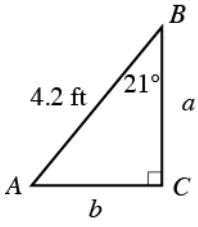
$$c \cos 66°54' = 88.22$$

$$c = \frac{88.22}{\cos 66°54'} \approx 224.9 \text{ cm}$$

$$\tan 66°54' = \frac{a}{88.22}$$

$$a = 88.22 \tan 66°54' \approx 206.8 \text{ cm}$$

30. Begin by drawing ΔABC and label missing information:



Note that $A = 90° - 21° = 69°$. Therefore:

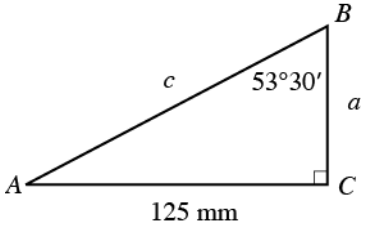
$$\cos 69° = \frac{b}{4.2}$$

$$b = 4.2 \cos 69° \approx 1.5 \text{ ft}$$

$$\sin 69° = \frac{a}{4.2}$$

$$a = 4.2 \sin 69° \approx 3.9 \text{ ft}$$

32. Begin by drawing ΔABC and label missing information:



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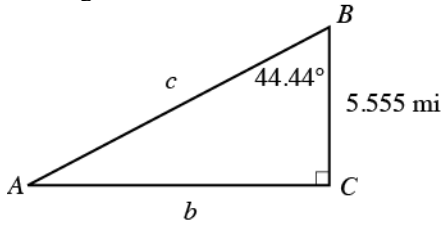
Note that $A = 90° - 53°30' = 89°60' - 53°30' = 36°30'$. So:

$$\cos 36°30' = \frac{125}{c}$$
$$c\cos 36°30' = 125$$
$$c = \frac{125}{\cos 36°30'} \approx 156 \text{ mm}$$

$$\tan 36°30' = \frac{a}{125}$$
$$a = 125\tan 36°30' \approx 92.5 \text{ mm}$$

34. Begin by drawing ΔABC and label missing information:

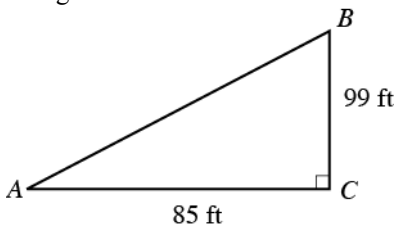


Note that $A = 90° - 44.44° = 45.56°$. Therefore:

$$\sin 45.56° = \frac{5.555}{c}$$
$$c\sin 45.56° = 5.555$$
$$c = \frac{5.555}{\sin 45.56°} \approx 7.780 \text{ mi}$$

$$\tan 45.56° = \frac{5.555}{b}$$
$$b\tan 45.56° = 5.555$$
$$b = \frac{5.555}{\tan 45.56°} \approx 5.447 \text{ mi}$$

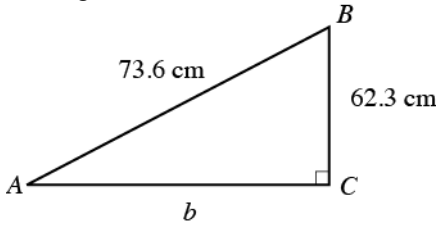
36. Begin by drawing ΔABC and label missing information:



Therefore: $c = \sqrt{85^2 + 99^2} \approx 130 \text{ ft}$. Finding the angles:

$$\tan A = \frac{99}{85}$$
$$A = \tan^{-1}\left(\frac{99}{85}\right) \approx 49°$$
$$B = 90° - 49° = 41°$$

38. Begin by drawing ΔABC and label missing information:



Therefore: $b = \sqrt{73.6^2 - 62.3^2} \approx 39.2 \text{ cm}$. Finding the angles:

$$\sin A = \frac{62.3}{73.6}$$
$$A = \sin^{-1}\left(\frac{62.3}{73.6}\right) \approx 57.8°$$
$$B = 90° - 57.8° = 32.2°$$

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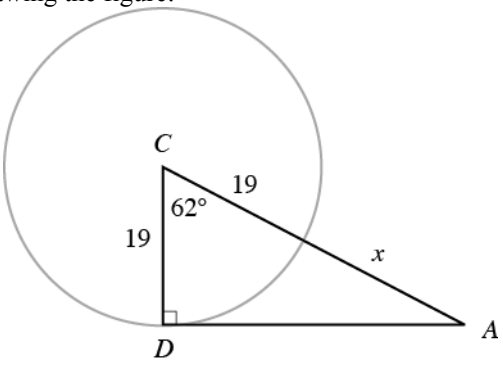
40. Since the right triangle is a 45°–45°–90° triangle, its height is 2.0. Therefore:

$$\tan A = \frac{2.0}{3.0}$$

$$A = \tan^{-1}\left(\frac{2.0}{3.0}\right) \approx 34°$$

42. Re-drawing the figure:



Using the right triangle:

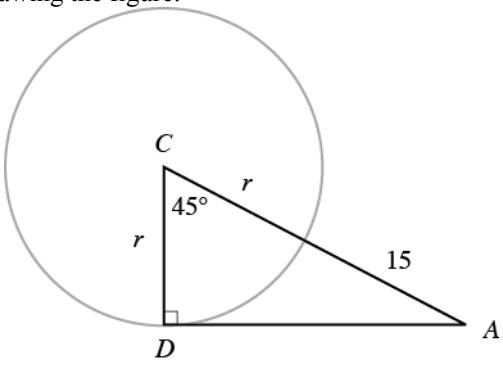
$$\cos 62° = \frac{19}{19+x}$$

$$(19+x)\cos 62° = 19$$

$$19 + x = \frac{19}{\cos 62°}$$

$$x = \frac{19}{\cos 62°} - 19 \approx 21$$

44. Re-drawing the figure:



Using the right triangle:

$$\cos 45° = \frac{r}{r+15}$$

$$(r+15)\cos 45° = r$$

$$r\cos 45° + 15\cos 45° = r$$

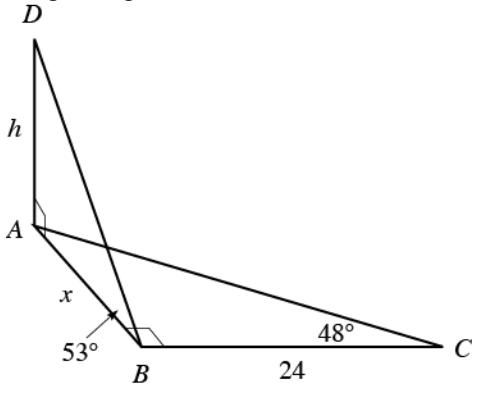
$$15\cos 45° = r - r\cos 45°$$

$$15\cos 45° = r(1-\cos 45°)$$

$$r = \frac{15\cos 45°}{1-\cos 45°}$$

$$r = \frac{15\left(\frac{\sqrt{2}}{2}\right)}{1-\frac{\sqrt{2}}{2}} = \frac{15\sqrt{2}}{2-\sqrt{2}} \approx 36$$

46. Re-drawing the figure:

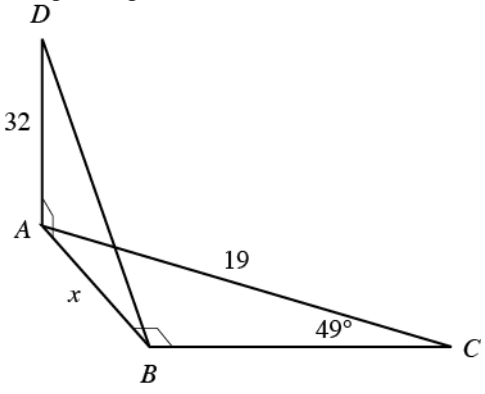


First find x:

$$\tan 48° = \frac{x}{24}$$

$$x = 24\tan 48° \approx 27$$

48. Re-drawing the figure:



First find x:

$$\sin 49° = \frac{x}{19}$$

$$x = 19\sin 49° \approx 14$$

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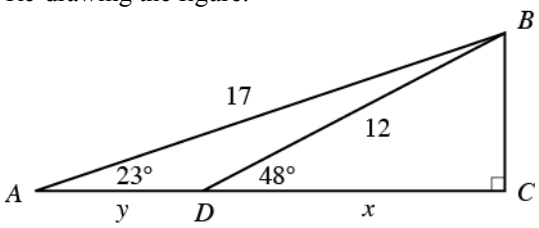
Now find h:

$$\tan 53° = \frac{h}{47}$$
$$h = 27 \tan 53° \approx 35$$

Now find $\angle ABD$:

$$\tan \angle ABD = \frac{32}{14}$$
$$\angle ABD = \tan^{-1}\left(\frac{32}{14}\right) \approx 66°$$

50. Re-drawing the figure:



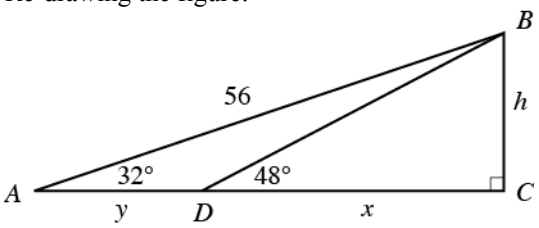
First find x:

$$\cos 48° = \frac{x}{12}$$
$$x = 12 \cos 48° \approx 8.0$$

Now find y:

$$\cos 23° = \frac{8 + y}{17}$$
$$8 + y = 17 \cos 23°$$
$$y = 17 \cos 23° - 8 \approx 7.6$$

52. Re-drawing the figure:



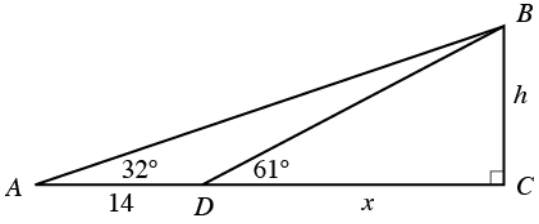
First find h:

$$\sin 32° = \frac{h}{56}$$
$$h = 56 \sin 32° \approx 30$$

Now find x:

$$\tan 48° = \frac{30}{x}$$
$$x \tan 48° = 30$$
$$x = \frac{30}{\tan 48°} \approx 27$$

54. Re-drawing the figure:



Note that $\tan 61° = \frac{h}{x}$, so $h = x \tan 61°$. Also note that $\tan 32° = \frac{h}{14 + x}$, so $h = (14 + x) \tan 32°$. Setting these two expressions equal:

$$x \tan 61° = (14 + x) \tan 32°$$
$$x \tan 61° = 14 \tan 32° + x \tan 32°$$
$$x \tan 61° - x \tan 32° = 14 \tan 32°$$
$$x(\tan 61° - \tan 32°) = 14 \tan 32°$$
$$x = \frac{14 \tan 32°}{\tan 61° - \tan 32°} \approx 7.4$$

56. Since $GC = CD = 3.00$, using the Pythagorean Theorem: $GD = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$. Therefore:

$$\tan \angle GDE = \frac{GE}{GD} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$$
$$\angle GDE = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right) \approx 35.3°$$

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58. First find $\angle CAB$:

$$\tan(\angle CAB) = \frac{66}{54}$$

$$\angle CAB = \tan^{-1}\left(\frac{66}{54}\right) \approx 50.71°$$

Now find $\angle EAB$:

$$\tan(\angle EAB) = \frac{78}{54}$$

$$\angle EAB = \tan^{-1}\left(\frac{78}{54}\right) \approx 55.30°$$

Therefore: $\angle CAE = \angle EAB - \angle CAB = 55.30° - 50.71° \approx 4.6°$

60. Let O represent the center of the goal.

First find $\angle OAD$:

$$\tan(\angle OAD) = \frac{6}{54}$$

$$\angle OAD = \tan^{-1}\left(\frac{6}{54}\right) \approx 6.34°$$

Now find $\angle OAF$:

$$\tan(\angle OAF) = \frac{12}{54}$$

$$\angle OAF = \tan^{-1}\left(\frac{12}{54}\right) \approx 12.53°$$

Therefore: $\angle DAF = \angle OAF - \angle OAD = 12.53° - 6.34° \approx 6.2°$

Since $\angle CAE$ is also 6.2°, the sum of the angles is 12.4°.

62. Since 12.4° is much greater than 4.6°, the chance of scoring is much better from the center than from the corner of the penalty area.

64. From Example 5, we have:

$$\cos 135° = \frac{139 - h}{125}$$

$$-\frac{1}{\sqrt{2}} = \frac{139 - h}{125}$$

$$-\frac{125}{\sqrt{2}} = 139 - h$$

$$h = 139 + \frac{125}{\sqrt{2}} \approx 227.4$$

The rider is approximately 230 ft above the ground.

66. First note that the distance from the ground to the low point of Colossus is 174 – 165 = 9 ft. The radius is 82.5 ft. Since $x + h = 82.5 + 9 = 91.5$, $x = 91.5 - h$. Therefore:

$$\cos\theta = \frac{x}{82.5} = \frac{91.5 - h}{82.5}$$

$$91.5 - h = 82.5\cos\theta$$

$$h = 91.5 - 82.5\cos\theta$$

a. Substituting $\theta = 150°$: $h = 91.5 - 82.5\cos 150° \approx 163$ ft

b. Substituting $\theta = 240°$: $h = 91.5 - 82.5\cos 240° \approx 133$ ft

c. Substituting $\theta = 315°$: $h = 91.5 - 82.5\cos 315° \approx 33.2$ ft

68. First note that the distance from the ground to the low point of the High Roller is 550 – 520 = 30 ft. The radius is 260 ft. Since $x + h = 260 + 30 = 290$, $x = 290 - h$. Therefore:

$$\cos\theta = \frac{x}{260} = \frac{290 - h}{260}$$

$$290 - h = 260\cos\theta$$

$$h = 290 - 260\cos\theta$$

Substituting $\theta = 110°$: $h = 290 - 260\cos 110° \approx 379$ ft ≈ 380 ft

70. Entering the functions $Y_1 = -\frac{7}{640}(X - 80)^2 + 70$ and $Y_2 = \tan^{-1}\left(\frac{Y_1}{X}\right)$, complete the table:

X	10	5	1	0.5	0.1	0.01
Y_1	16.4063	8.4766	1.7391	0.8723	0.1749	0.0175
Y_2	58.6°	59.5°	60.1°	60.2°	60.2°	60.3°

Based on these results, it appears the angle between the cannon and the horizontal is approximately 60.3°.

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72. Since $\csc B = 5$, $\sin B = \frac{1}{5}$, so $\sin^2 B = \left(\frac{1}{5}\right)^2 = \frac{1}{25}$.

74. Finding $\cos^2 A$: $\cos^2 A = 1 - \sin^2 A = 1 - \left(\frac{3}{4}\right)^2 = 1 - \frac{9}{16} = \frac{7}{16}$

So $\cos A = \pm\frac{\sqrt{7}}{4}$. Since A terminates in quadrant II, where $x < 0$, $\cos A < 0$. Thus $\cos A = -\frac{\sqrt{7}}{4}$.

76. First find $\sin\theta$ (note $\sin\theta < 0$ since θ terminates in quadrant IV):

$$\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\frac{1}{5}} = -\sqrt{\frac{4}{5}} = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

Now find the other four trigonometric ratios using $x = 1$, $y = -2$, and $r = \sqrt{5}$:

$$\sec\theta = \frac{r}{x} = \sqrt{5} \qquad \csc\theta = \frac{r}{y} = -\frac{\sqrt{5}}{2} \qquad \tan\theta = \frac{y}{x} = -2 \qquad \cot\theta = \frac{x}{y} = -\frac{1}{2}$$

78. Since $\csc\theta = -2$, $\sin\theta = -\frac{1}{2}$. Now find $\cos\theta$ (note that $\cos\theta < 0$ since θ terminates in quadrant III):

$$\cos\theta = -\sqrt{1-\sin^2\theta} = -\sqrt{1-\left(-\frac{1}{2}\right)^2} = -\sqrt{1-\frac{1}{4}} = -\sqrt{\frac{3}{4}} = -\frac{\sqrt{3}}{2}$$

Now find the other three trigonometric ratios using $x = -\sqrt{3}$, $y = -1$, and $r = 2$:

$$\sec\theta = \frac{r}{x} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3} \qquad \tan\theta = \frac{y}{x} = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \qquad \cot\theta = \frac{x}{y} = \frac{-\sqrt{3}}{-1} = \sqrt{3}$$

80. Finding side b:

$$\cos A = \frac{b}{c}$$

$$\cos 58° = \frac{b}{15}$$

$$b = 15\cos 58° \approx 7.9 \text{ ft}$$

The correct answer is c.

82. Let x represent the height of the rider above the center of the wheel (which is 51.5 feet above the ground). Since the point of the rider is 50° above the horizontal, we have:

$$\sin 50° = \frac{h - 51.5}{45}$$

$$h - 51.5 = 45\sin 50°$$

$$h = 51.5 + 45\sin 50° \approx 86 \text{ ft}$$

The correct answer is c.

ODD SOLUTIONS

1. left, right, first nonzero, not

3. sides, angles

5. **a.** 2 **b.** 3 **c.** 2 **d.** 2

7. **a.** 4 **b.** 6 **c.** 4 **d.** 4

9.

$\cos 42^\circ = \frac{b}{89}$	Cosine relationship
$b = 89\cos 42^\circ$	Multiply both sides by 89
$= 89(0.7431)$	Substitute value for $\cos 42^\circ$
$= 66\,\text{cm}$	Answer rounded to 2 significant digits

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11. $\sin 34^\circ = \dfrac{22}{c}$ Sine relationship

$c \sin 34^\circ = 22$ Multiply both sides by c

$c = \dfrac{22}{\sin 34^\circ}$ Divide both sides by $\sin 34^\circ$

$c = \dfrac{22}{0.5592}$ Substitute value for $\sin 34^\circ$

$c = 39 \text{ m}$ Answer rounded to 2 significant digits

13. $\sin 16.9^\circ = \dfrac{b}{7.55}$ Sine relationship

$b = 7.55 \sin 16.9^\circ$ Multiply both sides by 7.55

$= 7.55(0.2907)$ Substitute value for $\cos 24.5^\circ$

$= 2.19 \text{ cm}$ Answer rounded to 3 significant digits

15. $\tan 55.33^\circ = \dfrac{12.34}{a}$ Tangent relationship

$a \tan 55.33^\circ = 12.34$ Multiply both sides by a

$a = \dfrac{12.34}{\tan 55.33^\circ}$ Divide both sides by $\tan 55.33^\circ$

$a = \dfrac{12.34}{1.4458}$ Substitute value for $\tan 55.33^\circ$

$a = 8.535 \text{ yd}$ Answer rounded to 4 significant digits

17. $\tan B = \dfrac{32.4}{42.3}$ Tangent relationship

$= 0.7659$ Divide

$B = \tan^{-1}(0.7659)$ Use calculator to find angle

$B = 37.5^\circ$ Answer rounded to the nearest tenth of a degree

19. $\sin B = \dfrac{9.8}{12}$ Sine relationship

$= 0.8166$ Divide

$B = \cos^{-1}(0.8166)$ Use calculator to find angle

$= 55^\circ$ Answer rounded to the nearest degree

21. $\cos B = \dfrac{23.32}{45.54}$ Cosine relationship

$= 0.5120$ Divide 23.32 by 45.54

$B = \cos^{-1}(0.5120)$ Use calculator to find angle

$= 59.20^\circ$ Answer rounded to the nearest hundredth of a degree

23. First, we find $\angle B$: $\angle B = 90^\circ - \angle A = 90^\circ - 25^\circ = 65^\circ$

Next, we find side a:

$\sin 25^\circ = \dfrac{a}{24}$ Sine relationship

$a = 24 \sin 25^\circ$ Multiply both sides by 24

$a = 10 \text{ m}$ Answer rounded to 2 significant digits

Last, we find side b:

$\cos 25^\circ = \dfrac{b}{24}$ Cosine relationship

$b = 24 \sin 25^\circ$ Multiply both sides by 24

$b = 22 \text{ m}$ Answer rounded to 2 significant digits

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25. First, we find $\angle B$: $\angle B = 90^\circ - \angle A$

$= 90^\circ - 32.6^\circ = 57.4^\circ$

Next, we find side *c*:

$\sin 32.6^\circ = \dfrac{43.4}{c}$ Sine relationship

$c = \dfrac{43.4}{\sin 32.6^\circ}$ Multiply both sides by *c* then divide by $\sin 32.6^\circ$

$= 80.6 \text{ in}$ Answer rounded to 3 significant digits

Last, we find side *b*:

$\tan 57.4^\circ = \dfrac{b}{43.4}$ Tangent relationship

$b = 43.4 \tan 57.4^\circ$ Multiply both sides by 43.4

$= 67.9 \text{ in}$ Answer rounded to 2 significant digits

27. First, we find $\angle B$: $\angle B = 90^\circ - \angle A$

$= 90^\circ - 10^\circ 42'$

$= 79^\circ 18'$

Next, we find side *a*:

$\tan 10^\circ 42' = \dfrac{a}{5.932}$ Tangent relationship

$\tan 10.7^\circ = \dfrac{a}{5.932}$ Change angle to decimal degrees

$a = 5.932 \tan 10.7^\circ$ Multiply both sides by 5.932

$a = 1.121 \text{ cm}$ Answer rounded to 4 significant digits

Last, we find side *c*:

$\cos 10.7^\circ = \dfrac{5.932}{c}$ Cosine relationship

$c = \dfrac{5.932}{\cos 10.7^\circ}$ Multiply both sides by *c* then divide by $\cos 10.7^\circ$

$c = 6.037 \text{ cm}$ Answer rounded to 4 significant digits

29. First, we find $\angle A$: $\angle A = 90^\circ - 76^\circ$

$= 14^\circ$

Next, we find side *a*:

$\cos 76^\circ = \dfrac{a}{5.8}$ Cosine relationship

$a = 5.8 \cos 76^\circ$ Multiply both sides by 5.8

$= 1.4 \text{ ft}$ Answer rounded to 2 significant digits

Last, we find side *b*:

$\sin 76^\circ = \dfrac{b}{5.8}$ Sine relationship

$b = 5.8 \sin 76^\circ = 5.6 \text{ ft}$ Multiply both sides by 5.8 and round to 2 significant digits

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31. First, we find $\angle A$:

$$\angle A = 90^\circ - \angle B$$
$$= 90^\circ - 26^\circ 30'$$
$$= 63^\circ 30'$$

Next, we find side *a*:

$\tan 26^\circ 30' = \dfrac{324}{a}$ Tangent relationship

$\tan 26.5^\circ = \dfrac{324}{a}$ Change angle to decimal degrees

$a = \dfrac{324}{\tan 26.5^\circ}$ Multiply both sides by *a* then divide by $\tan 26.5^\circ$

$a = 650 \text{ mm}$ Answer rounded to 3 significant digits

Last, we find side *c*:

$\sin 26.5^\circ = \dfrac{324}{c}$ Sine relationship

$c = \dfrac{324}{\sin 26.5^\circ}$ Multiply both sides by *c* then divide by $\sin 26.5^\circ$

$= 726 \text{ mm}$ Answer rounded to 3 significant digits

33. First, we find $\angle A$:

$$\angle A = 90^\circ - 23.45^\circ$$
$$= 66.55^\circ$$

Next, we find side *b*:

$\tan 23.45^\circ = \dfrac{b}{5.432}$ Tangent relationship

$b = 5.432 \tan 23.45^\circ$ Multiply both sides by 5.432

$= 2.356 \text{ mi}$ Answer rounded to 4 significant digits

Last, we find side *c*:

$\cos 23.45^\circ = \dfrac{5.432}{c}$ Cosine relationship

$c = \dfrac{5.432}{\cos 23.45^\circ}$ Multiply both sides by *c* and then divide by $\cos 23.45^\circ$

$= 5.921 \text{ mi}$ Answer rounded to 4 significant digits

35. First, we find $\angle A$:

$\tan A = \dfrac{37}{87}$ Tangent relationship

$= 0.4253$ Divide 37 by 87

$A = \tan^{-1}(0.4253)$ Use calculator to find angle

$= 23^\circ$ Answer rounded to nearest degree

Next, we find $\angle B$:

$\angle B = 90^\circ - \angle A = 90^\circ - 23^\circ = 67^\circ$

Last, we find *c*:

$c^2 = 37^2 + 87^2$ Pythagorean Theorem

$= 1369 + 7569$ Simplify

$= 8938$ Simplify

$c = \pm 95$ Take square root of both sides

$= 95 \text{ ft}$ *c* must be positive

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37. First, we find $\angle A$:

$\cos A = \dfrac{377.3}{588.5}$ Cosine relationship

$= 0.6411$ Divide

$A = \cos^{-1}(0.6411)$ Use calculator to find angle

$= 50.12^\circ$ Answer rounded to nearest hundredth of a degree

Next, we find $\angle B$: $\angle B = 90^\circ - \angle A$

$= 90^\circ - 50.12^\circ$

$= 39.88^\circ$

Last, we find side *a*:

$a^2 + 377.3^2 = 588.5^2$ Pythagorean Theorem

$a^2 = 203{,}976.96$ Subtract and simplify

$a = \pm 451.6$ Take square root of both sides

$= 451.6$ in *a* must be positive

39. Using ΔBCD, we find *BD*:

$\sin 30^\circ = \dfrac{BD}{6.0}$ Sine relationship

$BD = 6.0 \sin 30^\circ$ Multiply both sides by 6

$= 3$ Exact answer

Next, we find $\angle A$

$\sin A = \dfrac{3}{4.0}$ Sine relationship

$= 0.75$ Divide

$A = \sin^{-1}(0.75)$ Use calculator to find angle

$A = 49^\circ$ Answer rounded to the nearest degree

41. $\sin 31^\circ = \dfrac{12}{x+12}$ Sine relationship

$(x+12)\sin 31^\circ = 12$ Multiply both sides by $x + 12$

$x + 12 = \dfrac{12}{\sin 31^\circ}$ Divide both sides by $\sin 31^\circ$

$x = \dfrac{12}{\sin 31^\circ} - 12 = 11$ Subtract 12 from both sides and round to 2 significant digits

43. $\cos 65^\circ = \dfrac{r}{r+22}$ Sine relationship

$r = (r+22)\cos 65^\circ$ Multiply both side by $r + 22$

$r = r\cos 65^\circ + 22\cos 65^\circ$ Use distributive property

$r - r\cos 65^\circ = 22\cos 65^\circ$ Subtract $r \cos 65^\circ$ from both sides

$r(1 - \cos 65^\circ) = 22\cos 65^\circ$ Factor left side

$r = \dfrac{22\cos 65^\circ}{1 - \cos 65^\circ}$ Divide both sides by $1 - \cos 65^\circ$

$= 16$ Answer rounded to 2 significant digits

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45. Using ΔABC, we find side x:

$$\tan 62^\circ = \frac{x}{42}$$ Tangent relationship

$$x = 42 \tan 62^\circ$$ Multiply both sides by 42

$$= 79$$ Answer rounded to 2 significant digits

Next, using ΔABD, we find side h:

$$\tan 27^\circ = \frac{h}{x}$$ Tangent relationship

$$= \frac{h}{79}$$ Substitute value for x

$$h = 79 \tan 27^\circ$$ Multiply both sides by 79

$$h = 40$$ Answer rounded to 2 significant digits

47. Using ΔABC, we find side x:

$$\sin 41^\circ = \frac{x}{32}$$ Sine relationship

$$x = 32 \sin 41^\circ$$ Multiply both sides by 32

$$= 21$$ Answer rounded to 2 significant digits

Next, using ΔABD, we find $\angle ABD$:

$$\tan \angle ABD = \frac{h}{x}$$ Tangent relationship

$$= \frac{19}{21}$$ Substitute known values

$$= 0.9047$$ Divide 19 by 21

$$\angle ABD = \tan^{-1}(0.9047)$$ Use calculator to find angle

$$\angle ABD = 42^\circ$$ Answer rounded to the nearest degree

49. Using ΔBCD, we find side x:

$$\cos 58^\circ = \frac{x}{14}$$ Cosine relationship

$$x = 14 \cos 58^\circ$$ Multiply both sides by 14

$$x = 7.4$$ Answer rounded to 2 significant digits

Next, using ΔABC, we find y:

$$\cos 41^\circ = \frac{x+y}{18}$$ Cosine relationship

$$x + y = 18 \cos 41^\circ$$ Multiply both sides by 18

$$x + y = 13.58$$ Evaluate right side

$$7.4 + y = 13.58$$ Substitute value for x

$$y = 6.18 \approx 6.2$$ Subtract 7.4 from both sides and round to 2 significant digits

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51. Using ΔABC, we find side *h*:

$\sin 41^\circ = \dfrac{h}{28}$ Sine relationship

$h = 28 \sin 41^\circ$ Multiply both sides by 28

$= 18$ Answer rounded to 2 significant digits

Next, using ΔBCD, we find side *x*:

$\tan 58^\circ = \dfrac{h}{x}$ Tangent relationship

$\tan 58^\circ = \dfrac{18}{x}$ Substitute value found for *h*

$x = \dfrac{18}{\tan 58^\circ} = 11$ Solve for *x* and round to 2 significant digits

53. Since *h* is in both ΔABC and ΔBCD, we will solve for *h* in the two triangles:

In ΔBCD, $\tan 57^\circ = \dfrac{h}{x}$ Tangent relationship

$h = x \tan 57^\circ$ Multiply both sides by *x*

In ΔABC, $\tan 43^\circ = \dfrac{h}{x+y}$ Tangent relationship

$h = (x+y)\tan 43^\circ$ Multiply both sides by $x + y$

$h = (x+11)\tan 43^\circ$ Substitute value for *y*

Therefore, $x \tan 57^\circ = (x+11)\tan 43^\circ$ Property of equality

$x \tan 57^\circ = x \tan 43 + 11 \tan 43^\circ$ Distribution Property

$x \tan 57^\circ - x \tan 43^\circ = 11 \tan 43^\circ$ Subtract $x \tan 43^\circ$ from both sides

$x(\tan 57^\circ - \tan 43^\circ) = 11 \tan 43^\circ$ Factor left side

$x = \dfrac{11 \tan 43^\circ}{\tan 57^\circ - \tan 43^\circ} = 17$ Divide both sides by $\tan 57^\circ - \tan 43^\circ$

55. From Problem 69 in Problem Set 2.1, we found that

$\sin\theta = \dfrac{1}{\sqrt{3}}$

$= 0.5774$

$\theta = \sin^{-1}(0.5774) = 35.3^\circ$

57. From Problem 69 in Problem Set 2.1, we found that

$\cos\theta = \dfrac{\sqrt{2}}{\sqrt{3}}$

$= 0.8165$

$\theta = \cos^{-1}(0.8165) = 35.3^\circ$

59. We know that $EC = DF = 6$ ft, $EB = 78$ ft, $CB = 72$ ft, $DB = 60$ ft, and $\angle FAB = 45^\circ$.

$\tan\angle EAB = \dfrac{78}{54}$ $\tan\angle CAB = \dfrac{72}{54}$ $\tan\angle DAB = \dfrac{60}{54}$

$\angle EAB = \tan^{-1}\dfrac{78}{54}$ $\angle CAB = \tan^{-1}\dfrac{72}{54}$ $\angle DAB = \tan^{-1}\dfrac{60}{54}$

$\angle EAB = 55.3^\circ$ $\angle CAB = 53.1^\circ$ $\angle DAB = 48.0^\circ$

$\angle EAC = \angle EAB - \angle CAB$ and $\angle DAF = \angle DAB - \angle FAB$

$= 55.3^\circ - 53.1^\circ = 2.2^\circ$ $= 48.0^\circ - 45^\circ = 3.0^\circ$

Therefore, the sum of the angles is 5.2°.

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63. $\cos 120^\circ = \dfrac{x}{125} = \dfrac{139-h}{125}$

$125\cos 120^\circ = 139 - h$

$h = 139 - 125\cos 120^\circ$ Solve for h

$= 201.5$

$= 200\text{ ft}$ Round to 2 significant digits

65. $r = 98.5$

a. $h = 12 + 98.5 + x$

$\cos 60.0^\circ = \dfrac{x}{98.5}$

$x = 98.5\cos 60.0^\circ$

$= 49.25$

$h = 12 + 98.5 + 49.25$

$= 159.8 \approx 160\text{ ft}$

b. $h = 12 + 98.5 + x$

$\cos 30.0^\circ = \dfrac{x}{98.5}$

$x = 98.5\cos 30.0^\circ$

$= 85.3$

$h = 12 + 98.5 + 85.3$

$= 195.8 \approx 196\text{ ft}$

c. $r + 12 = 98.5 + 12 = 110.5$

$h = 110.5 - x$

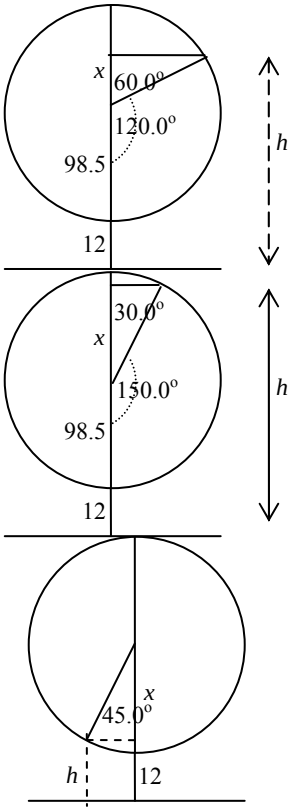
$\cos 45.0^\circ = \dfrac{x}{98.5}$

$x = 98.5\cos 45.0^\circ$

$= 69.7$

$h = 110.5 - 69.7$

$= 40.8\text{ ft}$



67. The radius of the London Eye is $\dfrac{135}{2} = 67.5$.

$\cos\theta = \dfrac{67.5 - 44.5}{67.5}$

$= \dfrac{23}{67.5}$

$\theta = \cos^{-1}(0.6592)$

$\theta = 70.1^\circ$

71. $\sec\theta = 2$

$\cos\theta = \dfrac{1}{\sec\theta}$ Reciprocal identity

$= \dfrac{1}{2}$ Substitute known value

$\cos^2\theta = \left(\dfrac{1}{2}\right)^2 = \dfrac{1}{4}$ Square both sides

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73. $\cos\theta = -\sqrt{1-\sin^2\theta}$ Pythagorean identity, θ in QIII

$= -\sqrt{1-\left(-\frac{2}{3}\right)^2} = -\sqrt{1-\frac{4}{9}}$

$= -\sqrt{\frac{5}{9}} = -\frac{\sqrt{5}}{3}$

75. $\cos\theta = -\sqrt{1-\sin^2\theta}$ Pythagorean identity, θ in QII

$= -\sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}$ Substitute known value

$= -\sqrt{1-\frac{3}{4}}$ Simplify

$= -\sqrt{\frac{1}{4}} = -\frac{1}{2}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ Ratio identity

$= \frac{\sqrt{3}/2}{-1/2}$ Substitute known values

$= -\sqrt{3}$ Simplify

$\csc\theta = \frac{1}{\sin\theta}$ Reciprocal identity

$= \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\sec\theta = \frac{1}{\cos\theta}$ Reciprocal identity

$= \frac{1}{-1/2} = -2$

$\cot\theta = \frac{1}{\tan\theta}$ Reciprocal identity

$= \frac{1}{-\sqrt{3}} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

77. $\cos\theta = \frac{1}{\sec\theta}$ Reciprocal identity

$= -\frac{1}{2}$ Substitute known values

$\sin\theta = -\sqrt{1-\cos^2\theta}$ Pythagorean identity, θ in QIII

$= -\sqrt{1-\left(-\frac{1}{2}\right)^2}$ Substitute value for $\cos\theta$

$= -\sqrt{1-\frac{1}{4}} = -\sqrt{\frac{3}{4}}$ Simplify

$= -\frac{\sqrt{3}}{2}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ Ratio identity

$= \frac{-\sqrt{3}/2}{-1/2}$ Substitute values

$= \sqrt{3}$

$\cot\theta = \frac{1}{\tan\theta}$ Reciprocal identity

$= \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\csc\theta = \frac{1}{\sin\theta}$ Reciprocal identity

$= \frac{1}{-\sqrt{3}/2} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

81. $\tan B = \frac{35}{58}$

$B = 31^\circ$

The answer is b.

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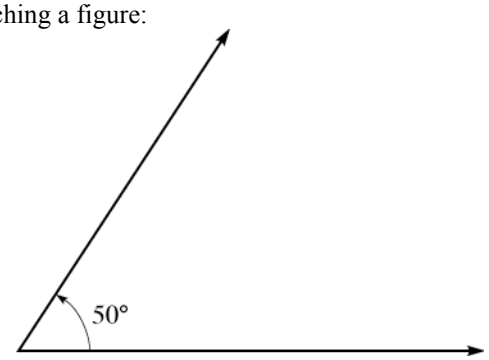
2.4 Applications

EVEN SOLUTIONS

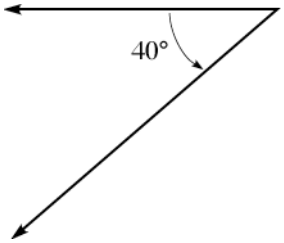
2. If an observer positioned at the vertex of an angle views an object in the direction of the non-horizontal side of the angle, then this side is called the line of sight of the observer.

4. The bearing of a line is always measured as an angle from the north or south rotating toward the east or west.

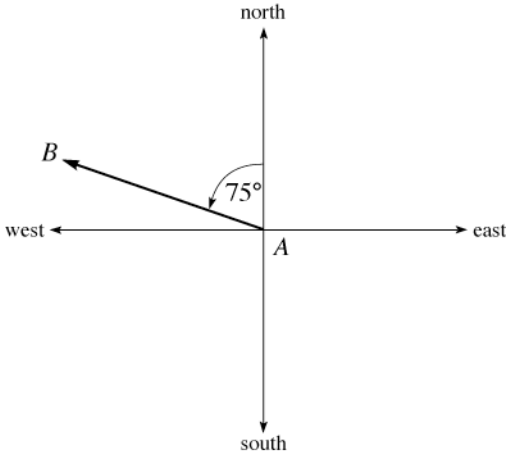
6. Sketching a figure:



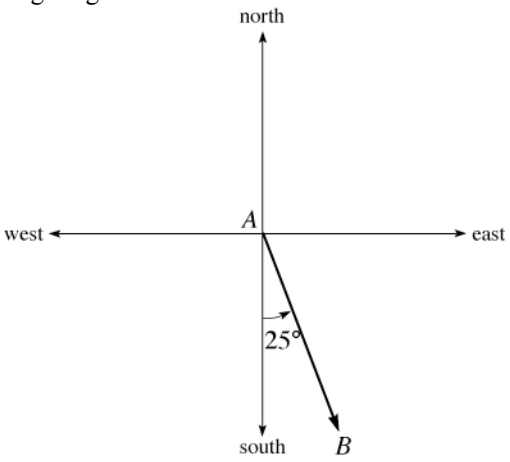
8. Sketching a figure:



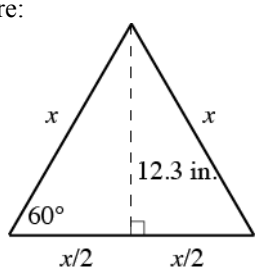
10. Sketching a figure:



12. Sketching a figure:



14. Call x the length of each side. Note the figure:



Therefore:

$$\sin 60° = \frac{12.3}{x}$$

$$x \sin 60° = 12.3$$

$$x = \frac{12.3}{\sin 60°} \approx 14.2$$

The length of each side is 14.2 in.

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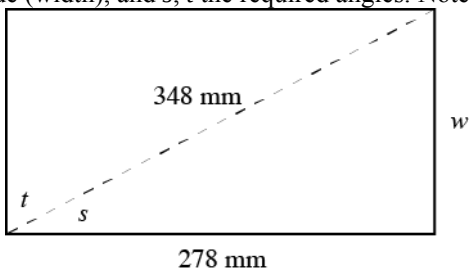
16. Call w the length of the shorter side (width), and s, t the required angles. Note the figure:



Find w using the Pythagorean Theorem:

$$278^2 + w^2 = 348^2$$

$$w^2 = 348^2 - 278^2 = 43820$$

$$w \approx 209$$

Now find angles s and t:

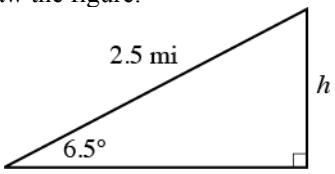
$$\cos s = \frac{278}{348}$$

$$s = \cos^{-1}\left(\frac{278}{348}\right) \approx 37.0°$$

$$t = 90° - 37.0° = 53.0°$$

The shorter side is 209 mm, and the two angles are 37.0° and 53.0°.

18. Let h represent the height of the hill. Draw the figure:



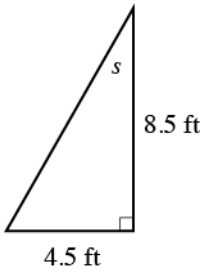
Therefore:

$$\sin 6.5° = \frac{h}{2.5}$$

$$h = 2.5 \sin 6.5° \approx 0.28$$

The hill is approximately 0.28 mi high, which is approximately 1,480 feet.

20. Let s represent the angle between the ladder and the wall. Draw the figure:



Therefore:

$$\tan s = \frac{4.5}{8.5}$$

$$s = \tan^{-1}\left(\frac{4.5}{8.5}\right) \approx 28°$$

The angle between the ladder and the wall is approximately 28°.

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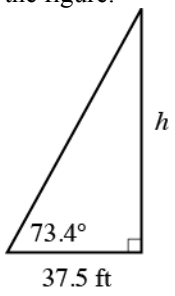
22. Let h represent the height of the building. Draw the figure:



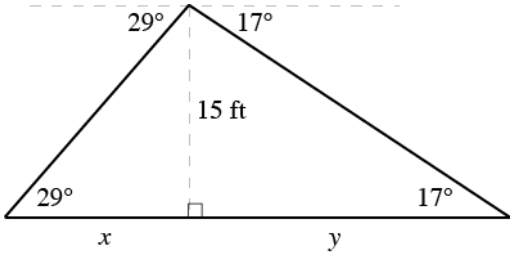
Therefore:

$$\tan 73.4° = \frac{h}{37.5}$$

$$h = 37.5 \tan 73.4° \approx 126$$

The height of the building is approximately 126 feet.

24. Draw the figure with the associated labels:



The sum $x + y$ represents the width of the sand pile. Using the two triangles:

$$\tan 29° = \frac{15}{x} \qquad \tan 17° = \frac{15}{y}$$

$$x \tan 29° = 15 \qquad y \tan 17° = 15$$

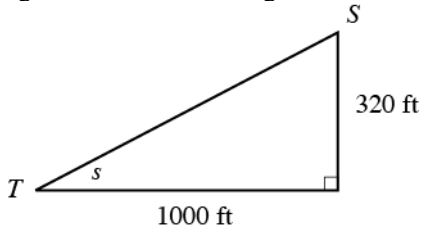
$$x = \frac{15}{\tan 29°} \approx 27.1 \qquad y = \frac{15}{\tan 17°} \approx 49.1$$

The width of the sand pile is therefore 27.1 + 49.1 ≈ 76 feet.

26. **a**. First note that $\frac{5}{8}$ in. $= \frac{5}{8} \cdot 1600 = 1000$ ft, which is the horizontal distance between Stacey and Travis.

b. There are 8 contour intervals between Stacey and Travis, which corresponds to a vertical distance of $8 \cdot 40 = 320$ ft.

c. Let s represent the elevation angle. Construct the triangle:



Therefore:

$$\tan s = \frac{320}{1000} = 0.32$$

$$s = \tan^{-1}(0.32) \approx 17.7°$$

The elevation angle from Travis to Stacey is approximately 17.7°.

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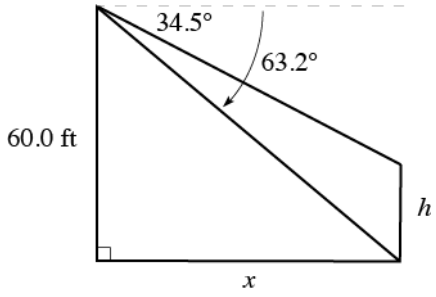
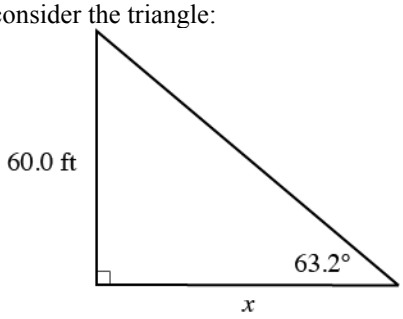
28. Construct the figure:



First consider the triangle:



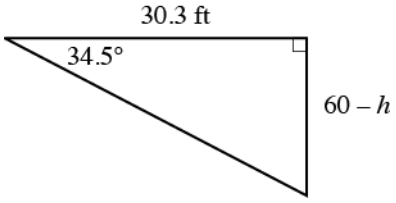
Therefore:

$$\tan 63.2° = \frac{60.0}{x}$$

$$x \tan 63.2° = 60.0$$

$$x = \frac{60.0}{\tan 63.2°} \approx 30.3 \text{ ft}$$

Now consider the triangle:



Therefore:

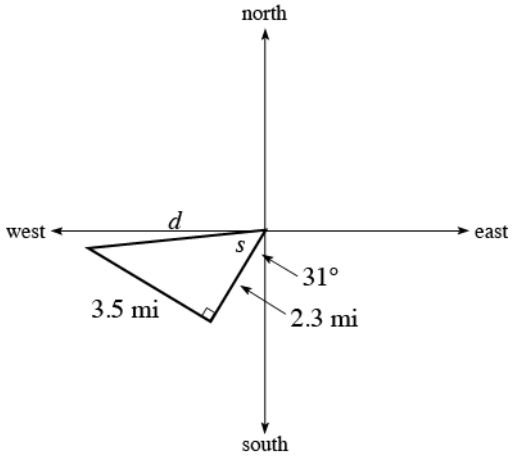
$$\tan 34.5° = \frac{60 - h}{30.3}$$

$$60 - h = 30.3 \tan 34.5°$$

$$h = 60 - 30.3 \tan 34.5° \approx 39.2 \text{ ft}$$

The building next door is approximately 39.2 feet tall.

30. Construct the figure:



To find his distance from the starting point, use the Pythagorean Theorem:

$$d^2 = 3.5^2 + 2.3^2 = 17.54$$

$$d = \sqrt{17.54} \approx 4.2 \text{ mi}$$

Now find angle s:

$$\tan s = \frac{3.5}{2.3}$$

$$s = \tan^{-1}\left(\frac{3.5}{2.3}\right) \approx 57°$$

His bearing is S 88° W.

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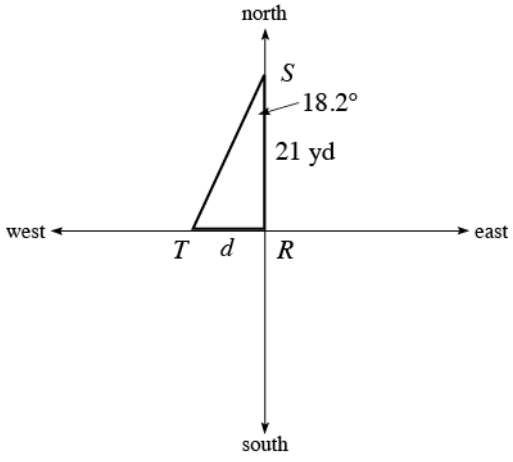
32. Construct the figure:



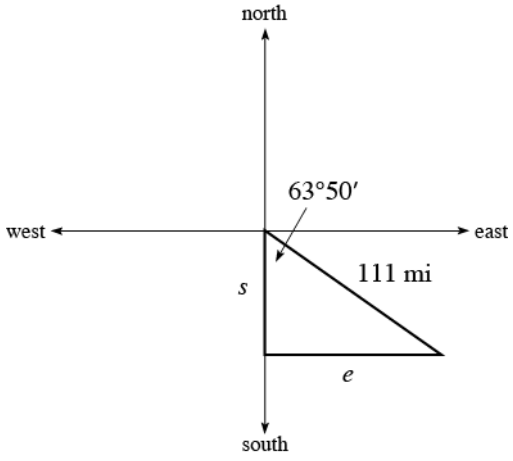
Therefore:

$$\tan 18.2° = \frac{d}{21.0}$$

$$d = 21.0 \tan 18.2° \approx 6.90$$

The distance from the tree to the rock is 6.90 yards.

34. Construct the figure:



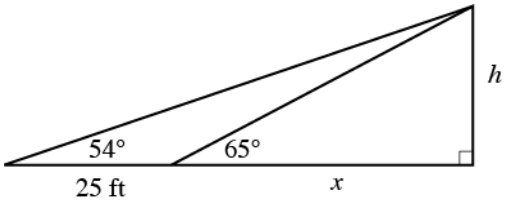
Therefore:

$$\sin 63°50' = \frac{e}{111} \qquad \cos 63°50' = \frac{s}{111}$$

$$e = 111 \sin 63°50' \approx 99.6 \text{ mi} \qquad s = 111 \cos 63°50' \approx 48.9 \text{ mi}$$

The boat travels 48.9 mi south and 99.6 mi east.

36. Draw the figure:



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From the smaller triangle:

$$\tan 65° = \frac{h}{x}$$
$$x \tan 65° = h$$
$$x = \frac{h}{\tan 65°}$$

From the larger triangle:

$$\tan 54° = \frac{h}{25 + x}$$
$$(25 + x)\tan 54° = h$$
$$25 + x = \frac{h}{\tan 54°}$$
$$x = \frac{h}{\tan 54°} - 25$$

Setting these two expressions equal:

$$\frac{h}{\tan 65°} = \frac{h}{\tan 54°} - 25$$
$$h \cot 65° = h \cot 54° - 25$$
$$h \cot 65° - h \cot 54° = -25$$
$$h(\cot 65° - \cot 54°) = -25$$
$$h = \frac{25}{\cot 54° - \cot 65°} \approx 96$$

The height of the obelisk is approximately 96 feet.

38. First find the length CB:

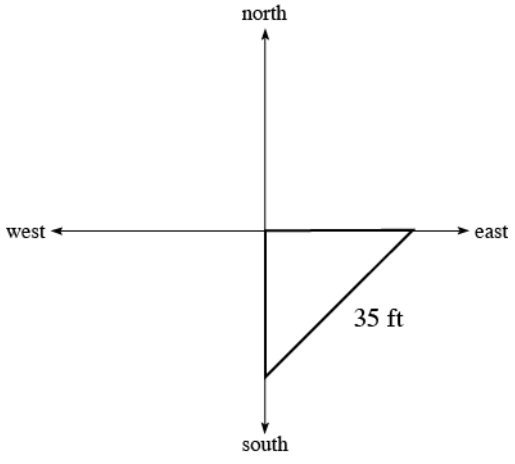
$$\tan 12.3° = \frac{426}{CB}$$
$$CB \tan 12.3° = 426$$
$$CB = \frac{426}{\tan 12.3°} \approx 1{,}954 \text{ ft}$$

Therefore:

$$\sin \angle BCA = \frac{AB}{CB}$$
$$\sin 57.5° = \frac{AB}{1954}$$
$$AB = 1954 \sin 57.5° \approx 1{,}650 \text{ ft}$$

A rescue boat at A will have to travel approximately 1,650 feet to reach any survivors at point B.

40. Construct a figure:



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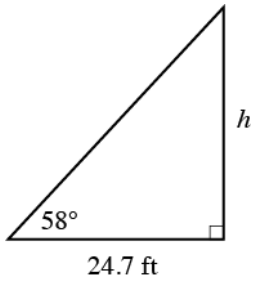
First note that each person is $\frac{35}{\sqrt{2}}$ ft ≈ 24.7 ft from the base of the tree. Let h represent the height of the tree. Now construct the triangle:



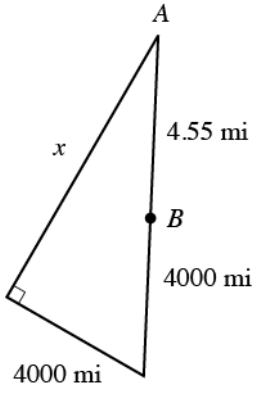
Therefore:

$$\tan 58° = \frac{h}{24.7}$$

$$h = 24.7 \tan 58° \approx 40 \text{ ft}$$

The tree is approximately 27 feet tall.

42. Construct the figure (not drawn to scale):



Using the Pythagorean Theorem:

$$x^2 + 3960^2 = 3964.55^2$$

$$x^2 + 15,681,600 = 15,717,656.7$$

$$x^2 = 36,056.7$$

$$x \approx 190$$

The plane is 190 miles from the horizon. Now find angle A:

$$\sin A = \frac{3960}{3964.55}$$

$$A = \sin^{-1}\left(\frac{3960}{3964.55}\right) \approx 87.3°$$

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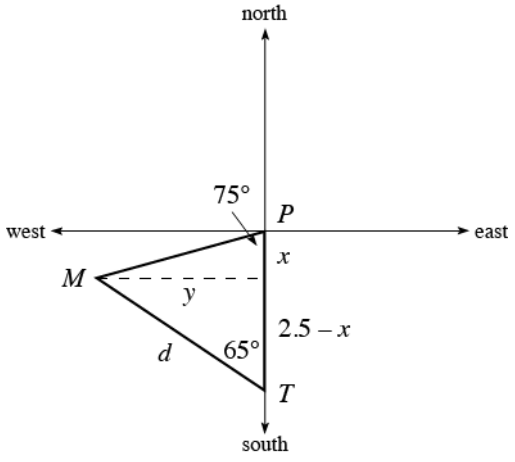
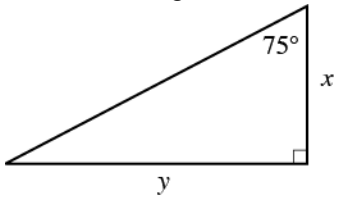
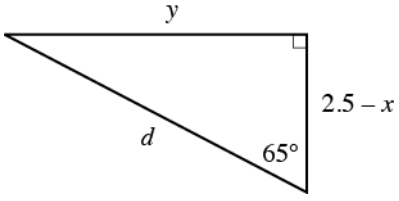
44. Construct the figure:



Consider the two triangles:





Therefore:

$$\tan 75° = \frac{y}{x} \qquad \tan 65° = \frac{y}{2.5 - x}$$

$$y = x \tan 75° \qquad y = (2.5 - x)\tan 65°$$

Setting these two expressions equal:

$$(2.5 - x)\tan 65° = x \tan 75°$$

$$2.5 \tan 65° - x \tan 65° = x \tan 75°$$

$$2.5 \tan 65° = x \tan 75° + x \tan 65°$$

$$2.5 \tan 65° = x\left(\tan 75° + \tan 65°\right)$$

$$x = \frac{2.5 \tan 65°}{\tan 75° + \tan 65°} \approx 0.91$$

Therefore:

$$\cos 65° = \frac{2.5 - x}{d} = \frac{2.5 - 0.91}{d} = \frac{1.59}{d}$$

$$d \cos 65° = 1.59$$

$$d = \frac{1.59}{\cos 65°} \approx 3.8 \text{ mi}$$

Tim is approximately 3.8 miles from the missile when it is launched.

46. Note that $\sin\theta_1 = \frac{1}{\sqrt{2}}, \sin\theta_2 = \frac{1}{\sqrt{3}}, \sin\theta_3 = \frac{1}{\sqrt{4}}$, ... , thus $\sin\theta_n = \frac{1}{\sqrt{n+1}}$.

48. **a**. Let x represent the height that is illuminated on the floor. Then:

$$\tan 84° = \frac{4}{x}$$

$$x = \frac{4}{\tan 84°} \approx 0.42$$

The illuminated area is then: $(0.42)(6.5) \approx 2.7 \text{ ft}^2$.

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b. Following the procedure from part **a**:

$$\tan 37° = \frac{4}{x}$$

$$x = \frac{4}{\tan 37°} \approx 5.31$$

The illuminated area is then: $(5.31)(6.5) \approx 34.5 \text{ ft}^2$. The area is much larger on the winter day.

50. Simplifying: $\frac{1}{\sin\theta} - \sin\theta = \frac{1}{\sin\theta} - \sin\theta \cdot \frac{\sin\theta}{\sin\theta} = \frac{1-\sin^2\theta}{\sin\theta} = \frac{\cos^2\theta}{\sin\theta}$

52. Working from the left side: $\cos\theta\csc\theta\tan\theta = \cos\theta \cdot \frac{1}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta} = \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta} = 1$

54. Working from the left side: $(1-\cos\theta)(1+\cos\theta) = 1-\cos\theta+\cos\theta-\cos^2\theta = 1-\cos^2\theta = \sin^2\theta$

56. Working from the left side: $1 - \frac{\cos\theta}{\sec\theta} = 1 - \frac{\cos\theta}{1/\cos\theta} = 1-\cos^2\theta = \sin^2\theta$

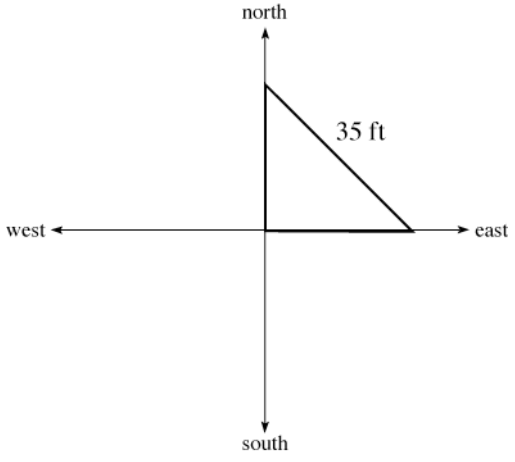
58. Let h represent the height of the flagpole. Then:

$$\tan 74.3° = \frac{h}{22.5}$$

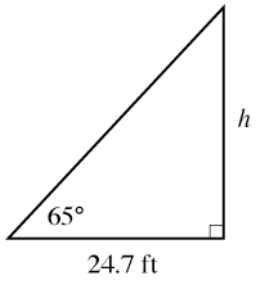
$$h = 22.5\tan 74.3° \approx 80.0$$

The flagpole is 80.0 feet tall. The correct answer is d.

60. Construct a figure:



First note that each person is $\frac{35}{\sqrt{2}}$ ft ≈ 24.7 ft from the base of the tree. Let h represent the height of the tree. Now construct the triangle:



Therefore:

$$\tan 65° = \frac{h}{24.7}$$

$$h = 24.7\tan 65° \approx 53 \text{ ft}$$

The tree is approximately 53 feet tall. The correct answer is a.

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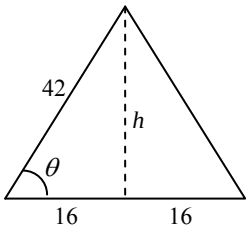
ODD SOLUTIONS

1. elevation, depression **3.** north-south

For problems 5 through 11, see diagrams in textbook answer section.

13. To find the height, *h,* we can use the Pythagorean Theorem:

$$h^2 + (16)^2 = (42)^2$$
$$h^2 + 256 = 1,764$$
$$h^2 = 1,508$$
$$h = \pm\sqrt{1,508} = 39\text{cm}$$



To find angle θ, we can use the cosine ratio:

$$\cos\theta = \frac{16}{42}$$
$$\theta = \cos^{-1}\left(\frac{16}{42}\right) = 68^\circ$$

The height is 39 cm and the two equal angles are 68°.

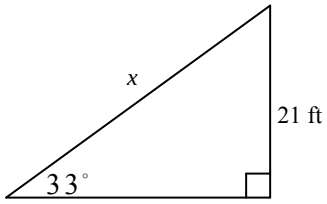
15. Consider the right triangle with sides of 25.3 cm and 5.2 cm (one-half of the diameter):

$$\tan\theta = \frac{25.3}{5.2}$$
$$= 4.8654$$
$$\theta = \tan^{-1}(4.8654)$$
$$= 78.4^\circ$$

The angle the side makes with the base is 78.4°.

17. To find the length of the escalator, *x*, we use the sine ratio:

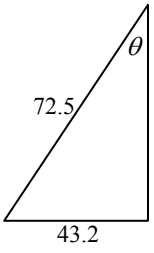
$$\sin 33^\circ = \frac{21}{x}$$
$$x = \frac{21}{\sin 33^\circ}$$
$$= \frac{21}{0.5446} = 39 \text{ ft}$$



The length of the escalator is 39 feet.

19.

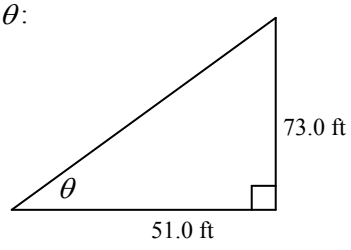
$$\sin\theta = \frac{43.2}{72.5}$$
$$= 0.5959$$
$$\theta = \sin^{-1}(0.5959)$$
$$= 36.6^\circ$$



The angle the rope makes with the pole is 36.6°

21. We use the tangent ratio to find the angle of elevation to the sun, θ:

$$\tan\theta = \frac{73.0}{51.0}$$
$$= 1.4313$$
$$\theta = \tan^{-1}(1.4313)$$
$$= 55.1^\circ$$



The angle of elevation to the sun is 55.1°.

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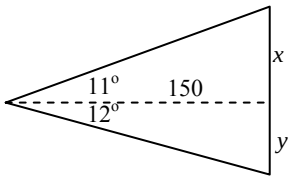
23. $\tan 11^\circ = \dfrac{x}{150}$

$x = 150 \tan 11^\circ = 29 \text{ cm}$



$\tan 12^\circ = \dfrac{y}{150}$

$y = 150 \tan 12^\circ$

$= 32 \text{ cm}$

The vertical dimension of the mirror is $x + y$ or 61 cm.

25. **a.** horizontal distance = 0.50(1,600) = 800 ft

b. vertical distance = (number of contour intervals)(40)

$= 5(40)$

$= 200 \text{ ft}$

c. $\tan\theta = \dfrac{\textit{vertical distance}}{\textit{horizontal distance}}$

$= \dfrac{200}{800}$

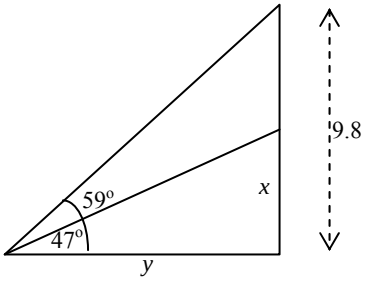
$= 0.25$

$\theta = \tan^{-1}(0.25)$

$= 14^\circ$

27. $\tan 59^\circ = \dfrac{9.8}{y}$

$y = \dfrac{9.8}{\tan 59^\circ} = \dfrac{9.8}{1.6643} = 5.9$



$\tan 47^\circ = \dfrac{9.8}{\tan 59^\circ}$

$x = y \tan 47^\circ$

$= 5.9(1.0724) = 6.3 \text{ ft}$

The vertical dimension of the door is 6.3 feet.

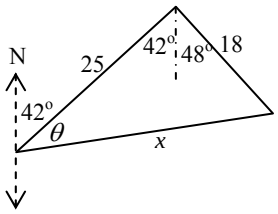
29. We use the Pythagorean Theorem to find the distance *x:*

$x^2 = 25^2 + 18^2$

$= 625 + 324$

$= 949$

$x = 31 \text{ mi}$



We use the tangent relationship to find angle θ:

$\tan\theta = \dfrac{18}{25}$

$= 0.72$

$\theta = \tan^{-1}(0.72)$

$= 36^\circ$

To find the bearing we add $42^\circ + 36^\circ = 78^\circ$. The boat is 31 miles from the harbor entrance and its bearing is N 78° E.

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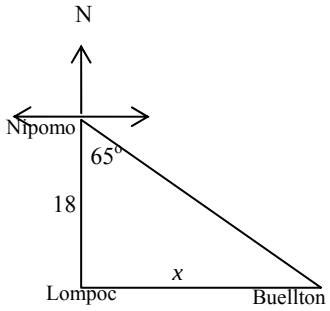
31. $\tan 65^\circ = \dfrac{x}{18}$

$x = 18 \tan 65^\circ$

$= 18(2.1445)$

$= 39 \text{ mi}$

The distance from Lompoc to Buellton is 39 miles.



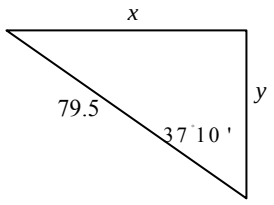
33. We will call the west distance, x and the north distance, y:

$\sin 37^\circ 10' = \dfrac{x}{79.5}$ $\qquad$ $\cos 37^\circ 10' = \dfrac{y}{79.5}$

$x = 79.5 \sin 37^\circ 10'$ $\qquad$ $y = 79.5 \cos 37^\circ 10'$

$= 48.0 \text{ mi}$ $\qquad$ $= 63.4 \text{ mi}$

The boat has traveled 48.0 miles west and 63.4 miles north.



35. In $\Delta ABC, \tan 42.17^\circ = \dfrac{h}{x+33}$

$h = (x+33)\tan 42.17^\circ$

In $\Delta BCD, \tan 47.5^\circ = \dfrac{h}{x}$

$h = x \tan 47.5^\circ$

Therefore, $x \tan 47.5^\circ = (x+33)\tan 42.17^\circ$

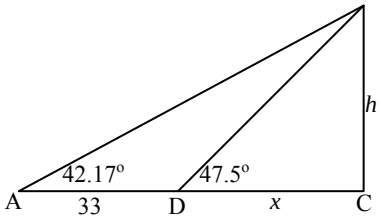
$x \tan 47.5^\circ = x \tan 42.17^\circ + 33 \tan 42.17^\circ$

$x \tan 47.5^\circ - x \tan 42.17^\circ = 33 \tan 42.17^\circ$

$x(\tan 47.5^\circ - \tan 42.17^\circ) = 33 \tan 42.17^\circ$

$$x = \frac{33 \tan 42.17^\circ}{\tan 47.5^\circ - \tan 42.17^\circ} = 161 \text{ ft}$$

The person at point A is 161 feet from the base of the antenna.



37. $\tan 86.6^\circ = \dfrac{x}{24.8}$

$x = 24.8 \tan 86.6^\circ$

$= 24.8(16.8319)$

$= 417.431$

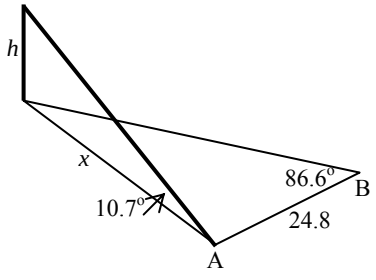
$\tan 10.7^\circ = \dfrac{h}{x}$

$h = x \tan 10.7^\circ$

$= (417.431)(0.18895)$

$= 78.9 \text{ ft}$

The tree is 78.9 feet high.



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39. First, we will find each person's distance from the pole, x, using the Pythagorean Theorem:

$$x^2 + x^2 = 25^2$$
$$2x^2 = 625$$
$$x^2 = 312.5$$
$$x = 17.678 \text{ ft}$$

Next, we will find the height of the pole, h, using the tangent relationship:

$$\tan 56^\circ = \frac{h}{17.678}$$
$$h = 17.678 \tan 56^\circ$$
$$= 26 \text{ ft}$$

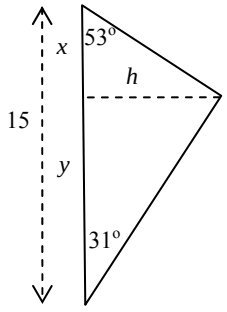
The height of the pole is 26 feet.

41.
$$\sin 76.6^\circ = \frac{r}{r+112}$$
$$r = (r+112)\sin 76.6^\circ$$
$$r = r \sin 76.6^\circ + 112 \sin 76.6^\circ$$
$$r - r \sin 76.6^\circ = 112 \sin 76.6^\circ$$
$$r(1 - \sin 76.6^\circ) = 112 \sin 76.6^\circ$$
$$r = \frac{112 \sin 76.6^\circ}{1 - \sin 76.6^\circ}$$
$$= \frac{112(0.9728)}{1 - 0.9728}$$
$$= \frac{108.9509}{0.02722}$$
$$= 4{,}000 \text{ mi}$$

The radius of the earth is 4,000 miles.

43. We want to find x and y in terms of h

$$\tan 53^\circ = \frac{h}{x} \qquad \tan 31^\circ = \frac{h}{y}$$
$$x \tan 53^\circ = h \qquad y \tan 31^\circ = h$$
$$x = \frac{h}{\tan 53^\circ} \qquad y = \frac{h}{\tan 31^\circ}$$



We know that $x + y = 15$. Therefore,

$$\frac{h}{\tan 53^\circ} + \frac{h}{\tan 31^\circ} = 15$$
$$h\left(\frac{1}{\tan 53^\circ} + \frac{1}{\tan 31^\circ}\right) = 15$$
$$h(0.7536 + 1.6643) = 15$$
$$2.4179h = 15$$
$$h = \frac{15}{2.4179} = 6.2 \text{ mi}$$

The ship is 6.2 miles from the shore.

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45.

$$\tan\theta_1 = \frac{1}{1} \qquad \tan\theta_2 = \frac{1}{\sqrt{2}} \qquad \tan\theta_3 = \frac{1}{\sqrt{3}}$$

$$= 1 \qquad = 0.7071 \qquad = 0.5774$$

$$\theta_1 = \tan^{-1}(1) \qquad \theta_2 = \tan^{-1}(0.7071) \qquad \theta_3 = \tan^{-1}(0.5774)$$

$$\theta_1 = 45.00^\circ \qquad \theta_2 = 35.26^\circ \qquad \theta_3 = 30.00^\circ$$

49.

$$\begin{aligned}(\sin\theta - \cos\theta)^2 &= (\sin\theta - \cos\theta)(\sin\theta - \cos\theta)\\ &= \sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta\\ &= \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta\\ &= 1 - 2\sin\theta\cos\theta\end{aligned}$$

51.

$$\sin\theta\cot\theta = \sin\theta \cdot \frac{\cos\theta}{\sin\theta}$$ Ratio identity

$$= \frac{\sin\theta\cos\theta}{\sin\theta}$$ Multiplication of fractions

$$= \cos\theta$$ Division of common factor

53.

$$\frac{\sec\theta}{\tan\theta} = \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}$$ Reciprocal and ratio identity

$$= \frac{1}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta}$$ Division of fractions

$$= \frac{1}{\sin\theta}$$ Multiplication of fractions and divide common factor

$$= \csc\theta$$

55.

$$\sec\theta - \cos\theta = \frac{1}{\cos\theta} - \cos\theta$$ Reciprocal identity

$$= \frac{1}{\cos\theta} - \cos\theta \cdot \frac{\cos\theta}{\cos\theta}$$ L.C.D. is $\cos\theta$

$$= \frac{1 - \cos^2\theta}{\cos\theta}$$ Subtraction of fractions

$$= \frac{\sin^2\theta}{\cos\theta}$$ Pythagorean identity

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2.5 Vectors: A Geometric Approach

EVEN SOLUTIONS

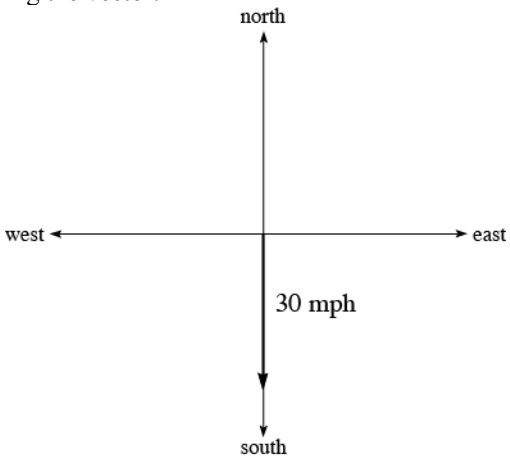
2. Two vectors are equivalent if they have the same magnitude and direction.

4. A vector is in standard position if the tail of the vector is placed at the origin of a rectangular coordinate system.

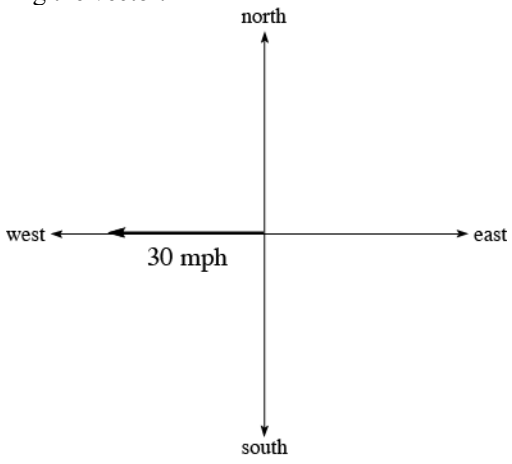
6. If **V** makes and angle θ with the positive *x*-axis when in standard position, then $|\mathbf{V}_x| = |\mathbf{V}|\cos\theta$ and $|\mathbf{V}_y| = |\mathbf{V}|\sin\theta$.

8. If a constant force **F** is applied to an object and moves the object in a straight line a distance *d* at an angle θ with the force, then the work performed by the force is found by multiplying $|\mathbf{F}|\cos\theta$ and *d*.

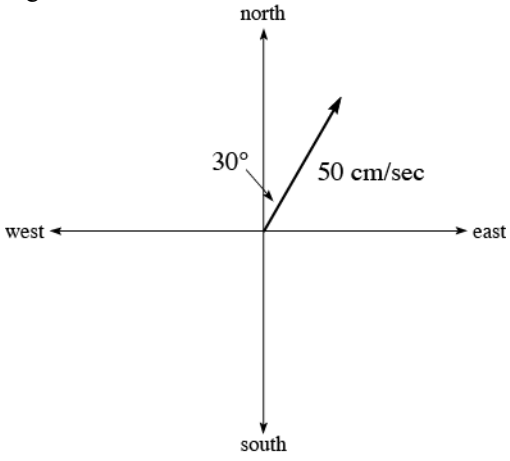
10. Sketching the vector:



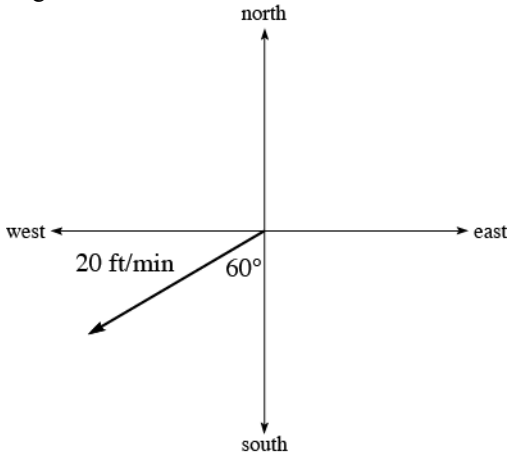
12. Sketching the vector:



14. Sketching the vector:



16. Sketching the vector:



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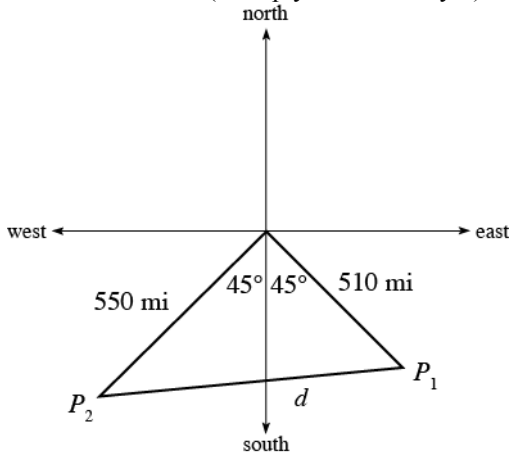
18. Construct the figure for their position after 2 hours (multiply their rates by 2):

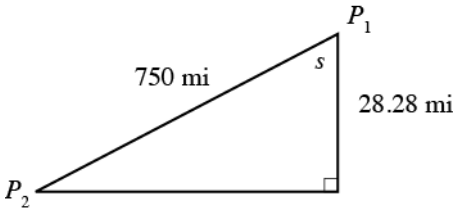


Using the Pythagorean Theorem: $d = \sqrt{550^2 + 510^2} \approx 750$ miles

To find the bearing from P_1 to P_2, first find the vertical (north-south) change in their positions. This is given by:

$$550\sin 45° - 510\sin 45° \approx 28.28 \text{ miles}$$

Construct the triangle:



Therefore:

$$\cos s = \frac{28.28}{750}$$

$$s = \cos^{-1}\left(\frac{28.28}{750}\right) \approx 87.8°$$

The bearing from P_1 to P_2 is S 87.8° W.

20. Computing the magnitudes of $\mathbf{V}_x$ and $\mathbf{V}_y$:

$$|\mathbf{V}_x| = 17.6\cos 72.6° \approx 5.26$$

$$|\mathbf{V}_y| = 17.6\sin 72.6° \approx 16.8$$

22. Computing the magnitudes of $\mathbf{V}_x$ and $\mathbf{V}_y$:

$$|\mathbf{V}_x| = 383\cos 12°20' \approx 374$$

$$|\mathbf{V}_y| = 383\sin 12°20' \approx 81.8$$

24. Computing the magnitudes of $\mathbf{V}_x$ and $\mathbf{V}_y$:

$$|\mathbf{V}_x| = 84\cos 90° = 0$$

$$|\mathbf{V}_y| = 84\sin 90° = 84$$

26. Using the Pythagorean Theorem: $|\mathbf{V}| = \sqrt{54.2^2 + 14.5^2} \approx 56.1$

28. Using the Pythagorean Theorem: $|\mathbf{V}| = \sqrt{2.2^2 + 8.8^2} \approx 9.1$

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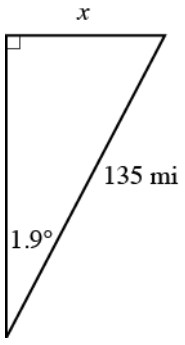
30. Construct the triangle:



Therefore:

$$\sin 1.9° = \frac{x}{135}$$

$$x = 135 \sin 1.9° \approx 4.48 \text{ miles}$$

The plane will be approximately 4.48 miles off course.

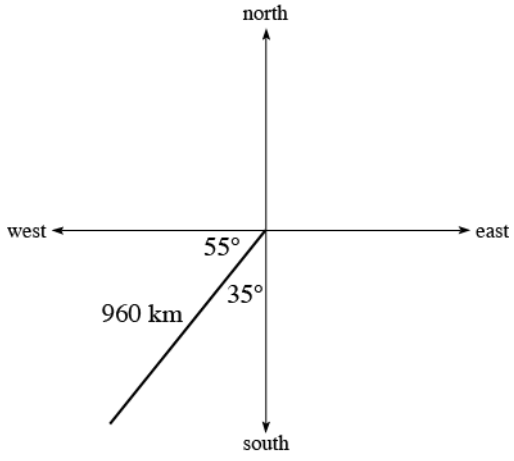
32. Computing the magnitudes of $\mathbf{V}_x$ and $\mathbf{V}_y$:

$$|\mathbf{V}_x| = 1,800 \cos 55° = 1,032 \frac{\text{ft}}{\text{sec}} \approx 1,000 \frac{\text{ft}}{\text{sec}}$$

$$|\mathbf{V}_y| = 1,800 \sin 55° \approx 1,474 \frac{\text{ft}}{\text{sec}} \approx 1,500 \frac{\text{ft}}{\text{sec}}$$

34. The horizontal distance traveled is $1.5 \cdot 1,032 = 1,548$ feet $\approx 1,500$ feet.

36. Draw the figure corresponding to $t = 3$ hours:



The west and south distances are given by:

west: $960 \cos 55° \approx 550$ km south: $960 \sin 55° \approx 790$ km

38. Using the Pythagorean Theorem: $|\mathbf{V}| = \sqrt{16.5^2 + 24.3^2} \approx 29.4$ ft/sec

The elevation angle is given by:

$$\tan\theta = \frac{24.3}{16.5}$$

$$\theta = \tan^{-1}\left(\frac{24.3}{16.5}\right) \approx 55.8°$$

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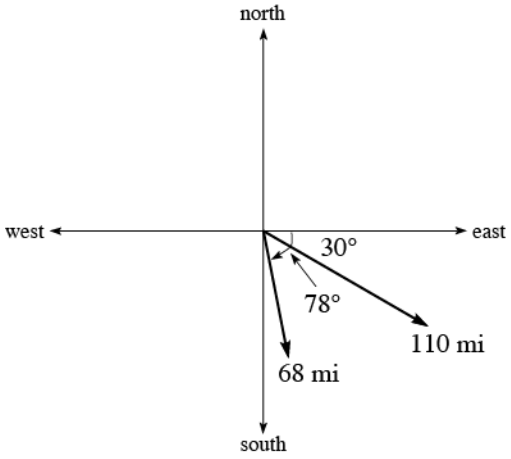
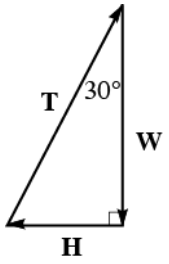
40. Construct the figure:



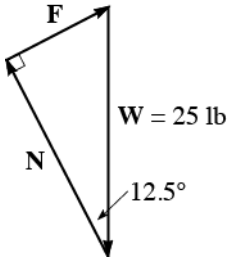
The total distance south and east is given by:

east: $68\cos 78° + 110\cos 30° \approx 110$ miles south: $68\sin 78° + 110\sin 30° \approx 120$ miles

42. The corresponding force diagram would be:



44. The corresponding force diagram would be:

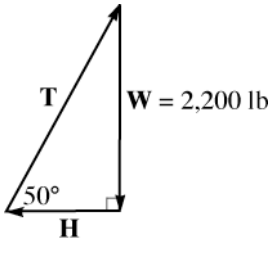


Therefore:

$$\sin 12.5° = \frac{|\mathbf{F}|}{25.0} \qquad \cos 12.5° = \frac{|\mathbf{N}|}{25.0}$$

$$|\mathbf{F}| = 25.0\sin 12.5° \approx 5.41 \text{ lb} \qquad |\mathbf{N}| = 25.0\cos 12.5° \approx 24.4 \text{ lb}$$

46. The corresponding force diagram would be:



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Therefore:

$$\sin 50° = \frac{2{,}200}{|\mathbf{T}|} \qquad \tan 50° = \frac{2{,}200}{|\mathbf{H}|}$$

$$|\mathbf{T}| \sin 50° = 2{,}200 \qquad |\mathbf{H}| \tan 50° = 2{,}200$$

$$|\mathbf{T}| = \frac{2{,}200}{\sin 50°} \approx 2{,}900 \text{ lb} \qquad |\mathbf{H}| = \frac{2{,}200}{\tan 50°} \approx 1{,}800 \text{ lb}$$

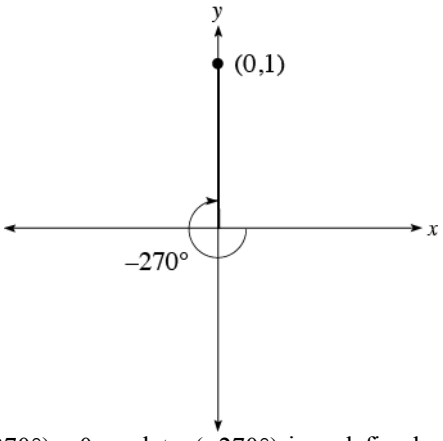
48. The horizontal portion of the force is given by: $|\mathbf{F}_x| = |\mathbf{F}| \cos 35° = 15 \cos 35°$ lb

The work is then given by: Work $= (15 \cos 35°)(52) \approx 640$ ft-lb

50. The horizontal portion of the force is given by: $|\mathbf{F}_x| = |\mathbf{F}| \cos 15° = 85 \cos 15°$ lb

The work is then given by: Work $= (85 \cos 15°)(110) \approx 9{,}000$ ft-lb

52. Drawing the angle in standard position:



Since $r = 1$, $\sin(-270°) = 1$, $\cos(-270°) = 0$, and $\tan(-270°)$ is undefined.

54. Choose $(-1,1)$ as a point on the terminal side of θ. Then $r = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$. Therefore:

$$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \cos\theta = \frac{x}{r} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

56. Since $\cos\theta = \frac{x}{r} = -\frac{3}{5} = -\frac{6}{10}$, choose $x = -6$ and $r = 10$. Now find y:

$$(-6)^2 + y^2 = 10^2$$
$$36 + y^2 = 100$$
$$y^2 = 64$$
$$y = \pm 8$$

58. Using the Pythagorean Theorem: $|\mathbf{V}| = \sqrt{9.6^2 + 2.3^2} \approx 9.9$

Finding the angle:

$$\tan\theta = \frac{2.3}{9.6}$$
$$\theta = \tan^{-1}\left(\frac{2.3}{9.6}\right) \approx 13°$$

The correct answer is c.

60. The horizontal portion of the force is given by: $|\mathbf{F}_x| = |\mathbf{F}| \cos 35° = 28 \cos 35°$ lb

The work is then given by: Work $= (28 \cos 35°)(150) \approx 3{,}400$ ft-lb

The correct answer is b.

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ODD SOLUTIONS

1. scalar, vector

3. resultant, diagonal

5. horizontal, component, vertical, component

7. zero, static equilibrium

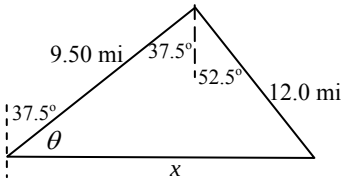
For problems 9 through 15, see textbook answer section for diagrams.

17. The first hour, the distance traveled is

(9.50 mph)(1 hr) = 9.50 miles

The next hour and a half, the distance traveled is

(8.00 mph)(1.5 hr) = 12.0 miles



We will use the Pythagorean Theorem to find x:

$$x^2 = 9.50^2 + 12.0^2$$
$$x^2 = 234.25$$
$$x = 15.3 \text{ mi}$$

We will use the tangent ratio to find θ and then add 37.5°:

$$\tan\theta = \frac{12.0}{9.50}$$
$$= 1.2632$$
$$\theta = \tan^{-1}(1.2632)$$
$$= 51.6^\circ \qquad 51.6^\circ + 37.5^\circ = 89.1^\circ$$

The balloon is 15.3 miles from its starting point. The bearing is N 89.1° E.

19. $|V_x| = |V|\cos\theta$ $\qquad$ $|V_y| = |V|\sin\theta$

$= 13.8\cos 24.2^\circ$ $\qquad$ $= 13.8\sin 24.2^\circ$

$= 12.6$ $\qquad$ $= 5.66$

21. $|V_x| = |V|\cos\theta$ $\qquad$ $|V_y| = |V|\sin\theta$

$= 425\ \cos 36^\circ 10'$ $\qquad$ $= 425\ \sin 36^\circ 10'$

$= 425\ \cos 36.17^\circ$ $\qquad$ $= 425\ \sin 36.17^\circ$

$= 425(0.8073)$ $\qquad$ $= 425(0.5901)$

$= 343$ $\qquad$ $= 251$

23. $|V_x| = |V|\cos\theta$ $\qquad$ $|V_y| = |V|\sin\theta$

$= 64\cos 0^\circ$ $\qquad$ $= 64\sin 0^\circ$

$= 64(1) = 64$ $\qquad$ $= 64(0) = 0$

25. $|V| = \sqrt{|V_x|^2 + |V_y|^2}$

$= \sqrt{(35.0)^2 + (26.0)^2}$

$= \sqrt{1{,}225 + 676}$

$= \sqrt{1{,}901}$

$= 43.6$

27. $|V| = \sqrt{|V_x|^2 + |V_y|^2}$

$= \sqrt{(4.5)^2 + (3.8)^2}$

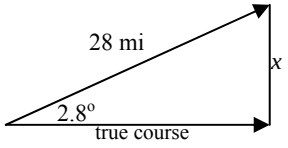
$= \sqrt{20.25 + 14.44}$

$= \sqrt{34.69}$

$= 5.9$

29. To find the distance, x, the plane has flown off its course, we can use the sine ratio:

$$\sin 2.8^\circ = \frac{x}{28}$$
$$x = 28\sin 2.8^\circ$$
$$= 1.37 \text{ miles}$$



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31. $|V_x| = |V|\cos\theta$ $\quad$ $|V_y| = |V|\sin\theta$

$= 1{,}200\cos 45^\circ$ $\quad$ $= 1{,}200\sin 45^\circ$

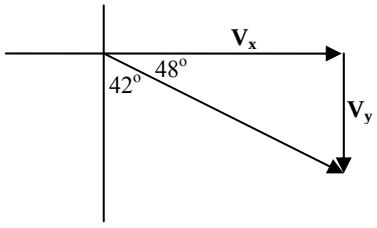
$= 1200(0.7071)$ $\quad$ $= 1200(0.7071)$

$= 850$ feet per second $\quad$ $= 850$ feet per second

33. In 3 seconds, the bullet travels 3(850 ft/sec) = 2,550 ft.

35. $|V_x| = 130\cos 48^\circ$ $\quad$ $|V_y| = 130\sin 48^\circ$

$= 87$ $\quad$ $= 97$



The ship has traveled 97 km south and 87 km east.

37. We are given that $|V_x| = 35.0$ and $|V_y| = 15.0$

$|V| = \sqrt{|V_x|^2 + |V_y|^2}$ $\quad$ $\tan\theta = \dfrac{|V_y|}{|V_x|}$

$= \sqrt{(35.0)^2 + (15.0)^2}$

$= \sqrt{1{,}225 + 225}$ $\quad$ $= \dfrac{15.0}{35.0}$

$= \sqrt{1{,}450}$ $\quad$ $= 0.4285$

$= 38.1$ feet per second $\quad$ $\theta = 23.2^\circ$

Therefore, the velocity of the arrow is 38.1 feet per second at an elevation of 23.2°.

39. To find the total distance traveled north, we must find the sum of $|V_y|$ and $|W_y|$ and to find the total distance traveled east, we must find the sum of $|V_x|$ and $|W_x|$.

We are given that $|V|$ is 170 mi. at an angle of inclination of $90^\circ - 18^\circ$ or 72° and also that $|W|$ is 120 mi. at an angle of inclination of $90^\circ - 49^\circ$ or 41°

$|V_x| = |V|\cos\theta_1$ $\quad$ $|V_y| = |V|\sin\theta_1$

$= 170\cos 72^\circ$ $\quad$ $= 170\sin 72^\circ$

$= 170(0.3090)$ $\quad$ $= 170(0.9510)$

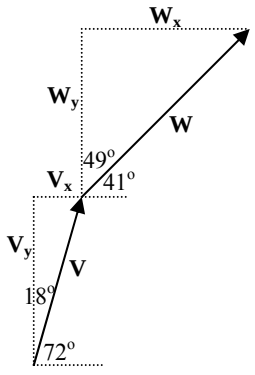
$= 53$ mi $\quad$ $= 162$ mi

$|W_x| = |W|\cos\theta_2$ $\quad$ $|W_y| = |W|\sin\theta_2$

$= 120\cos 41^\circ$ $\quad$ $= 120\sin 41^\circ$

$= 120(0.7547)$ $\quad$ $= 120(0.6560)$

$= 90$ mi $\quad$ $= 80$ mi



$|V_y| + |W_y| = 162 + 80 = 242$ and $|V_x| + |W_x| = 53 + 90 = 143$

The total distance north is 240 miles and the total distance east is 140 miles, rounded to 2 significant digits.

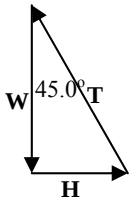
41. $|W| = 42.0$

$\cos 45.0^\circ = \dfrac{|W|}{|T|}$ $\quad$ $\tan 45.0^\circ = \dfrac{|H|}{|W|}$

$|T| = \dfrac{|W|}{\cos 45.0^\circ}$ $\quad$ $|H| = |W|\tan 45.0^\circ$

$= \dfrac{42.0}{\cos 45.0^\circ}$ $\quad$ $= 42.0\tan 45.0^\circ$

$= 59.4$ lb. $\quad$ $= 42.0$ lb.



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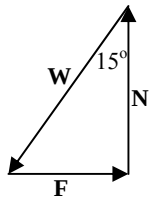
43. We are given that $|W| = 8.0$

$\cos 15^\circ = \dfrac{|N|}{|W|}$ $\qquad$ $\sin 15^\circ = \dfrac{|F|}{|W|}$

$|N| = |W| \cos 15^\circ$ $\qquad$ $|F| = |W| \sin 15^\circ$

$= 8.0(0.9659)$ $\qquad$ $= 8.0(0.2588)$

$= 7.7$ pounds $\qquad$ $= 2.1$ pounds



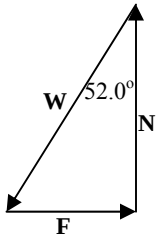
45. $|W| = 42.0$

$\sin 52.0^\circ = \dfrac{|F|}{|W|}$

$|F| = |W| \sin 52.0^\circ$

$= 42.0 \sin 52.0^\circ$

$= 33.1$ lb



47. $\theta = 20^\circ, |F| = 40$ lb, and $d = 75$ ft

$|F_x| = |F| \cos\theta$ $\qquad$ Work $= |F_x| \cdot d$

$= 41 \cos 20^\circ$ $\qquad$ $= (41 \cos 20^\circ)(75)$

$= 2900$ ft - lb.

49. $\theta = 30^\circ, |F| = 25$ lb, and $d = 350$ ft

$|F_x| = |F| \cos\theta$ $\qquad$ Work $= |F_x| \cdot d$

$= 25 \cos 30^\circ$ $\qquad$ $= (25 \cos 30^\circ)(350)$

$= 7{,}600$ ft - lb.

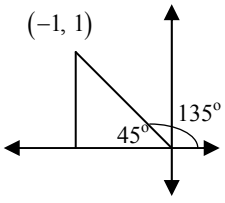
51. $(x, y) = (-1, 1)$

$x = -1, y = 1$ and $r = \sqrt{2}$

$\sin 135^\circ = \dfrac{y}{r} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

$\cos 135^\circ = \dfrac{x}{r} = -\dfrac{1}{\sqrt{2}} = -\dfrac{\sqrt{2}}{2}$

$\tan 135^\circ = \dfrac{y}{x} = \dfrac{1}{-1} = -1$



53. A point on the line $y = 2x$ in quadrant I is (1, 2). $x = 1$, $y = 2$, and $r = \sqrt{1^2 + 2^2} = \sqrt{5}$

$\sin\theta = \dfrac{y}{r} = \dfrac{2}{\sqrt{5}} = \dfrac{2\sqrt{5}}{5}$ $\qquad$ $\cos\theta = \dfrac{x}{r} = \dfrac{1}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$

55. $\sin\theta = \dfrac{y}{r} = \dfrac{-4}{5} = \dfrac{-8}{10}$

$y = -8$ and $r = 10$

$x^2 + y^2 = r^2$

$x^2 + (-8)^2 = 10^2$

$x^2 + 64 = 100$

$x^2 = 36$

$x = \pm 6$

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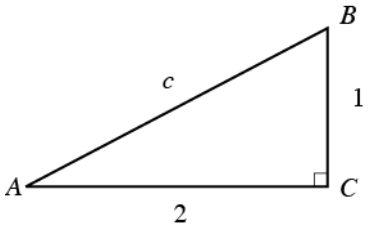
Chapter 2 Test

1. First draw the triangle:

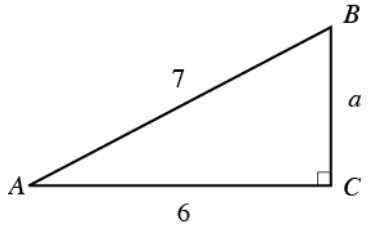


Note that $c = \sqrt{2^2 + 1^2} = \sqrt{5}$. Therefore:

$\sin A = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ $\quad \cos A = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ $\quad \tan A = \frac{1}{2}$

$\sin B = \cos A = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ $\quad \cos B = \sin A = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ $\quad \tan B = \cot A = \frac{2}{1} = 2$

2. First draw the triangle:

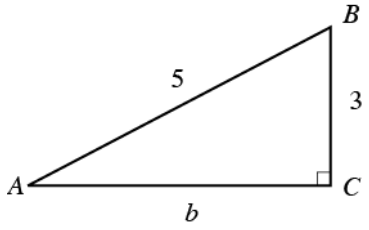


Note that $a = \sqrt{7^2 - 6^2} = \sqrt{13}$. Therefore:

$\sin A = \frac{\sqrt{13}}{7}$ $\quad \cos A = \frac{6}{7}$ $\quad \tan A = \frac{\sqrt{13}}{6}$

$\sin B = \cos A = \frac{6}{7}$ $\quad \cos B = \sin A = \frac{\sqrt{13}}{7}$ $\quad \tan B = \cot A = \frac{6}{\sqrt{13}} = \frac{6\sqrt{13}}{13}$

3. First draw the triangle:



Note that $b = \sqrt{5^2 - 3^2} = \sqrt{16} = 4$. Therefore:

$\sin A = \frac{3}{5}$ $\quad \cos A = \frac{4}{5}$ $\quad \tan A = \frac{3}{4}$

$\sin B = \cos A = \frac{4}{5}$ $\quad \cos B = \sin A = \frac{3}{5}$ $\quad \tan B = \cot A = \frac{4}{3}$

4. Since $y \le r$, $\frac{y}{r} \le 1$. Therefore $\sin\theta = \frac{y}{r} \le 1$, so it is impossible for $\sin\theta = 2$.

5. $\sin 14° = \cos(90° - 14°) = \cos 76°$

6. Simplifying: $\sin^2 45° + \cos^2 60° = \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

7. Simplifying: $\tan 45° + \cot 45° = 1 + 1 = 2$

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8. Simplifying: $\sin^2 60° - \cos^2 30° = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4} - \frac{3}{4} = 0$

9. Simplifying: $\frac{1}{\csc 30°} = \sin 30° = \frac{1}{2}$

10. Adding: $48°31' + 24°52' = 72°83' = 73°23'$

11. Converting to degrees and minutes: $73.2° = 73° + 0.2° = 73° + 0.2(60') = 73°12'$

12. Converting to decimal degrees: $2°48' = 2° + 48' = 2° + \left(\frac{48}{60}\right)° = 2.8°$

13. Calculating the value: $\sin 24°20' = \sin\left(24\frac{1}{3}\right)° \approx 0.4120$

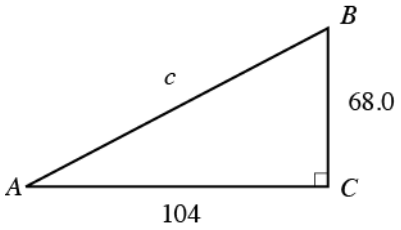
14. Calculating the value: $\cos 48.3° \approx 0.6652$

15. Calculating the value: $\cot 71°20' = \cot\left(71\frac{1}{3}\right)° = \frac{1}{\tan\left(71\frac{1}{3}\right)°} \approx 0.3378$

16. Since $\sin\theta = 0.6459$, $\theta = \sin^{-1}(0.6459) \approx 40.2°$.

17. Since $\sec\theta = 1.923$, $\cos\theta = \frac{1}{1.923}$, so $\theta = \cos^{-1}\left(\frac{1}{1.923}\right) \approx 58.7°$.

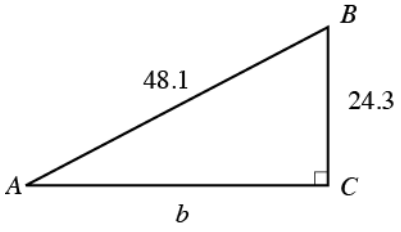
18. First sketch the triangle:



Using the Pythagorean Theorem: $c = \sqrt{104^2 + 68^2} \approx 124$. Therefore:

$$\tan A = \frac{68}{104}$$
$$A = \tan^{-1}\left(\frac{68}{104}\right) \approx 33.2°$$
$$B = 90° - 33.2° = 56.8°$$

19. First sketch the triangle:



Using the Pythagorean Theorem: $b = \sqrt{48.1^2 - 24.3^2} \approx 41.5$. Therefore:

$$\sin A = \frac{24.3}{48.1}$$
$$A = \sin^{-1}\left(\frac{24.3}{48.1}\right) \approx 30.3°$$
$$B = 90° - 30.3° = 59.7°$$

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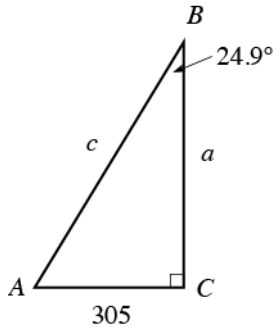
20. First sketch the triangle:

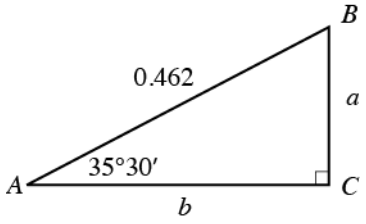


Note that $A = 90° - 24.9° = 65.1°$. Therefore:

$$\tan 65.1° = \frac{a}{305}$$
$$a = 305 \tan 65.1° \approx 657$$

$$\cos 65.1° = \frac{305}{c}$$
$$c \cos 65.1° = 305$$
$$c = \frac{305}{\cos 65.1°} \approx 724$$

21. First sketch the triangle:

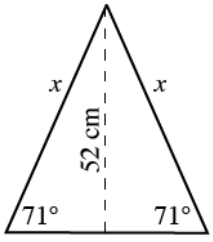


Note that $B = 90° - 35°30' = 89°60' - 35°30' = 54°30'$. Also:

$$\sin 35.5° = \frac{a}{0.462}$$
$$a = 0.462 \sin 35.5° \approx 0.268$$

$$\cos 35.5° = \frac{b}{0.462}$$
$$b = 0.462 \cos 35.5° \approx 0.376$$

22. First sketch the triangle:



Therefore:

$$\sin 71° = \frac{52}{x}$$
$$x \sin 71° = 52$$
$$x = \frac{52}{\sin 71°} \approx 55 \text{ cm}$$

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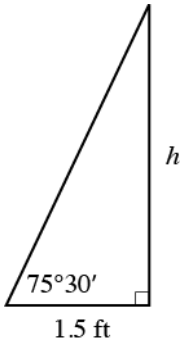
23. Sketch the figure:



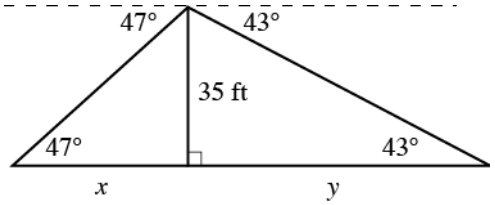
Therefore:

$$\tan 75.5° = \frac{h}{1.5}$$

$$h = 1.5 \tan 75.5° \approx 5.8$$

The post is approximately 5.8 feet tall.

24. Draw the figure:



Therefore:

$$\tan 47° = \frac{35}{x} \qquad \tan 43° = \frac{35}{y}$$

$$x \tan 47° = 35 \qquad y \tan 43° = 35$$

$$x = \frac{35}{\tan 47°} \approx 32.64 \text{ feet} \qquad y = \frac{35}{\tan 43°} \approx 37.53 \text{ feet}$$

The stakes are 32.6 + 37.5 ≈ 70 feet apart.

25. Let θ represent the required angle. Then:

$$\tan\theta = \frac{31}{11}$$

$$\theta = \tan^{-1}\left(\frac{31}{11}\right) \approx 70°$$

26. The magnitudes are given by:

$$|\mathbf{V}_x| = 850 \cos 52° \approx 523 \text{ ft/sec} \approx 520 \text{ ft/sec}$$

$$|\mathbf{V}_y| = 850 \sin 52° \approx 670 \text{ ft/sec}$$

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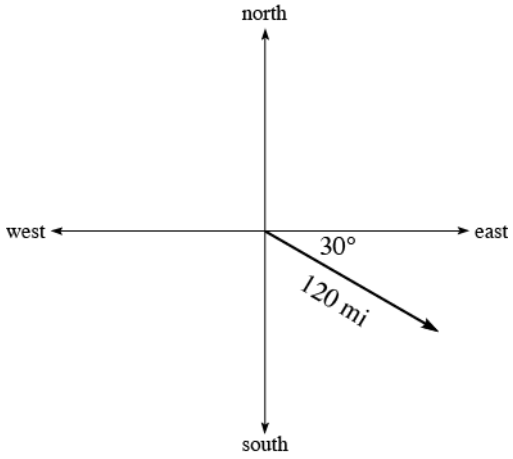
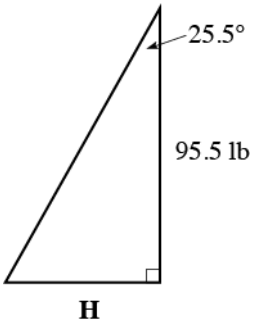
27. Draw the figure:



Therefore the distances the ship has traveled are:

east: $120\cos 30° \approx 100$ miles south: $120\sin 30° = 60$ miles

28. Drawing the figure:



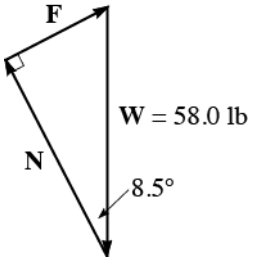
Now find the magnitude of **H**:

$$\tan 25.5° = \frac{|\mathbf{H}|}{95.5}$$

$$|\mathbf{H}| = 95.5\tan 25.5° \approx 45.6$$

Kelly must push horizontally with a force of 45.6 lb.

29. The corresponding force diagram would be:



Now find the magnitude of **F**:

$$\sin 8.5° = \frac{|\mathbf{F}|}{58.0}$$

$$|\mathbf{F}| = 58.0\sin 8.5° \approx 8.57 \text{ lb}$$

Tyler must push with a force of 8.57 lb.

30. The horizontal portion of the force is given by: $|\mathbf{F}_x| = |\mathbf{F}|\cos 40° = 44\cos 40°$ lb

The work is then given by: $\text{Work} = (44\cos 40°)(85) \approx 2{,}865$ ft-lb $\approx 2{,}900$ ft-lb ft-lb

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Chapter 3
Radian Measure

3.1 Reference Angle

EVEN SOLUTIONS

2. The only possible difference between a trigonometric function of an angle and its reference angle will be the sign of the value.

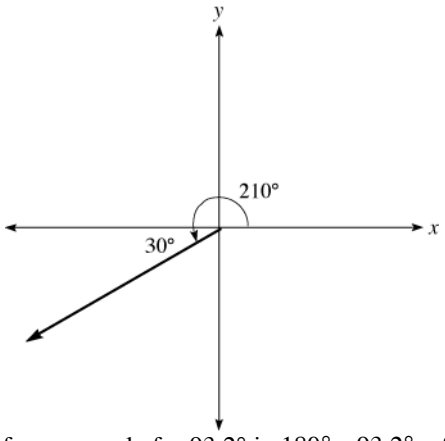
4. **a**. If $\theta \in \text{QI}$, then $\hat{\theta} = \theta$.

b. If $\theta \in \text{QII}$, then $\hat{\theta} = 180° - \theta$.

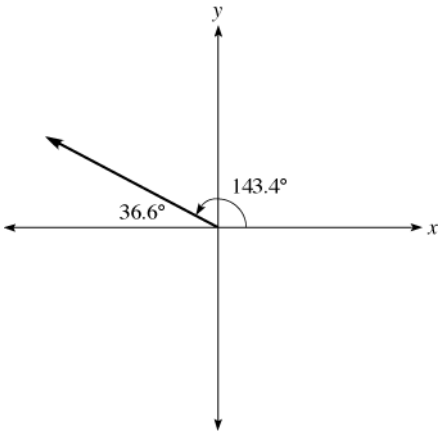
c. If $\theta \in \text{QIII}$, then $\hat{\theta} = \theta - 180°$.

d. If $\theta \in \text{QIV}$, then $\hat{\theta} = 360° - \theta$.

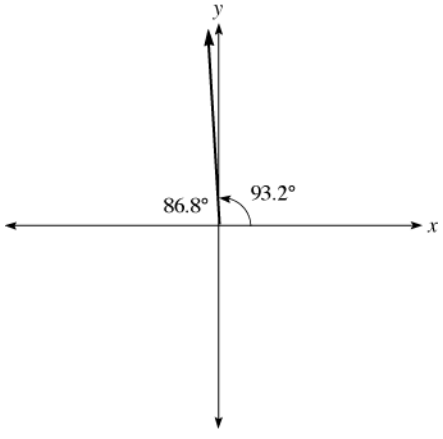
6. The reference angle for $210°$ is $210° - 180° = 30°$:



8. The reference angle for $143.4°$ is $180° - 143.4° = 36.6°$:



10. The reference angle for $93.2°$ is $180° - 93.2° = 86.8°$:



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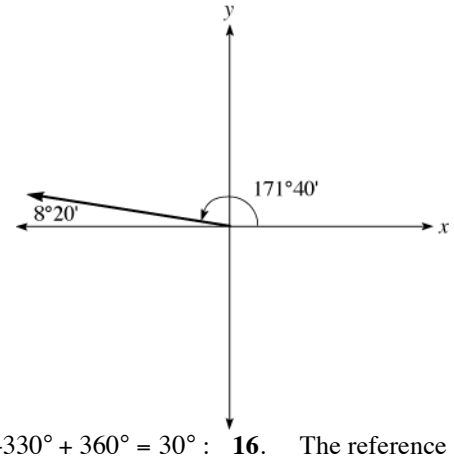
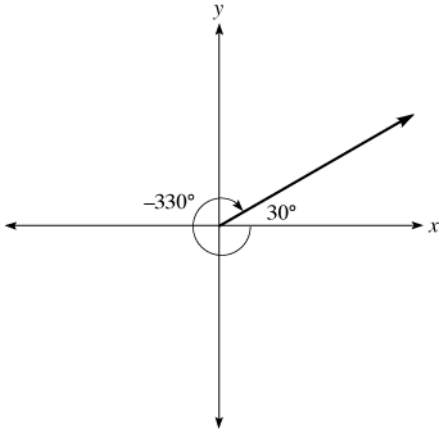
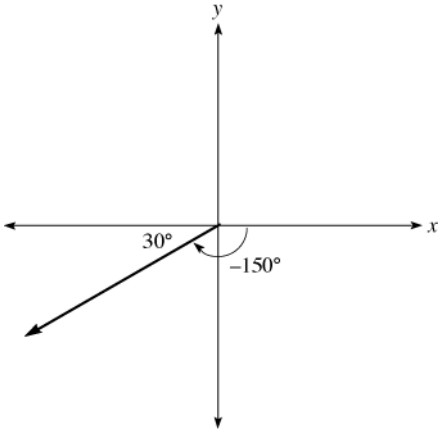
12. The reference angle for $171°40'$ is $180° - 171°40' = 8°20'$:



14. The reference angle for $-330°$ is $-330° + 360° = 30°$:



16. The reference angle for $-150°$ is $-150° + 180° = 30°$:



18. The reference angle for 225° is 45°. Since 225° terminates in quadrant III, its cosine will be negative. Therefore:

$$\cos 225° = -\cos 45° = -\frac{\sqrt{2}}{2}$$

20. The reference angle for 300° is 60°. Since 300° terminates in quadrant IV, its sine will be negative. Therefore:

$$\sin 300° = -\sin 60° = -\frac{\sqrt{3}}{2}$$

22. The reference angle for 315° is 45°. Since 315° terminates in quadrant IV, its tangent will be negative. Therefore:

$$\tan 315° = -\tan 45° = -1$$

24. The reference angle for –30° is 30°. Since –30° terminates in quadrant IV, its cosine will be positive. Therefore:

$$\cos(-30°) = \cos 30° = \frac{\sqrt{3}}{2}$$

26. The reference angle for –330° is 30°. Since –330° terminates in quadrant I, its cosecant is positive. Therefore:

$$\csc(-330°) = \csc 30° = \frac{1}{\sin 30°} = \frac{1}{1/2} = 2$$

28. The reference angle for 240° is 60°. Since 240° terminates in quadrant III, its secant is negative. Therefore:

$$\sec 240° = -\sec 60° = -\frac{1}{\cos 60°} = -\frac{1}{1/2} = -2$$

30. The reference angle for 420° is 60°. Since 420° terminates in quadrant I, its cosine is positive. Therefore:

$$\cos 420° = \cos 60° = \frac{1}{2}$$

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32. The reference angle for 510° is 30°. Since 510° terminates in quadrant II, its tangent is negative. Therefore:

$$\tan 510° = -\tan 30° = -\frac{\sin 30°}{\cos 30°} = -\frac{1/2}{\sqrt{3}/2} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

34. Calculating the value: $\sin 238° \approx -0.8480$

36. Calculating the value: $\csc 316.7° = \dfrac{1}{\sin 316.7°} \approx -1.4581$

38. Calculating the value: $\cot 253.8° = \dfrac{1}{\tan 253.8°} \approx 0.2905$

40. Calculating the value: $\csc 113.2° = \dfrac{1}{\sin 113.2°} \approx 1.0880$

42. Calculating the value: $\tan 420° \approx 1.7321$

44. Calculating the value: $\sec 909.5° = \dfrac{1}{\cos 909.5°} \approx -1.0139$

46. Calculating the value: $\cos(-345°) \approx 0.9659$

48. Calculating the value: $\cot 171°40' = \cot\left(171\frac{2}{3}\right)^\circ = \dfrac{1}{\tan\left(171\frac{2}{3}\right)^\circ} \approx -6.8269$

50. Calculating the value: $\sec 341°10' = \sec\left(341\frac{1}{6}\right)^\circ = \dfrac{1}{\cos\left(341\frac{1}{6}\right)^\circ} \approx 1.0566$

52. Calculating the value: $\cos(-100°) \approx -0.1736$

54. The reference angle is given by $\hat{\theta} = \sin^{-1}(0.7660) \approx 50.0°$. Since θ terminates in quadrant IV, $\theta = 360° - 50.0° = 310.0°$.

56. The reference angle is given by $\hat{\theta} = \cos^{-1}(0.3090) \approx 72.0°$. Since θ terminates in quadrant III, $\theta = 180° + 72.0° = 252.0°$.

58. The reference angle is given by $\hat{\theta} = \tan^{-1}(0.5890) \approx 30.5°$. Since θ terminates in quadrant I, $\theta = 30.5°$.

60. The reference angle is given by $\hat{\theta} = \cos^{-1}(0.9652) \approx 15.2°$. Since θ terminates in quadrant IV, $\theta = 360° - 15.2° = 344.8°$.

62. The reference angle is given by $\hat{\theta} = \sin^{-1}(0.2644) \approx 15.3°$. Since θ terminates in quadrant I, $\theta = 15.3°$.

64. Since $\csc\theta = 4.3152$, $\sin\theta = \dfrac{1}{4.3152}$. The reference angle is given by $\hat{\theta} = \sin^{-1}\left(\dfrac{1}{4.3152}\right) \approx 13.4°$.
Since θ terminates in quadrant II, $\theta = 180° - 13.4° = 166.6°$.

66. Since $\sec\theta = -9.4135$, $\cos\theta = -\dfrac{1}{9.4135}$. The reference angle is given by $\hat{\theta} = \cos^{-1}\left(\dfrac{1}{9.4135}\right) \approx 83.9°$.
Since θ terminates in quadrant II, $\theta = 180° - 83.9° = 96.1°$.

68. Since $\cot\theta = -0.4321$, $\tan\theta = -\dfrac{1}{0.4321}$. The reference angle is given by $\hat{\theta} = \tan^{-1}\left(\dfrac{1}{0.4321}\right) \approx 66.6°$.
Since θ terminates in quadrant IV, $\theta = 360° - 66.6° = 293.4°$.

70. Since $\csc\theta = -7.1768$, $\sin\theta = -\dfrac{1}{7.1768}$. The reference angle is given by $\hat{\theta} = \sin^{-1}\left(\dfrac{1}{7.1768}\right) \approx 8.0°$.
Since θ terminates in quadrant III, $\theta = 180° + 8.0° = 188.0°$.

72. The reference angle is 30°. Since θ terminates in quadrant III, $\theta = 180° + 30° = 210°$.

74. The reference angle is 60°. Since θ terminates in quadrant III, $\theta = 180° + 60° = 240°$.

76. The reference angle is 45°. Since θ terminates in quadrant II, $\theta = 180° - 45° = 135°$.

78. The reference angle is 45°. Since θ terminates in quadrant III, $\theta = 180° + 45° = 225°$.

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80. Since $\csc\theta = -2$, $\sin\theta = -\frac{1}{2}$, so the reference angle is 30°. Since θ terminates in quadrant III, $\theta = 180° + 30° = 210°$.

82. Since $\sec\theta = \sqrt{2}$, $\cos\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$, so the reference angle is 45°. Since θ terminates in quadrant IV, $\theta = 360° - 45° = 315°$.

84. Since $\cot\theta = \frac{\sqrt{3}}{3}$, $\tan\theta = \frac{3}{\sqrt{3}} = \sqrt{3}$, so the reference angle is 60°. Since θ terminates in quadrant III, $\theta = 180° + 60° = 240°$.

86. The complement of 120° is $90° - 120° = -30°$, and the supplement is $180° - 120° = 60°$.

88. The complement of $90° - y$ is $90° - (90° - y) = y$, and the supplement is $180° - (90° - y) = 90° + y$.

90. The hypotenuse is $\frac{3\sqrt{2}}{4}$.

92. Simplifying the expression: $4\sin 60° - 2\cos 30° = 4 \cdot \frac{\sqrt{3}}{2} - 2 \cdot \frac{\sqrt{3}}{2} = 2\sqrt{3} - \sqrt{3} = \sqrt{3}$

94. Simplifying the expression: $(\sin 45° + \cos 45°)^2 = \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)^2 = \left(\frac{2}{\sqrt{2}}\right)^2 = \left(\sqrt{2}\right)^2 = 2$

96. The reference angle for 210° is 30°. Since 210° terminates in quadrant III, its cosine will be negative. Therefore:

$$\cos 210° = -\cos 30° = -\frac{\sqrt{3}}{2}$$

The correct answer is b.

98. The reference angle is given by $\hat{\theta} = \cos^{-1}(0.3256) \approx 71.0°$. Since θ terminates in quadrant III, $\theta = 180° + 71° = 251°$. The correct answer is c.

ODD SOLUTIONS

1. reference, x

3. positive

5. $\hat{\theta} = 180° - 150° = 30°$

7. $\hat{\theta} = 253.8° - 180° = 73.8°$

9. $\hat{\theta} = 360° - 311.7° = 48.3°$

11. $\hat{\theta} = 195°10' - 180° = 15°10'$

13. $\hat{\theta} = -300° + 360° = 60°$

15. $\hat{\theta} = -120° + 180° = 60°$

17. $\hat{\theta} = 180° - 135°$
$= 45°$
Since θ terminates in QII, $\cos\theta$ is negative.
$\cos 135° = -\cos 45° = -\frac{\sqrt{2}}{2}$

19. $\hat{\theta} = 210° - 180°$
$= 30°$
Since θ terminates in QIII, $\sin\theta$ is negative.
$\sin 210° = -\sin 30° = -\frac{1}{2}$

21. $\hat{\theta} = 180° - 135°$
$= 45°$
Since θ terminates in QII, $\tan\theta$ is negative.
$\tan 135° = -\tan 45° = -1$

23. $\theta = -240° + 360° = 120°$
$\hat{\theta} = 180° - 120° = 60°$
Since θ terminates in QII, $\cos\theta$ is negative.
$\cos(-240°) = -\cos 60° = -\frac{1}{2}$

25. $\theta = -330° + 360° = 30°$
Since θ terminates in QI, $\sec\theta$ is positive.
$\sec(-330°) = \sec 30° = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

27. $\hat{\theta} = 360° - 300° = 60°$
Since θ terminates in QIV, $\csc\theta$ is negative.
$\csc 300° = -\csc 60° = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

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29. $\hat{\theta} = 390^\circ - 360^\circ = 30^\circ$

Since θ terminates in QI, $\sin\theta$ is positive.

$$\sin 390^\circ = \sin 30^\circ = \frac{1}{2}$$

31. $\theta = 480^\circ - 360^\circ = 120^\circ$

$\hat{\theta} = 180^\circ - 120^\circ = 60^\circ$

Since θ terminates in QII, $\cot\theta$ is negative.

$$\cot 480^\circ = -\cot 60^\circ = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

For Problems 33 to 51, see answer section of textbook.

53. First, we find $\hat{\theta}$: $\sin\hat{\theta} = 0.3090$

$\hat{\theta} = \sin^{-1}(0.3090)$

$\hat{\theta} = 18.0^\circ$

Scientific Calculator: 0.3090 [*INV*] [*SIN*]

Graphing Calculator: [*2nd*] [*SIN*] [(] 0.3090 [)] [*ENTER*]

Since θ is in QIII, $\theta = 180^\circ + 18.0^\circ$

$= 198.0^\circ$

55. First, we find $\hat{\theta}$: $\cos\hat{\theta} = 0.7660$

$\hat{\theta} = \cos^{-1}(0.7660)$

$= 40.0^\circ$

Scientific Calculator: 0.7660 [*INV*] [*COS*]

Graphing Calculator: [*2nd*] [*COS*] [(] 0.7660 [)] [*ENTER*]

Since θ is in QII, $\theta = 180^\circ - 40.0^\circ$

$= 140.0^\circ$

57. First we find $\hat{\theta}$: $\tan\hat{\theta} = 0.5890$

$\hat{\theta} = \tan^{-1}(0.5890)$

$\hat{\theta} = 30.5^\circ$

Scientific Calculator: 0.5890 [*INV*] [*TAN*]

Graphing Calculator: [*2nd*] [*TAN*] [(] 0.5890 [)] [*ENTER*]

Since θ is in QIII, $\theta = 180^\circ + 30.5^\circ$

$= 210.5^\circ$

59. Since θ is in QI, $\hat{\theta} = \theta$ $\cos\theta = 0.2644$

$\theta = \cos^{-1}(0.2664)$

$= 74.7^\circ$

Scientific Calculator: 0.2644 [*INV*] [*COS*]

Graphing Calculator: [*2nd*] [*COS*] [(] 0.2644 [)] [*ENTER*]

61. First , we find $\hat{\theta}$: $\sin\hat{\theta} = 0.9652$

$\hat{\theta} = 74.8^\circ$

Scientific Calculator: 0.9652 [*INV*] [*SIN*]

Graphing Calculator: [*2nd*] [*SIN*] [(] 0.9652 [)] [*ENTER*]

Since θ is in QII, $\theta = 180^\circ - 74.8^\circ = 105.2^\circ$

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63. First, we find $\hat{\theta}$: $\sec\hat{\theta} = 1.4325$

$$\cos\hat{\theta} = \frac{1}{1.4325}$$

$$\hat{\theta} = \cos^{-1}\left(\frac{1}{1.4325}\right)$$

$$= 45.7^\circ$$

Scientific Calculator: 1.4325 $\boxed{1/x}$ $\boxed{INV}$ $\boxed{COS}$

Graphing Calculator: $\boxed{2nd}$ $\boxed{COS}$ $\boxed{(}$ 1.4325 $\boxed{x^{-1}}$ $\boxed{)}$ $\boxed{ENTER}$

Since θ is in QIV, $\theta = 360^\circ - 45.7^\circ$

$= 314.3^\circ$

65. First, we find $\hat{\theta}$: $\csc\hat{\theta} = 2.4957$

$$\sin\hat{\theta} = \frac{1}{2.4957}$$

$$\hat{\theta} = \sin^{-1}\left(\frac{1}{2.4957}\right)$$

$$\hat{\theta} = 23.6^\circ$$

Scientific Calculator: 2.4957 $\boxed{1/x}$ $\boxed{INV}$ $\boxed{SIN}$

Graphing Calculator: $\boxed{2nd}$ $\boxed{SIN}$ $\boxed{(}$ 2.4957 $\boxed{x^{-1}}$ $\boxed{)}$ $\boxed{ENTER}$

Since θ is in QII, $\theta = 180^\circ - 23.6^\circ$

$= 156.4^\circ$

67. First, we find $\hat{\theta}$: $\cot\hat{\theta} = 0.7366$

$$\tan\hat{\theta} = \frac{1}{0.7366}$$

$$\hat{\theta} = \tan^{-1}\left(\frac{1}{0.7366}\right)$$

$$\hat{\theta} = 53.6^\circ$$

Scientific Calculator: 0.7366 $\boxed{1/x}$ $\boxed{INV}$ $\boxed{TAN}$

Graphing Calculator: $\boxed{2nd}$ $\boxed{TAN}$ $\boxed{(}$ 0.7366 $\boxed{x^{-1}}$ $\boxed{)}$ $\boxed{ENTER}$

Since θ is in QII, $\theta = 180^\circ - 53.6^\circ$

$= 126.4^\circ$

69. First, we find $\hat{\theta}$: $\sec\hat{\theta} = 1.7876$

$$\cos\hat{\theta} = \frac{1}{1.7876}$$

$$\hat{\theta} = \cos^{-1}\left(\frac{1}{1.7876}\right)$$

$$\hat{\theta} = 56.0^\circ$$

Scientific Calculator: $0.1.7876$ $\boxed{1/x}$ $\boxed{INV}$ $\boxed{COS}$

Graphing Calculator: $\boxed{2nd}$ $\boxed{COS}$ $\boxed{(}$ 1.7876 $\boxed{x^{-1}}$ $\boxed{)}$ $\boxed{ENTER}$

Since θ is in QIII, $\theta = 180^\circ + 56.0^\circ = 236.0^\circ$

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71. First, we find $\hat{\theta}$: $\sin\hat{\theta} = \frac{\sqrt{3}}{2}$

$\hat{\theta} = 60^\circ$

Since θ is in QIII, $\theta = 180^\circ + 60^\circ$

$= 240^\circ$

73. First, we find $\hat{\theta}$: $\cos\hat{\theta} = \frac{\sqrt{2}}{2}$

$\hat{\theta} = 45^\circ$

Since θ is in QII, $\theta = 180^\circ - 45^\circ$

$= 135^\circ$

75. First, we find $\hat{\theta}$: $\sin\hat{\theta} = \frac{\sqrt{3}}{2}$

$\hat{\theta} = 60^\circ$

Since θ is in QIV, $\theta = 360^\circ - 60^\circ$

$= 300^\circ$

77. First, we find $\hat{\theta}$: $\tan\hat{\theta} = \sqrt{3}$

$\hat{\theta} = 60^\circ$

Since θ is in QIII, $\theta = 180^\circ + 60^\circ$

$= 240^\circ$

79. First, we find $\hat{\theta}$: $\sec\hat{\theta} = 2$

$\cos\hat{\theta} = \frac{1}{2}$

$\hat{\theta} = 60^\circ$

Since θ is in QII, $\theta = 180^\circ - 60^\circ$

$= 120^\circ$

81. First, we find $\hat{\theta}$: $\csc\hat{\theta} = \sqrt{2}$

$\sin\hat{\theta} = \frac{1}{\sqrt{2}}$

$\hat{\theta} = 45^\circ$

Since θ is in QII, $\theta = 180^\circ - 45^\circ$

$= 135^\circ$

83. First, we find $\hat{\theta}$: $\cot\hat{\theta} = 1$

$\tan\hat{\theta} = 1$

$\hat{\theta} = 45^\circ$

Since θ is in QIV, $\theta = 360^\circ - 45^\circ$

$= 315^\circ$

85. The complement of 70° *is* 20° *because* $70^\circ + 20^\circ = 90^\circ$.

The supplement of 70° *is* 110° *because* $70^\circ + 110^\circ = 180^\circ$.

87. The complement of x is $90^\circ - x$.

The supplement of x is $180^\circ - x$.

89. The side opposite the 30° angle is one-half of the longest side, or $\frac{1}{2} \cdot 10 = 5$.

The side opposite the 60° angle is $\sqrt{3}$ times the shortest side, or $5\sqrt{3}$.

91. $\sin 30^\circ \cos 60^\circ = \frac{1}{2} \cdot \frac{1}{2}$

$= \frac{1}{4}$

93. $\sin^2 45^\circ + \cos^2 45^\circ = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{1}{2} + \frac{1}{2} = 1$

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3.2 Radians and Degrees

EVEN SOLUTIONS

2. When converting between degree measure and radian measure, use the fact that 180 degrees equals π radians.

4. **a**. $30° = \frac{\pi}{6}$ radians (iii) **b**. $45° = \frac{\pi}{4}$ radians (iv)

c. $60° = \frac{\pi}{3}$ radians (ii) **d**. $90° = \frac{\pi}{2}$ radians (i)

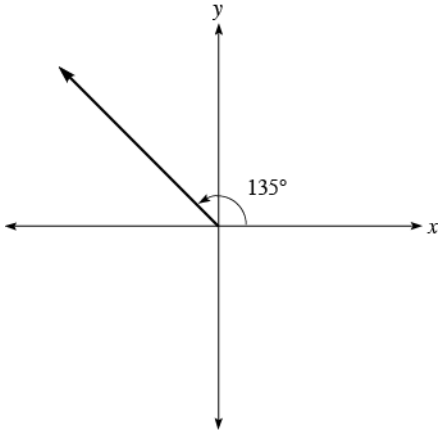
6. Computing the radian measure: $\theta = \frac{s}{r} = \frac{5}{20} = \frac{1}{4}$ radian

8. Computing the radian measure: $\theta = \frac{s}{r} = \frac{12\pi}{3} = 4\pi$ radians

10. Computing the radian measure: $\theta = \frac{s}{r} = \frac{1/16}{1/4} = \frac{1}{16} \cdot \frac{4}{1} = \frac{1}{4}$ radian

12. Computing the radian measure: $\theta = \frac{s}{r} = \frac{2500}{4000} = \frac{5}{8}$ radian $= 0.625$ radian

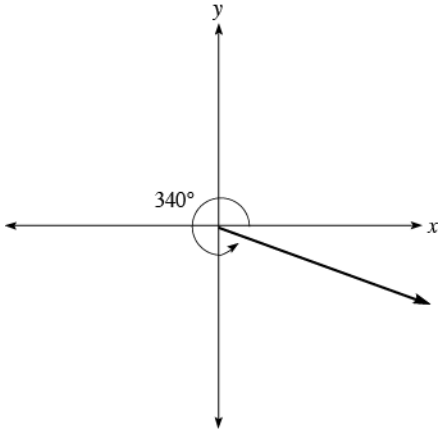
14. **a**. Drawing the angle in standard position:



b. Converting to radian measure: $135° = 135 \cdot \frac{\pi}{180} = \frac{3\pi}{4}$ radians

c. The reference angle is $45° = \frac{\pi}{4}$ radians .

16. **a**. Drawing the angle in standard position:



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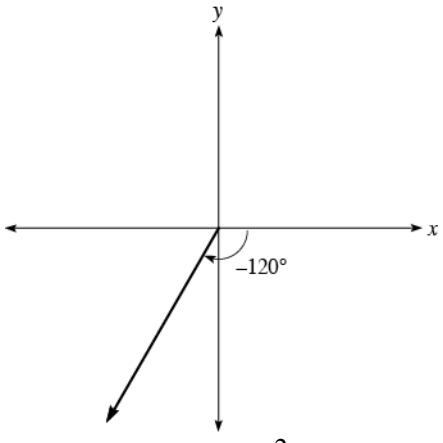
b. Converting to radian measure: $340° = 340 \cdot \frac{\pi}{180} = \frac{17\pi}{9}$ radians

c. The reference angle is $20° = \frac{\pi}{9}$ radians .

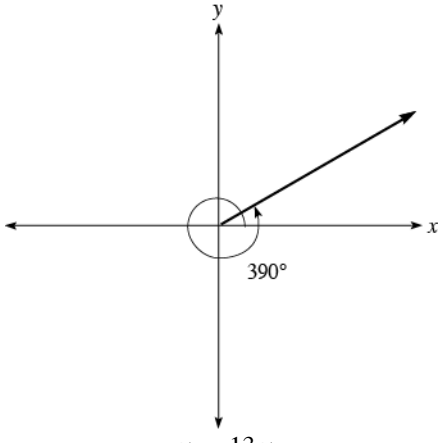
18. **a**. Drawing the angle in standard position:



b. Converting to radian measure: $-120° = -120 \cdot \frac{\pi}{180} = -\frac{2\pi}{3}$ radians

c. The reference angle is $60° = \frac{\pi}{3}$ radians .

20. **a**. Drawing the angle in standard position:



b. Converting to radian measure: $390° = 390 \cdot \frac{\pi}{180} = \frac{13\pi}{6}$ radians

c. The reference angle is $30° = \frac{\pi}{6}$ radians .

22. Converting to radian measure: $216°10' = 216\frac{1^{\circ}}{6} = 216\frac{1}{6} \cdot \frac{\pi}{180} \approx 3.77$ radians

24. Converting to radian measure: $1° = 1 \cdot \frac{\pi}{180} \approx 0.0175$ radians

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26. Let S represent the distance (in statute miles) apart. Since $\theta = 30' = \frac{1}{2}^\circ$, convert to radians:

$\theta = \frac{1}{2}^\circ = \frac{1}{2} \bullet \frac{\pi}{180} = \frac{\pi}{360}$ radians. Therefore:

$$\frac{\pi}{360} = \frac{S}{4{,}000}$$
$$360S = 4{,}000\pi$$
$$S = \frac{4{,}000\pi}{360} \approx 34.8$$

The ships are approximately 34.8 statute miles apart.

28. First note the conversion: $1' = \frac{1}{60}^\circ = \frac{1}{60} \bullet \frac{\pi}{180} = \frac{\pi}{10{,}800}$ radians

So 1 nautical mile corresponds to an angle of $\frac{\pi}{10{,}800}$ radians. Then 1 radian = $\frac{10{,}800}{\pi}$ nautical miles, so

0.1 radian = $\frac{1{,}080}{\pi} \approx 343.8$ nautical miles.

30. Since a 35 minute period is $\frac{35}{60} = \frac{7}{12}$ of a complete revolution, the number of radians is: $\frac{7}{12} \bullet 2\pi = \frac{7\pi}{6}$

32. Simplifying: $\pi + \frac{\pi}{6} = \frac{6\pi}{6} + \frac{\pi}{6} = \frac{7\pi}{6}$

34. Simplifying: $2\pi - \frac{\pi}{2} = \frac{4\pi}{2} - \frac{\pi}{2} = \frac{3\pi}{2}$

36. Simplifying: $\frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{6} + \frac{2\pi}{6} = \frac{3\pi}{6} = \frac{\pi}{2}$

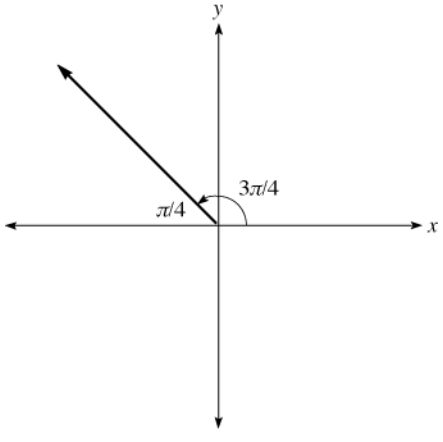
38. Simplifying: $\frac{3\pi}{4} - \frac{\pi}{2} = \frac{3\pi}{4} - \frac{2\pi}{4} = \frac{\pi}{4}$

40. Writing as a sum: $\frac{4\pi}{3} = \frac{3\pi}{3} + \frac{\pi}{3} = \pi + \frac{\pi}{3}$

42. Writing as a difference: $\frac{\pi}{4} = \frac{4\pi}{4} - \frac{3\pi}{4} = \pi - \frac{3\pi}{4}$

44. Writing as a difference: $\frac{11\pi}{6} = \frac{12\pi}{6} - \frac{\pi}{6} = 2\pi - \frac{\pi}{6}$

46. **a**. Drawing the angle in standard position:



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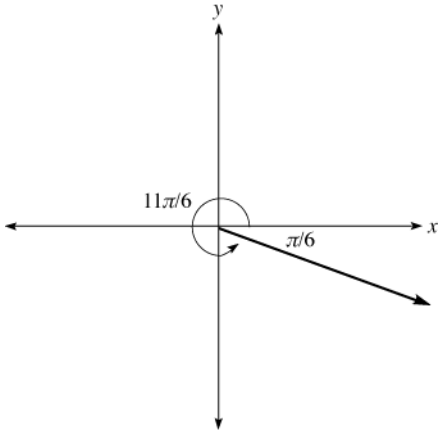
b. Converting to degree measure: $\frac{3\pi}{4}$ radians $= \frac{3\pi}{4} \cdot \frac{180°}{\pi} = 135°$

c. The reference angle is $45° = \frac{\pi}{4}$ radians .

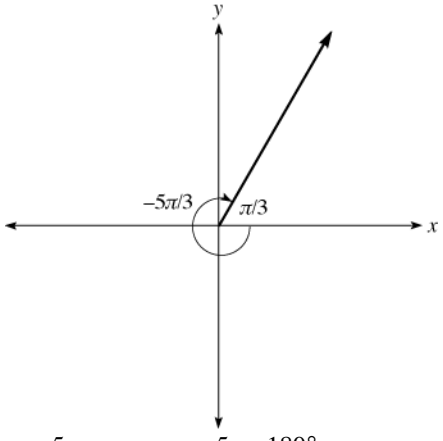
48. **a**. Drawing the angle in standard position:



b. Converting to degree measure: $\frac{11\pi}{6}$ radians $= \frac{11\pi}{6} \cdot \frac{180°}{\pi} = 330°$

c. The reference angle is $30° = \frac{\pi}{6}$ radians .

50. **a**. Drawing the angle in standard position:



b. Converting to degree measure: $-\frac{5\pi}{3}$ radians $= -\frac{5\pi}{3} \cdot \frac{180°}{\pi} = -300°$

c. The reference angle is $60° = \frac{\pi}{3}$ radians .

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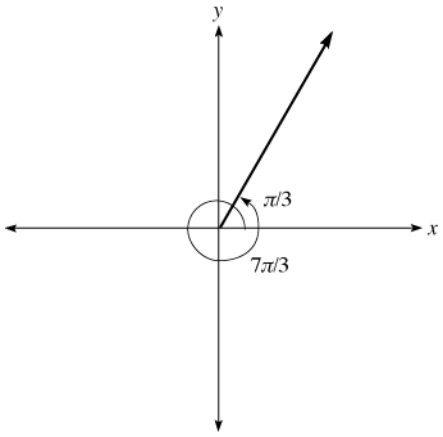
52. **a**. Drawing the angle in standard position:



b. Converting to degree measure: $\frac{7\pi}{3}$ radians $= \frac{7\pi}{3} \bullet \frac{180°}{\pi} = 420°$

c. The reference angle is $60° = \frac{\pi}{3}$ radians .

54. Converting to degree measure: $4.2 \text{ radians} = 4.2 \bullet \frac{180°}{\pi} = \frac{756°}{\pi} \approx 240.6°$

56. Converting to degree measure: $2 \text{ radians} = 2 \bullet \frac{180°}{\pi} = \frac{360°}{\pi} \approx 114.6°$

58. Since $\frac{5\pi}{3}$ terminates in quadrant IV, its cosine will be positive. The reference angle is $2\pi - \frac{5\pi}{3} = \frac{\pi}{3}$, therefore:

$$\cos\frac{5\pi}{3} = \cos\frac{\pi}{3} = \frac{1}{2}$$

60. Since $\frac{\pi}{4}$ terminates in quadrant I, its cotangent will be positive. The reference angle is $\frac{\pi}{4}$, therefore:

$$\cot\frac{\pi}{4} = \frac{\cos\frac{\pi}{4}}{\sin\frac{\pi}{4}} = \frac{\sqrt{2}/2}{\sqrt{2}/2} = 1$$

62. Using the point $(0,-1)$ as a point on the terminal side of $\frac{3\pi}{2}$: $\csc\frac{3\pi}{2} = \frac{1}{\sin\frac{3\pi}{2}} = \frac{1}{-1} = -1$

64. Since $\frac{5\pi}{6}$ terminates in quadrant II, its secant will be negative. The reference angle is $\pi - \frac{5\pi}{6} = \frac{\pi}{6}$, therefore:

$$\sec\frac{5\pi}{6} = -\sec\frac{\pi}{6} = -\frac{1}{\cos\frac{\pi}{6}} = -\frac{1}{\sqrt{3}/2} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

66. Computing the value: $-\cos\frac{\pi}{3} = -\frac{1}{2}$

68. Since $-\frac{\pi}{6}$ terminates in quadrant IV, its sine will be negative. The reference angle is $\frac{\pi}{6}$, therefore:

$$4\sin\left(-\frac{\pi}{6}\right) = -4\sin\frac{\pi}{6} = -4 \bullet \frac{1}{2} = -2$$

70. Evaluating the expression: $\cos\left(4 \bullet \frac{\pi}{6}\right) = \cos\frac{2\pi}{3} = -\frac{1}{2}$

72. Evaluating the expression: $5\sin\left(3 \bullet \frac{\pi}{6}\right) = 5\sin\frac{\pi}{2} = 5 \bullet 1 = 5$

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74. Evaluating the expression: $\frac{1}{2}+\cos\frac{\pi}{6}=\frac{1}{2}+\frac{\sqrt{3}}{2}=\frac{1+\sqrt{3}}{2}$

76. Evaluating the expression: $\cos\left(\frac{\pi}{6}-\frac{\pi}{3}\right)=\cos\left(-\frac{\pi}{6}\right)=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$

78. Evaluating the expression: $-\frac{1}{2}\sin\left(3\bullet\frac{\pi}{6}+\frac{\pi}{6}\right)=-\frac{1}{2}\sin\frac{2\pi}{3}=-\frac{1}{2}\sin\frac{\pi}{3}=-\frac{1}{2}\bullet\frac{\sqrt{3}}{2}=-\frac{\sqrt{3}}{4}$

80. Evaluating the expression: $-1+\frac{3}{4}\cos\left(4\bullet\frac{\pi}{6}-\frac{\pi}{2}\right)=-1+\frac{3}{4}\cos\left(\frac{2\pi}{3}-\frac{\pi}{2}\right)=-1+\frac{3}{4}\cos\frac{\pi}{6}=-1+\frac{3}{4}\bullet\frac{\sqrt{3}}{2}=\frac{-8+3\sqrt{3}}{8}$

82. Computing the *y*-coordinates: $\cos 0=1,\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2},\cos\frac{\pi}{2}=0,\cos\frac{3\pi}{4}=-\frac{\sqrt{2}}{2},\cos\pi=-1$

The ordered pairs are $(0,1),\left(\frac{\pi}{4},\frac{\sqrt{2}}{2}\right),\left(\frac{\pi}{2},0\right),\left(\frac{3\pi}{4},-\frac{\sqrt{2}}{2}\right)$, and $(\pi,-1)$.

84. Computing the *y*-coordinates: $-\sin 0=0,-\sin\frac{\pi}{2}=-1,-\sin\pi=0,-\sin\frac{3\pi}{2}=1,-\sin 2\pi=0$

The ordered pairs are $(0,0),\left(\frac{\pi}{2},-1\right),(\pi,0),\left(\frac{3\pi}{2},1\right)$, and $(2\pi,0)$.

86. Computing the *y*-coordinates: $3\sin 0=0,3\sin\frac{\pi}{2}=3,3\sin\pi=0,3\sin\frac{3\pi}{2}=-3,3\sin 2\pi=0$

The ordered pairs are $(0,0),\left(\frac{\pi}{2},3\right),(\pi,0),\left(\frac{3\pi}{2},-3\right)$, and $(2\pi,0)$.

88. Computing the *y*-coordinates:

$$\cos\left(\frac{1}{2}\bullet 0\right)=\cos 0=1,\cos\left(\frac{1}{2}\bullet\pi\right)=\cos\frac{\pi}{2}=0,\cos\left(\frac{1}{2}\bullet 2\pi\right)=\cos\pi=-1,$$

$$\cos\left(\frac{1}{2}\bullet 3\pi\right)=\cos\frac{3\pi}{2}=0,\cos\left(\frac{1}{2}\bullet 4\pi\right)=\cos 2\pi=1$$

The ordered pairs are $(0,1),(\pi,0),(2\pi,-1),(3\pi,0)$, and $(4\pi,1)$.

90. Computing the *y*-coordinates:

$$\sin\left(-\frac{\pi}{4}+\frac{\pi}{4}\right)=\sin 0=0,\sin\left(0+\frac{\pi}{4}\right)=\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2},\sin\left(\frac{\pi}{4}+\frac{\pi}{4}\right)=\sin\frac{\pi}{2}=1,$$

$$\sin\left(\frac{\pi}{2}+\frac{\pi}{4}\right)=\sin\frac{3\pi}{4}=\frac{\sqrt{2}}{2},\sin\left(\frac{3\pi}{4}+\frac{\pi}{4}\right)=\sin\pi=0$$

The ordered pairs are $\left(-\frac{\pi}{4},0\right),\left(0,\frac{\sqrt{2}}{2}\right),\left(\frac{\pi}{4},1\right),\left(\frac{\pi}{2},\frac{\sqrt{2}}{2}\right)$, and $\left(\frac{3\pi}{4},0\right)$.

92. Computing the *y*-coordinates:

$$-3+\sin 0=-3,-3+\sin\frac{\pi}{2}=-2,-3+\sin\pi=-3,-3+\sin\frac{3\pi}{2}=-4,-3+\sin 2\pi=-3$$

The ordered pairs are $(0,-3),\left(\frac{\pi}{2},-2\right),(\pi,-3),\left(\frac{3\pi}{2},-4\right)$, and $(2\pi,-3)$.

94. Computing the *y*-coordinates:

$$5\cos\left(2\bullet\frac{\pi}{6}-\frac{\pi}{3}\right)=5\cos 0=5,5\cos\left(2\bullet\frac{\pi}{3}-\frac{\pi}{3}\right)=5\cos\frac{\pi}{3}=2.5,5\cos\left(2\bullet\frac{2\pi}{3}-\frac{\pi}{3}\right)=5\cos\pi=-5,$$

$$5\cos\left(2\bullet\pi-\frac{\pi}{3}\right)=5\cos\frac{5\pi}{3}=2.5,5\cos\left(2\bullet\frac{7\pi}{6}-\frac{\pi}{3}\right)=5\cos 2\pi=5$$

The ordered pairs are $\left(\frac{\pi}{6},5\right),\left(\frac{\pi}{3},2.5\right),\left(\frac{2\pi}{3},-5\right),(\pi,2.5)$, and $\left(\frac{7\pi}{6},5\right)$.

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96. The central angle measurement is: $\frac{2\pi}{16} = \frac{\pi}{8}$

98. First convert each angle to radians:

$$LT_1 = 35°51.449' = 35° + \left(\frac{51.449}{60}\right)° \approx 35.8575° = 35.8575\left(\frac{\pi}{180}\right) \text{ rad} \approx 0.626 \text{ rad}$$

$$LN_1 = 14°28.668' = 14° + \left(\frac{28.668}{60}\right)° = 14.4778° = 14.4778\left(\frac{\pi}{180}\right) \text{ rad} \approx 0.253 \text{ rad}$$

$$LT_2 = 36°25.155' = 36° + \left(\frac{24.155}{60}\right)° \approx 36.4026° = 36.4026\left(\frac{\pi}{180}\right) \text{ rad} \approx 0.635 \text{ rad}$$

$$LN_2 = 25°28.776' = 25° + \left(\frac{28.776}{60}\right)° = 25.4796° = 25.4796\left(\frac{\pi}{180}\right) \text{ rad} \approx 0.445 \text{ rad}$$

The great circle distance is then given as:

$$\begin{aligned}&\sin(LT_1)\sin(LT_2) + \cos(LT_1)\cos(LT_2)\cos(LN_1 - LN_2)\\ &= \sin(0.626)\sin(0.635) + \cos(0.626)\cos(0.635)\cos(0.253 - 0.445)\\ &\approx 0.9880\end{aligned}$$

100. First find $r = \sqrt{(-1)^2 + 3^2} = \sqrt{1+9} = \sqrt{10}$. Using $x = -1$, $y = 3$, and $r = \sqrt{10}$ in the definitions:

$$\sin\theta = \frac{y}{r} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10} \qquad \cos\theta = \frac{x}{r} = -\frac{1}{\sqrt{10}} = -\frac{\sqrt{10}}{10} \qquad \tan\theta = \frac{y}{x} = \frac{3}{-1} = -3$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{10}}{3} \qquad \sec\theta = \frac{r}{x} = -\sqrt{10} \qquad \cot\theta = \frac{x}{y} = -\frac{1}{3}$$

102. Choose $x = -1$ and $r = \sqrt{2}$. Find y using $x^2 + y^2 = r^2$:

$$\begin{aligned}(-1)^2 + y^2 &= \left(\sqrt{2}\right)^2\\ 1 + y^2 &= 2\\ y^2 &= 1\\ y &= \pm 1\end{aligned}$$

Since θ terminates in quadrant II, $y > 0$ and thus $y = 1$. Using $x = -1$, $y = 1$, and $r = \sqrt{2}$ in the definitions:

$$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \tan\theta = \frac{y}{x} = \frac{1}{-1} = -1 \qquad \sec\theta = \frac{r}{x} = \frac{\sqrt{2}}{-1} = -\sqrt{2}$$

$$\csc\theta = \frac{r}{y} = \sqrt{2} \qquad \cot\theta = \frac{x}{y} = \frac{-1}{1} = -1$$

104. Since θ terminates in quadrant III, $x < 0$ and $y < 0$. Choose $x = -1$ and $y = -2$. Find $r = \sqrt{(-1)^2 + (-2)^2} = \sqrt{1+4} = \sqrt{5}$. Using $x = -1$, $y = -2$, and $r = \sqrt{5}$ in the definitions:

$$\sin\theta = \frac{y}{r} = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5} \qquad \cos\theta = \frac{x}{r} = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5} \qquad \tan\theta = \frac{y}{x} = \frac{-2}{-1} = 2$$

$$\csc\theta = \frac{r}{y} = -\frac{\sqrt{5}}{2} \qquad \sec\theta = \frac{r}{x} = -\sqrt{5} \qquad \cot\theta = \frac{x}{y} = \frac{-1}{-2} = \frac{1}{2}$$

106. Converting to radian measure: $80° = 80 \bullet \frac{\pi}{180} = \frac{4\pi}{9}$. The correct answer is b.

108. The reference angle is: $2\pi - \frac{11\pi}{6} = \frac{12\pi}{6} - \frac{11\pi}{6} = \frac{\pi}{6}$. The correct answer is d.

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ODD SOLUTIONS

1. radius

3. **a.** θ **b.** $\pi-\theta$ **c.** $\theta-\pi$ **d.** $2\pi-\theta$

5.
$\theta=\dfrac{s}{r}$ Def of radian measure

$=\dfrac{9\,cm}{3\,cm}$ Substitute given values

$=3$ Divide

7.
$\theta=\dfrac{s}{r}$ Def of radian measure

$=\dfrac{3\pi\ inches}{12\ inches}$ Substitute given values

$=\dfrac{\pi}{4}$ Divide

9.
$\theta=\dfrac{s}{r}$ Def of radian measure

$=\dfrac{\frac{1}{2}\,cm}{\frac{1}{4}\,cm}$ Substitute given values

$=2$ Divide

11.
$\theta=\dfrac{s}{r}$ Def of radian measure

$=\dfrac{450}{4000}$ Substitute given values

$=0.1125$ Divide

13. **b.** $30^\circ=30\left(\dfrac{\pi}{180}\right)$

$=\dfrac{\pi}{6}$ radians

c. Reference angle is itself :

$30^\circ=\dfrac{\pi}{6}$ radians

15. **b.** $260^\circ=260\left(\dfrac{\pi}{180}\right)$

$=\dfrac{13\pi}{9}\ radians$

c. $\hat{\theta}=260^\circ-180^\circ=80^\circ$

$80^\circ=80\left(\dfrac{\pi}{180}\right)$

$=\dfrac{4\pi}{9}\ radians$

17. **b.** $-150^\circ=-150\left(\dfrac{\pi}{180}\right)$

$=-\dfrac{5\pi}{6}$

c. $-150^\circ=-150+360^\circ$

$=210^\circ$

$\hat{\theta}=210^\circ-180^\circ=30^\circ$

$30^\circ=30\left(\dfrac{\pi}{180}\right)$

$=\dfrac{\pi}{6}\ radians$

19. **b.** $420^\circ=420\left(\dfrac{\pi}{180}\right)$

$=\dfrac{7\pi}{3}\ radians$

c. $\hat{\theta}=420^\circ-360^\circ=60^\circ$

$60^\circ=60\left(\dfrac{\pi}{180}\right)=\dfrac{\pi}{3}\ radians$

21. $120^\circ 40'=\left(120+\dfrac{40}{60}\right)^\circ=120.67^\circ$

$120.67^\circ=120.67\left(\dfrac{\pi}{180}\right)$

$=2.11$

23. 1 minute $=\dfrac{1}{60}$ degree

$=\dfrac{1}{60}\left(\dfrac{\pi}{180}\right)$

$=\dfrac{\pi}{10{,}800}=0.000291$

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25. From problem 23, we know that $1' = \dfrac{\pi}{10{,}800}$.

$$\theta = \frac{s}{r}$$
$$s = r\theta$$
$$= 4{,}000\left(\frac{\pi}{10{,}800}\right)$$
$$= 1.16 \text{ miles}$$

27. $$\theta = 70\left(\frac{\pi}{10{,}800}\right)$$
$$= \frac{7\pi}{1080} = 0.02 \text{ radians}$$

29. $$\theta = \frac{5 \text{ min}}{60 \text{ min}} \cdot 2\pi$$
$$= \frac{1}{12} \cdot 2\pi = \frac{\pi}{6}$$

31. $$\pi - \frac{\pi}{3} = \frac{3\pi}{3} - \frac{\pi}{3} = \frac{2\pi}{3}$$

33. $$2\pi - \frac{\pi}{4} = \frac{8\pi}{4} - \frac{\pi}{4} = \frac{7\pi}{4}$$

35. $$\frac{\pi}{6} + \frac{\pi}{2} = \frac{\pi}{6} + \frac{3\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$$

37. $$\frac{5\pi}{4} - \frac{\pi}{2} = \frac{5\pi}{4} - \frac{2\pi}{4} = \frac{3\pi}{4}$$

39. $$\frac{2\pi}{3} = \frac{3\pi}{3} - \frac{\pi}{3} = \pi - \frac{\pi}{3}$$

41. $$\frac{7\pi}{6} = \frac{6\pi}{6} + \frac{\pi}{6} = \pi + \frac{\pi}{6}$$

43. $$\frac{7\pi}{4} = \frac{8\pi}{4} - \frac{\pi}{4} = 2\pi - \frac{\pi}{4}$$

45. **a.** $$\frac{\pi}{3} = \frac{\pi}{3}\left(\frac{180}{\pi}\right)^\circ$$
$$= 60^\circ$$

c. Reference angle is itself:
$$\frac{\pi}{3} = 60^\circ$$

47. **a.** $$\frac{4\pi}{3} = \frac{4\pi}{3}\left(\frac{180}{\pi}\right)^\circ$$
$$= 240^\circ$$

c. $$\hat{\theta} = 240^\circ - 180^\circ = 60^\circ$$
$$60^\circ = \frac{\pi}{3}$$

49. **a.** $$-\frac{7\pi}{6} = -\frac{7\pi}{6}\left(\frac{180}{\pi}\right)^\circ$$
$$= -210^\circ$$

c. $$-210^\circ = -210^\circ + 360^\circ = 150^\circ$$
$$\hat{\theta} = 180^\circ - 150^\circ = 30^\circ$$
$$30^\circ = 30\left(\frac{\pi}{180}\right) = \frac{\pi}{6}$$

51. **a.** $$\frac{11\pi}{4} = \frac{11\pi}{4}\left(\frac{180}{\pi}\right)^\circ$$
$$= 495^\circ$$
$$495^\circ - 360^\circ = 135^\circ$$

c. $$\hat{\theta} = 180^\circ - 135^\circ = 45^\circ$$
$$45^\circ = \frac{\pi}{4}$$

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53. $1 = 1\left(\frac{180}{\pi}\right)^\circ$

$= \left(\frac{180}{\pi}\right)^\circ$

$= 57.3^\circ$

55. $0.25 = 0.25\left(\frac{180}{\pi}\right)^\circ$

$= \left(\frac{45}{\pi}\right)^\circ$

$= 14.3^\circ$

57. Since $\frac{4\pi}{3}$ terminates in QIII, its sine will be negative.

$\hat{\theta} = \frac{4\pi}{3} - \pi = \frac{\pi}{3}$

$\sin\frac{4\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2} \qquad \left(\frac{\pi}{3} = 60^\circ\right)$

59. $\tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$

61. Since $\frac{2\pi}{3}$ terminates in QII, its secant will be negative.

$\hat{\theta} = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$

$\sec\frac{2\pi}{3} = -\sec\frac{\pi}{3} = -2$

63. Since $\frac{5\pi}{6}$ terminates in QII, its cosecant is positive.

$\hat{\theta} = \pi - \frac{5\pi}{6} = \frac{\pi}{6}$

$\csc\frac{5\pi}{6} = \csc\frac{\pi}{6}$

$= \frac{1}{\sin\frac{\pi}{6}} = \frac{1}{\frac{1}{2}} = 2$

65. $-\sin\frac{\pi}{6} = -\frac{1}{2}$

67. Since $-\frac{\pi}{4}$ terminates in QIV, its cosine will be positive.

$\theta = -\frac{\pi}{4} + 2\pi = \frac{7\pi}{4}$

$\hat{\theta} = 2\pi - \frac{7\pi}{4} = \frac{\pi}{4}$

$4\cos\left(-\frac{\pi}{4}\right) = 4\cos\frac{\pi}{4}$

$= 4\left(\frac{\sqrt{2}}{2}\right)$

$= 2\sqrt{2}$

69. $\sin 2x = \sin 2\left(\frac{\pi}{6}\right)$

$= \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

71. $\frac{1}{2}\cos 2\left(\frac{\pi}{6}\right) = \frac{1}{2}\cos\frac{\pi}{3}$

$= \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$

73. $3 - \sin\left(\frac{\pi}{6}\right) = 3 - \frac{1}{2} = \frac{5}{2}$

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75. $\sin\left(x+\frac{\pi}{2}\right)=\sin\left(\frac{\pi}{6}+\frac{\pi}{2}\right)$

$=\sin\frac{2\pi}{3}$

Since $\frac{2\pi}{3}$ terminates in QII, its sine is positive.

$\hat{\theta}=\pi-\frac{2\pi}{3}=\frac{\pi}{3}$

$\sin\frac{2\pi}{3}=\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$

77. $4\cos\left[2\left(\frac{\pi}{6}\right)+\frac{\pi}{3}\right]=4\cos\left(\frac{\pi}{3}+\frac{\pi}{3}\right)=4\cos\frac{2\pi}{3}$

Since $\frac{2\pi}{3}$ terminates in QII, its cosine will be negative.

$\hat{\theta}=\pi-\frac{2\pi}{3}=\frac{\pi}{3}$ $\qquad 4\cos\frac{2\pi}{3}=-4\cos\frac{\pi}{3}=-4\left(\frac{1}{2}\right)=-2$

79. $4-\frac{2}{3}\sin\left(3\cdot\frac{\pi}{6}-\pi\right)=4-\frac{2}{3}\sin\left(\frac{\pi}{2}-\pi\right)$

$=4-\frac{2}{3}\sin\left(-\frac{\pi}{2}\right)$

$=4-\frac{2}{3}(-1)=\frac{14}{3}$

81. For $x=0$, $\quad y=\sin 0=0$ $\quad (x,y)=(0,0)$

For $x=\frac{\pi}{4}$, $\quad y=\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}$ $\quad (x,y)=\left(\frac{\pi}{4},\frac{\sqrt{2}}{2}\right)$

For $x=\frac{\pi}{2}$, $\quad y=\sin\frac{\pi}{2}=1$ $\quad (x,y)=\left(\frac{\pi}{2},1\right)$

For $x=\frac{3\pi}{4}$, $\quad y=\sin\frac{3\pi}{4}=\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}$ $\quad (x,y)=\left(\frac{3\pi}{4},\frac{\sqrt{2}}{2}\right)$

For $x=\pi$, $\quad y=\sin\pi=0$ $\quad (x,y)=(\pi,0)$

83. For $x=0$, $\quad y=-\cos 0=-1$ $\quad (x,y)=(0,-1)$

For $x=\frac{\pi}{2}$, $\quad y=-\cos\frac{\pi}{2}=0$ $\quad (x,y)=\left(\frac{\pi}{2},0\right)$

For $x=\pi$, $\quad y=-\cos\pi=-(-1)=1$ $\quad (x,y)=(\pi,1)$

For $x=\frac{3\pi}{2}$, $\quad y=-\cos\frac{3\pi}{2}=0$ $\quad (x,y)=\left(\frac{3\pi}{2},0\right)$

For $x=2\pi$, $\quad y=-\cos(2\pi)=-1$ $\quad (x,y)=(2\pi,-1)$

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85. For $x = 0$ $\quad y = \frac{1}{2}\cos 0 = \frac{1}{2}(1) = \frac{1}{2}$ $\quad (x,y) = \left(0, \frac{1}{2}\right)$

For $x = \frac{\pi}{2}$, $\quad y = \frac{1}{2}\cos\frac{\pi}{2} = \frac{1}{2}(0) = 0$ $\quad (x,y) = \left(\frac{\pi}{2}, 0\right)$

For $x = \pi$, $\quad y = \frac{1}{2}\cos\pi = \frac{1}{2}(-1) = -\frac{1}{2}$ $\quad (x,y) = \left(\pi, -\frac{1}{2}\right)$

For $x = \frac{3\pi}{2}$, $\quad y = \frac{1}{2}\cos\frac{3\pi}{2} = \frac{1}{2}(0) = 0$ $\quad (x,y) = \left(\frac{3\pi}{2}, 0\right)$

For $x = 2\pi$, $\quad y = \frac{1}{2}\cos 2\pi = \frac{1}{2}(1) = \frac{1}{2}$ $\quad (x,y) = \left(2\pi, \frac{1}{2}\right)$

87. For $x = 0$, $\quad y = \sin 2(0) = \sin 0 = 0$ $\quad (x,y) = (0,0)$

For $x = \frac{\pi}{4}$, $\quad y = \sin 2\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{2} = 1$ $\quad (x,y) = \left(\frac{\pi}{4}, 1\right)$

For $x = \frac{\pi}{2}$, $\quad y = \sin 2\left(\frac{\pi}{2}\right) = \sin\pi = 0$ $\quad (x,y) = \left(\frac{\pi}{2}, 0\right)$

For $x = \frac{3\pi}{4}$, $\quad y = \sin 2\left(\frac{3\pi}{4}\right) = \sin\frac{3\pi}{2} = -1$ $\quad (x,y) = \left(\frac{3\pi}{4}, -1\right)$

For $x = \pi$, $\quad y = \sin 2(\pi) = 0$ $\quad (x,y) = (\pi, 0)$

89. For $x = \frac{\pi}{6}$, $\quad y = \cos\left(\frac{\pi}{6} - \frac{\pi}{6}\right) = \cos(0) = 1$ $\quad (x, y) = \left(\frac{\pi}{6}, 1\right)$

For $x = \frac{\pi}{3}$, $\quad y = \cos\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ $\quad (x, y) = \left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$

For $x = \frac{2\pi}{3}$, $\quad y = \cos\left(\frac{2\pi}{3} - \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{2}\right) = 0$ $\quad (x, y) = \left(\frac{2\pi}{3}, 0\right)$

For $x = \pi$, $\quad y = \cos\left(\pi - \frac{\pi}{6}\right) = \cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$ $\quad (x, y) = \left(\pi, -\frac{\sqrt{3}}{2}\right)$

For $x = \frac{7\pi}{6}$, $\quad y = \cos\left(\frac{7\pi}{6} - \frac{\pi}{6}\right) = \cos(\pi) = -1$ $\quad (x, y) = \left(\frac{7\pi}{6}, -1\right)$

91. For $x = 0$ $\quad y = 2 + \cos 0$

$= 2 + 1 = 3$ $\quad (x,y) = (0,3)$

For $x = \frac{\pi}{2}$, $\quad y = 2 + \cos\frac{\pi}{2}$

$= 2 + 0 = 2 = 2$ $\quad (x,y) = \left(\frac{\pi}{2}, 2\right)$

For $x = \pi$, $\quad y = 2 + \cos\pi$

$= 2 + (-1) = 1$ $\quad (x,y) = (\pi, 1)$

For $x = \frac{3\pi}{2}$, $\quad y = 2 + \cos\frac{3\pi}{2}$

$= 2 + 0 = 2$ $\quad (x,y) = \left(\frac{3\pi}{2}, 2\right)$

For $x = 2\pi$, $\quad y = 2 + \cos 2\pi$

$= 2 + 1 = 3$ $\quad (x,y) = (2\pi, 3)$

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93. For $x=-\frac{\pi}{4}$, $y=3\sin\left[2\left(-\frac{\pi}{4}\right)+\frac{\pi}{2}\right]$ $(x,y)=\left(-\frac{\pi}{4},0\right)$

$=3\sin\left(-\frac{\pi}{2}+\frac{\pi}{2}\right)$

$=3\sin 0=3\cdot 0=0$

For $x=0$, $y=3\sin\left[2(0)+\frac{\pi}{2}\right]$ $(x,y)=(0,3)$

$=3\sin\frac{\pi}{2}=3(1)=3$

For $x=\frac{\pi}{4}$, $y=3\sin\left[2\left(\frac{\pi}{4}\right)+\frac{\pi}{2}\right]$ $(x,y)=\left(\frac{\pi}{4},0\right)$

$=3\sin\left(\frac{\pi}{2}+\frac{\pi}{2}\right)=3\sin\pi=3(0)=0$

For $x=\frac{\pi}{2}$, $y=3\sin\left[2\left(\frac{\pi}{2}\right)+\frac{\pi}{2}\right]$ $(x,y)=\left(\frac{\pi}{2},-3\right)$

$=3\sin\left(\pi+\frac{\pi}{2}\right)=3\sin\frac{3\pi}{2}=3(-1)=-3$

For $x=\frac{3\pi}{4}$, $y=3\sin\left[2\left(\frac{3\pi}{4}\right)+\frac{\pi}{2}\right]$ $(x,y)=\left(\frac{3\pi}{4},0\right)$

$=3\sin\left(\frac{3\pi}{2}+\frac{\pi}{2}\right)=3\sin 2\pi=3(0)=0$

95. $\theta=\frac{2\pi}{22}=\frac{\pi}{11}$ *radians*

97. $\sin(21^\circ 53.896')\sin(19^\circ 43.219')+\cos(21^\circ 53.896')\cos(19^\circ 43.219')\cos(159^\circ 36.336'-155^\circ 2.930')=$

$(.3729)(.3374)+(.9278)(.9413)(.9968)=0.9965$

99. $(x,y)=(1,-3)$

$r=\sqrt{x^2+y^2}=\sqrt{(1)^2+(-3)^2}=\sqrt{1+9}=\sqrt{10}$

$\sin\theta=\frac{y}{r}=\frac{-3}{\sqrt{10}}=-\frac{3\sqrt{10}}{10}$ $\cot\theta=\frac{x}{y}=\frac{1}{-3}=-\frac{1}{3}$

$\cos\theta=\frac{x}{r}=\frac{1}{\sqrt{10}}=\frac{\sqrt{10}}{10}$ $\sec\theta=\frac{r}{x}=\frac{\sqrt{10}}{1}=\sqrt{10}$

$\tan\theta=\frac{y}{x}=\frac{-3}{1}=-3$ $\csc\theta=\frac{r}{y}=\frac{\sqrt{10}}{-3}=-\frac{\sqrt{10}}{3}$

101. $\sin\theta=\frac{1}{2}$ and θ terminates in QII.

$\cos\theta=-\sqrt{1-\sin^2\theta}$ because $\cos\theta$ is negative in QII

$=-\sqrt{1-\left(\frac{1}{2}\right)^2}=-\sqrt{\frac{3}{4}}=-\frac{\sqrt{3}}{2}$

$\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{1/2}{-\sqrt{3}/2}=-\frac{\sqrt{3}}{3}$ $\cot\theta=\frac{1}{\tan\theta}=\frac{1}{-1/\sqrt{3}}=-\sqrt{3}$ $\sec\theta=\frac{1}{\cos\theta}=\frac{1}{-\sqrt{3}/2}=-\frac{2\sqrt{3}}{3}$ $\csc\theta=\frac{1}{\sin\theta}=\frac{1}{1/2}=2$

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103. A point on the line $y = 2x$ in QI is (1, 2).

$(x, y) = (1, 2)$

$r = \sqrt{1^2 + 2^2}$

$= \sqrt{1+4} = \sqrt{5}$

$\sin\theta = \frac{y}{r} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ $\qquad$ $\cot\theta = \frac{x}{y} = \frac{1}{2}$

$\cos\theta = \frac{x}{r} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ $\qquad$ $\sec\theta = \frac{r}{x} = \frac{\sqrt{5}}{1} = \sqrt{5}$

$\tan\theta = \frac{y}{x} = \frac{2}{1} = 2$ $\qquad$ $\csc\theta = \frac{r}{y} = \frac{\sqrt{5}}{2}$

3.3 Definition III: Circular Functions

EVEN SOLUTIONS

2. On the unit circle, the radian measure of a central angle and the length of arc it cuts off are exactly the same value.

4. The circular functions are functions of real numbers, so we must always use a calculator set to radian mode when evaluating them.

6. The largest possible value for the sine or cosine function is 1 and the smallest possible value is –1.

8. From the unit circle: $\cos 135° = -\frac{\sqrt{2}}{2}$

10. From the unit circle: $\tan 330° = \frac{\sin 330°}{\cos 330°} = \frac{-1/2}{\sqrt{3}/2} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

12. From the unit circle: $\csc 300° = \frac{1}{\sin 300°} = \frac{1}{-\sqrt{3}/2} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

14. From the unit circle: $\sin\frac{3\pi}{2} = \sin 270° = -1$

16. From the unit circle: $\sec\frac{5\pi}{4} = \frac{1}{\cos\frac{5\pi}{4}} = \frac{1}{\cos 225°} = \frac{1}{-\sqrt{2}/2} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$

18. From the unit circle: $\cot\frac{2\pi}{3} = \frac{\cos\frac{2\pi}{3}}{\sin\frac{2\pi}{3}} = \frac{\cos 120°}{\sin 120°} = \frac{-1/2}{\sqrt{3}/2} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

20. Finding the six trigonometric functions of 240°:

$\sin 240° = -\frac{\sqrt{3}}{2}$ $\qquad$ $\csc 240° = \frac{1}{\sin 240°} = \frac{1}{-\sqrt{3}/2} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

$\cos 240° = -\frac{1}{2}$ $\qquad$ $\sec 240° = \frac{1}{\cos 240°} = \frac{1}{-1/2} = -2$

$\tan 240° = \frac{\sin 240°}{\cos 240°} = \frac{-\sqrt{3}/2}{-1/2} = \sqrt{3}$ $\qquad$ $\cot 240° = \frac{\cos 240°}{\sin 240°} = \frac{-1/2}{-\sqrt{3}/2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

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22. Finding the six trigonometric functions of $\frac{7\pi}{6}$:

$$\sin\frac{7\pi}{6} = -\frac{1}{2} \qquad \csc\frac{7\pi}{6} = \frac{1}{\sin\frac{7\pi}{6}} = \frac{1}{-1/2} = -2$$

$$\cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2} \qquad \sec\frac{7\pi}{6} = \frac{1}{\cos\frac{7\pi}{6}} = \frac{1}{-\sqrt{3}/2} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

$$\tan\frac{7\pi}{6} = \frac{\sin\frac{7\pi}{6}}{\cos\frac{7\pi}{6}} = \frac{-1/2}{-\sqrt{3}/2} = \frac{\sqrt{3}}{3} \qquad \cot\frac{7\pi}{6} = \frac{\cos\frac{7\pi}{6}}{\sin\frac{7\pi}{6}} = \frac{-\sqrt{3}/2}{-1/2} = \sqrt{3}$$

24. Finding the six trigonometric functions of 270°:

$$\sin 270° = -1 \qquad \csc 270° = \frac{1}{\sin 270°} = \frac{1}{-1} = -1$$

$$\cos 270° = 0 \qquad \sec 270° = \frac{1}{\cos 270°} = \frac{1}{0}, \text{ which is undefined}$$

$$\tan 270° = \frac{\sin 270°}{\cos 270°} = \frac{-1}{0}, \text{ which is undefined} \qquad \cot 270° = \frac{\cos 270°}{\sin 270°} = \frac{0}{-1} = 0$$

26. Finding the six trigonometric functions of $\frac{7\pi}{4}$:

$$\sin\frac{7\pi}{4} = -\frac{\sqrt{2}}{2} \qquad \csc\frac{7\pi}{4} = \frac{1}{\sin\frac{7\pi}{4}} = \frac{1}{-\sqrt{2}/2} = -\sqrt{2}$$

$$\cos\frac{7\pi}{4} = \frac{\sqrt{2}}{2} \qquad \sec\frac{7\pi}{4} = \frac{1}{\cos\frac{7\pi}{4}} = \frac{1}{\sqrt{2}/2} = \sqrt{2}$$

$$\tan\frac{7\pi}{4} = \frac{\sin\frac{7\pi}{4}}{\cos\frac{7\pi}{4}} = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1 \qquad \cot\frac{7\pi}{4} = \frac{\cos\frac{7\pi}{4}}{\sin\frac{7\pi}{4}} = \frac{\sqrt{2}/2}{-\sqrt{2}/2} = -1$$

28. From the unit circle, $\cos\theta = -\frac{1}{2}$ when $\theta = \frac{2\pi}{3}$ and $\theta = \frac{4\pi}{3}$.

30. From the unit circle, $\csc\theta = 1$ when $\sin\theta = 1$, which occurs when $\theta = \frac{\pi}{2}$.

32. From the unit circle, $\cot\theta = -1$ when $\theta = \frac{3\pi}{4}$ and $\theta = \frac{7\pi}{4}$.

34. Finding the values: $\sin\frac{5\pi}{4} \approx -0.7071$, $\cos\frac{5\pi}{4} \approx -0.7071$

36. Finding the values: $\sin\frac{17\pi}{12} \approx -0.9659$, $\cos\frac{17\pi}{12} \approx -0.2588$

38. Finding the values: $t \approx 0.5236,\ 2.6180$

40. Finding the value: $t \approx 3.1416$

42. Finding the values: $\sin 15° \approx 0.2588$, $\cos 15° \approx 0.9659$

44. Finding the values: $\sin 295° \approx -0.9063$, $\cos 295° \approx 0.4226$

46. Finding the values: $t = 90°,\ 270°$

48. Finding the values: $t = 135°,\ 315°$

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50. Since $x = \cos\theta$ and $y = \sin\theta$, $\csc\theta = \dfrac{1}{-3/\sqrt{10}} = -\dfrac{\sqrt{10}}{3}$, $\sec\theta = \dfrac{1}{-1/\sqrt{10}} = -\sqrt{10}$, and $\cot\theta = \dfrac{-1/\sqrt{10}}{-3/\sqrt{10}} = \dfrac{1}{3}$.

52. We have: $\csc t = \dfrac{1}{0.3350} \approx 2.9851$, $\sec t = \dfrac{1}{-0.9422} \approx -1.0613$, and $\cot t = \dfrac{-0.9422}{0.3350} \approx -2.8125$.

54. The coordinates are: $(\cos 3.5, \sin 3.5) \approx (-0.9365, -0.3508)$

56. $\cos 0.8 \approx 0.7$

58. $\csc 5.5 = \dfrac{1}{\sin 5.5} \approx -1.4$

60. $\cot 2.2 = \dfrac{-0.6}{0.8} \approx -0.75$

62. $\theta \approx 0.8,\ 2.35$

64. The argument is θ.

66. The argument is $\dfrac{A}{2}$.

68. The argument is $2x - \pi$.

70. The value is $\sin\dfrac{11\pi}{6} = -\dfrac{1}{2}$. The function is sine, the argument is $\dfrac{11\pi}{6}$, and the value is $-\dfrac{1}{2}$.

72. Evaluating: $\cos(-2.5) \approx -0.8011$

74. Evaluating: $\cot 60 = \dfrac{1}{\tan 60} \approx 3.1246$

76. Evaluating: $\csc\dfrac{\sqrt{3}}{2} = \dfrac{1}{\sin\dfrac{\sqrt{3}}{2}} \approx 1.3127$

78. Computing the value: $\cos\left(2\pi + \dfrac{\pi}{4}\right) = \cos\dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$

80. Computing the value: $\tan\left(2\pi + \dfrac{\pi}{3}\right) = \tan\dfrac{\pi}{3} = \dfrac{\sqrt{3}/2}{1/2} = \sqrt{3}$

82. Since $\cos\dfrac{\pi}{2} = 0$, $\sec\dfrac{\pi}{2}$ and $\tan\dfrac{\pi}{2}$ are both undefined.

84. This is not possible, since $\csc 0$ is undefined.

86. This is possible.

88. This is not possible, since $\cot\pi$ is undefined.

90. This is not possible, since $-1 \le \cos z \le 1$.

92. This is possible.

94. This is not possible, since $\csc z \le -1$ or $\csc z \ge 1$.

96. The value of cot *t* is undefined at *t* = 0. For values of *t* near 0, cot *t* will be a large positive number. As *t* increases to $\dfrac{\pi}{2}$, the values of cot *t* will decrease to 0.

98. The value of cos *t* is 0 at $t = \dfrac{\pi}{2}$. As *t* increases to π, the values of cos *t* will decrease to –1.

100. Since ΔOAF is similar to ΔACO, $\dfrac{OF}{OA} = \dfrac{AO}{AC}$. Since $AO = 1$, $AC = \sin\theta$, and $OA = 1$, $\dfrac{OF}{1} = \dfrac{1}{\sin\theta}$, so $OF = \dfrac{1}{\sin\theta} = \csc\theta$.

102. Point *C* will never be outside the circle. *OC* will be longest when either $\theta = 0$ or $\theta = \pi$, when this distance is the radius of the circle (1). For all other locations, *OC* is shorter than 1. So $OC = |\cos\theta| \le 1$.

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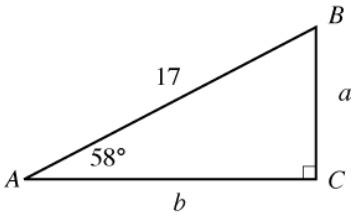
104. **a**. $\sin\theta$ is close to 1. **b**. $\cos\theta$ is close to 0.
c. $\tan\theta$ is becoming infinitely large. **d**. $\csc\theta$ is close to 1.
e. $\sec\theta$ is becoming infinitely large. **f**. $\cot\theta$ is close to 0.

106. **a**. Since $\csc\theta$ decreases, $\csc\dfrac{\pi}{6}$ is greater.

b. Since $\sec\theta$ increases, $\sec\dfrac{\pi}{4}$ is greater.

c. Since $\cot\theta$ decreases, $\cot\dfrac{\pi}{6}$ is greater.

108. Sketching the triangle:



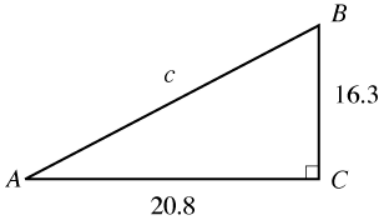
Note that $B = 90° - 58° = 32°$. Finding a:

$$\sin 58° = \frac{a}{17}$$
$$a = 17\sin 58° \approx 14$$

Finding b:

$$\cos 58° = \frac{b}{17}$$
$$b = 17\cos 58° \approx 9.0$$

110. Sketching the triangle:

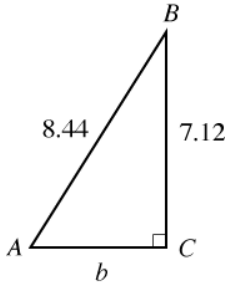


Using the Pythagorean Theorem: $c = \sqrt{(20.8)^2 + (16.3)^2} = \sqrt{432.64 + 265.69} = \sqrt{698.33} \approx 26.4$

Finding angle A:

$$\tan A = \frac{16.3}{20.8}$$
$$A = \tan^{-1}\left(\frac{16.3}{20.8}\right) \approx 38.1°$$
$$B = 90° - 38.1° = 51.9°$$

112. Sketching the triangle:



Using the Pythagorean Theorem: $b = \sqrt{(8.44)^2 - (7.12)^2} = \sqrt{71.2336 - 50.6944} = \sqrt{20.5392} \approx 4.53$

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Finding angle A:

$$\sin A = \frac{7.12}{8.44}$$

$$A = \sin^{-1}\left(\frac{7.12}{8.44}\right) \approx 57.5°$$

$$B = 90° - 57.5° = 32.5°$$

114. Finding the tangent: $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{4\sqrt{17}/17}{-\sqrt{17}/17} = -4$. The correct answer is a.

116. The correct answer is b, since if $\cos z = \dfrac{1}{2}$ then $\sec z = 2$. None of the other alternatives are possible.

ODD SOLUTIONS

1. cosine, sine

3. argument

5. sine, cosine, tangent, cotangent

7. The point on the unit circle is $\left(\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right)$

$$\sin 30° = y = \frac{1}{2}$$

9. The point on the unit circle is $(0,1)$

$$\cot 90° = \frac{x}{y} = \frac{0}{1} = 0$$

11. The point on the unit circle is $\left(-\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$

$$\sec 120° = \frac{1}{x} = \frac{1}{-1/2} = -2$$

13. The point on the unit circle is $\left(\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$

$$\cos\frac{\pi}{3} = x = \frac{1}{2}$$

15. The point on the unit circle is $\left(-\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right)$

$$\tan\frac{5\pi}{6} = \frac{1/2}{-\sqrt{3}/2} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

17. The point on the unit circle is $\left(\dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2}\right)$

$$\csc\frac{7\pi}{4} = \frac{1}{y} = \frac{1}{-\sqrt{2}/2} = -\sqrt{2}$$

19. The point on the unit circle is $\left(-\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right)$

$$\sin 150° = \frac{1}{2}$$

$$\cos 150° = -\frac{\sqrt{3}}{2}$$

$$\tan 150° = \frac{\sin 150°}{\cos 150°} = \frac{1/2}{-\sqrt{3}/2} = -\frac{\sqrt{3}}{3}$$

$$\cot 150° = \frac{1}{\tan 150°} = \frac{1}{-1/\sqrt{3}} = -\sqrt{3}$$

$$\sec 150° = \frac{1}{\cos 150°} = \frac{1}{-\sqrt{3}/2} = -\frac{2\sqrt{3}}{3}$$

$$\csc 150° = \frac{1}{\sin 150°} = \frac{1}{1/2} = 2$$

21. The point on the unit circle is $\left(\dfrac{1}{2}, -\dfrac{\sqrt{3}}{2},\right)$

$$\sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$\cos\frac{5\pi}{3} = \frac{1}{2}$$

$$\tan\frac{5\pi}{3} = \frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$$

$$\cot\frac{5\pi}{3} = \frac{1}{-\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$\sec\frac{5\pi}{3} = \frac{1}{1/2} = 2$$

$$\csc\frac{5\pi}{3} = \frac{1}{-\sqrt{3}/2} = -\frac{2\sqrt{3}}{3}$$

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23. The point on the unit circle is $(-1,0)$.

$\sin 180^\circ = 0$

$\cos 180^\circ = -1$

$\tan 180^\circ = \dfrac{0}{-1} = 0$

$\cot 180^\circ = \dfrac{-1}{0}$ (undefined)

$\sec 180^\circ = \dfrac{1}{-1} = -1$

$\csc 180^\circ = \dfrac{1}{0}$ (undefined)

25. The point on the unit circle is $\left(-\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$

$\sin \dfrac{3\pi}{4} = \dfrac{\sqrt{2}}{2}$

$\cos \dfrac{3\pi}{4} = -\dfrac{\sqrt{2}}{2}$

$\tan \dfrac{3\pi}{4} = \dfrac{\sqrt{2}/2}{-\sqrt{2}/2} = -1$

$\cot \dfrac{3\pi}{4} = \dfrac{1}{-1} = -1$

$\sec \dfrac{3\pi}{4} = \dfrac{1}{-\sqrt{2}/2} = -\sqrt{2}$

$\csc \dfrac{3\pi}{4} = \dfrac{1}{\sqrt{2}/2} = \sqrt{2}$

27. On the unit circle, we locate all points with a *y*-coordinate of $\dfrac{1}{2}$. The angles associated with these points are $\dfrac{\pi}{6}$ *and* $\dfrac{5\pi}{6}$.

29. On the unit circle, we locate all points with an *x*-coordinate of $-\dfrac{\sqrt{3}}{2}$. The angles associated with these points are $\dfrac{5\pi}{6}$ *and* $\dfrac{7\pi}{6}$.

31. We look for points on the unit circle where the ratio, $\dfrac{y}{x}$, equals $-\sqrt{3}$. The angles associated with these points are $\dfrac{2\pi}{3}$ *and* $\dfrac{5\pi}{3}$.

For Problems 33 to 47, see the answer section of your textbook.

49. $(x,y) = \left(\dfrac{\sqrt{5}}{5}, -\dfrac{2\sqrt{5}}{5}\right)$

$\sin\theta = y = -\dfrac{2\sqrt{5}}{5}$ $\qquad$ $\cos\theta = x = \dfrac{\sqrt{5}}{5}$ $\qquad$ $\tan\theta = \dfrac{y}{x} = \dfrac{-2\sqrt{5}/5}{\sqrt{5}/5} = -2$

51. $(x,y) = (0.5403, 0.8415)$

$\sin t = y = 0.8415$

$\cos t = x = 0.5403$

$\tan t = \dfrac{y}{x} = \dfrac{0.8415}{0.5403} = 1.5575$

55. $\sin 2.7 = y = 0.4$

57. $\sec 4.6 = \dfrac{1}{\cos 4.6} = \dfrac{1}{x} = \dfrac{1}{-0.1} = -10$

59. $\tan 3.9 = \dfrac{\sin 3.9}{\cos 3.9} = \dfrac{y}{x} = \dfrac{-0.7}{-0.7} = 1$

61. $\cos\theta = x = -0.8$ and $0 \le \theta < 2\pi$

$x = -.08$ corresponds to the points 2.5 and 3.75 along the unit circle.

Therefore, $\theta = 2.5$ or $\theta = 3.75$.

For Problems 63 to 75, see the answer section of your textbook.

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77. $\sin\left(2\pi+\frac{\pi}{2}\right)=\sin\frac{\pi}{2}$

$=1$

79. $\sin\left(2\pi+\frac{\pi}{6}\right)=\sin\frac{\pi}{6}$

$=\frac{1}{2}$

81. The point on the unit circle is $(1,0)$

If we look at the definitions of the six trigonometric functions, we find that for cot *t* and csc *t*, there is a *y* in the denominator. Therefore, cot 0 and csc 0 are not defined.

83. The point on the unit circle is $(1,0)$

$\sec\frac{\pi}{2}=\frac{1}{x}=\frac{1}{0}$ which is not defined. The statement is not possible.

85. The point on the unit circle is $(-1,0)$

$\cos\pi=x=-1$ The statement is possible.

87. The point on the unit circle is $(0,1)$

$\tan\frac{\pi}{2}=\frac{y}{x}=\frac{1}{0}$ which is not defined. The statement is not possible.

89. $-1\le\sin z\le 1$

$\sin z=1.2$ is not possible.

91. $\tan z=\frac{y}{x}$ as long as $x\neq 0$

This is possible.

93. $\sec z=\frac{1}{x}=\frac{1}{2}$

If this is true, then $x=\cos t$ would have to be 2. This is not possible because $-1\le\cos t\le 1$.

95. When $t=0$, OQ is vertical so $\csc t$ will not be defined. As t increases, $\csc t$ decreases. At $t=\frac{\pi}{2}$, $OQ=\csc t=1$. Over this interval, OQ decreases from a very large number to 1.

97. When $t=\frac{\pi}{2}$, $AP=\sin t=1$. As *t* increases from $\frac{\pi}{2}$ to π, $\sin t$ decreases from 1 to 0.

99. Triangle *OAE* is similar to triangle *AOC*. Therefore, their corresponding sides are in proportion:

$$\frac{AE}{OA}=\frac{AC}{OC}$$

We also know that $OA=1$, $AC=\sin t$, and $OC=\cos t$.

Therefore, $AE=\frac{\sin t}{\cos t}=\tan t$

101. A point on the line $y=2x$ is (1,2). If we solve for *r*, we get $r=\sqrt{1^2+2^2}=\sqrt{5}$.

$\sin\theta=\frac{y}{r}=\frac{2}{\sqrt{5}}$ $\quad\csc\theta=\frac{r}{y}=\frac{\sqrt{5}}{2}$

$\cos\theta=\frac{x}{r}=\frac{1}{\sqrt{5}}$ $\quad\sec\theta=\frac{r}{x}=\frac{\sqrt{5}}{1}=\sqrt{5}$

$\tan\theta=\frac{y}{x}=\frac{2}{1}=2$ $\quad\cot\theta=\frac{x}{y}=\frac{1}{2}$

103.
a. If θ is close to 0, then AC (or $\sin\theta$) is close to 0.
b. If θ is close to 0, then OC (or $\cos\theta$) is close to 1.
c. If θ is close to 0, then AE (or $\tan\theta$) is close to 0.
d. If θ is close to 0, then OF (or $\csc\theta$) becomes infinitely large.
e. If θ is close to 0, then OE (or $\sec\theta$) is close to 1.
f. If θ is close to 0, then AF (or $\cot\theta$) becomes infinitely large.

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105. **a.** $\sin\theta$ increases from 0 to $\frac{\pi}{2}$, so $\sin\frac{\pi}{3} > \sin\frac{\pi}{6}$

b. $\cos\theta$ decreases from 0 to $\frac{\pi}{2}$, so $\cos\frac{\pi}{6} > \cos\frac{\pi}{3}$

c. $\tan\theta$ increases from 0 to $\frac{\pi}{2}$, so $\tan\frac{\pi}{3} > \tan\frac{\pi}{6}$

107. $B = 90^\circ - 42^\circ = 48^\circ$

$\sin 42^\circ = \frac{a}{36}$ $\qquad$ $\cos 42^\circ = \frac{b}{36}$

$a = 36\sin 42^\circ$ $\qquad$ $b = 36\cos 42^\circ$

$= 24$ $\qquad$ $= 27$

109. $A = 90^\circ - 22^\circ = 68^\circ$

$\tan 22^\circ = \frac{320}{a}$ $\qquad$ $\sin 22^\circ = \frac{320}{c}$

$a = \frac{320}{\tan 22^\circ} = 790$ $\qquad$ $c = \frac{320}{\sin 22^\circ} = 850$

111. $b = \sqrt{c^2 - a^2}$

$= \sqrt{(6.21)^2 - (4.37)^2}$

$= \sqrt{19.4672}$

$= 4.41$

$\sin A = \frac{4.37}{6.21}$ $\qquad$ $B = 90^\circ - 44.7^\circ$

$= 0.7037$ $\qquad$ $= 45.3^\circ$

$A = 44.7^\circ$

3.4 Arc Length and Area of a Sector

EVEN SOLUTIONS

2. Arc length is proportional to the central angle. An angle three times as large would cut off an arc three times as long.

4. When calculating arc length or sector area, the angle must be measured in radians.

6. Finding the arc length: $s = r\theta = 5 \cdot 3 = 15$ inches

8. Finding the arc length: $s = r\theta = 1.8 \cdot 4.2 = 7.56$ feet

10. Finding the arc length: $s = r\theta = 24 \cdot \frac{\pi}{3} = 8\pi \approx 25.1$ cm

12. Note that $120^\circ = \frac{2\pi}{3}$ radians, so: $s = r\theta = 4 \cdot \frac{2\pi}{3} = \frac{8\pi}{3} \approx 8.38$ mm

14. Note that $240^\circ = \frac{4\pi}{3}$ radians, so: $s = r\theta = 16 \cdot \frac{4\pi}{3} = \frac{64\pi}{3} \approx 67.0$ in.

16. Since 20 minutes $= \frac{20}{60} \cdot 2\pi = \frac{2\pi}{3}$ radians, the arc length is given by $s = r\theta = 4.5 \cdot \frac{2\pi}{3} = 3\pi \approx 9.42$ cm.

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18. Using $r = 4200$ mi, $\theta = \frac{s}{r} = \frac{8400}{4200} = 2$ radians . Set up the proportion:

$$\frac{2 \text{ radians}}{2\pi \text{ radians}} = \frac{x \text{ hours}}{6 \text{ hours}}$$

$$2\pi x = 12$$

$$x = \frac{12}{2\pi} = \frac{6}{\pi} \approx 1.91 \text{ hours}$$

20. Since $20° = 20 \bullet \frac{\pi}{180} = \frac{\pi}{9}$ radians , the length the pendulum swings in 1 second is $s = r\theta = 4 \bullet \frac{\pi}{9} = \frac{4\pi}{9}$ feet . In 60 seconds, the total distance traveled is $60 \bullet \frac{4\pi}{9} = \frac{80\pi}{3} \approx 83.8$ feet .

22. Since $270° = 270 \bullet \frac{\pi}{180} = \frac{3\pi}{2}$ radians , the length of the cable riding on one of the drive sheaves is:

$$s = r\theta = 6 \bullet \frac{3\pi}{2} = 9\pi \approx 28.3 \text{ feet}$$

24. Using $s = r\theta$, we have:

$$335 = 160\theta$$

$$\theta = \frac{335}{160} \approx 2.09 \text{ radians}$$

Converting to degrees: $\theta = 2.09 \bullet \frac{180°}{\pi} \approx 120°$

26. Since $0.5° = 0.5 \bullet \frac{\pi}{180} = \frac{\pi}{360}$ radians , the diameter of the sun can be approximated by the arc length:

$$s = r\theta = 93{,}000{,}000 \bullet \frac{\pi}{360} \approx 810{,}000 \text{ miles}$$

28. Since $\theta = 120° = \frac{2\pi}{3}$ radians : $s = r\theta = 125 \bullet \frac{2\pi}{3} = \frac{250\pi}{3} \approx 262$ feet

30. Since $\theta = 315° = \frac{7\pi}{4}$ radians : $s = r\theta = 125 \bullet \frac{7\pi}{4} = \frac{875\pi}{4} \approx 687$ feet

32. **a**. Since $\theta = 150° = \frac{5\pi}{6}$ radians : $s = r\theta = 82.5 \bullet \frac{5\pi}{6} \approx 216$ feet

b. Since $\theta = 240° = \frac{4\pi}{3}$ radians : $s = r\theta = 82.5 \bullet \frac{4\pi}{3} \approx 346$ feet

c. Since $\theta = 345° = 345 \bullet \frac{\pi}{180} = \frac{23\pi}{12}$ radians : $s = r\theta = 82.5 \bullet \frac{23\pi}{12} \approx 497$ feet

34. Using $s = r\theta$:

$$4 = r \bullet 1$$

$$r = 4 \text{ feet}$$

36. Using $s = r\theta$:

$$13 = r \bullet 5.2$$

$$r = 2.5 \text{ inches}$$

38. Using $s = r\theta$:

$$2\pi = r \bullet \frac{3\pi}{4}$$

$$8\pi = r \bullet 3\pi$$

$$r = \frac{8}{3} \text{ cm}$$

40. Note that $\theta = 180° = \pi$ radians . Using $s = r\theta$:

$$\frac{\pi}{3} = r \bullet \pi$$

$$r = \frac{1}{3} \text{ m}$$

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42. Note that $\theta = 240° = \frac{4\pi}{3}$ radians . Using $s = r\theta$:

$$4 = r \cdot \frac{4\pi}{3}$$
$$12 = r \cdot 4\pi$$
$$r = \frac{3}{\pi} \approx 0.955 \text{ km}$$

44. Finding the sector area: $A = \frac{1}{2}r^2\theta = \frac{1}{2}(5)^2 \cdot 3 = 37.5 \text{ cm}^2$

46. Finding the sector area: $A = \frac{1}{2}r^2\theta = \frac{1}{2}(2)^2 \cdot 5.8 = 11.6 \text{ in.}^2$

48. Finding the sector area: $A = \frac{1}{2}r^2\theta = \frac{1}{2}(6)^2 \cdot \frac{\pi}{5} = \frac{18\pi}{5} \approx 11.3 \text{ m}^2$

50. Note that $\theta = 75° = 75 \cdot \frac{\pi}{180} = \frac{5\pi}{12}$ radians . Finding the sector area: $A = \frac{1}{2}r^2\theta = \frac{1}{2}(10)^2 \cdot \frac{5\pi}{12} = \frac{125\pi}{6} \approx 65.4 \text{ m}^2$

52. First find the radius of the circle. Given s = 12 in. and θ = 2 radians :

$$12 = r \cdot 2$$
$$r = 6 \text{ in.}$$

Finding the area of the sector: $A = \frac{1}{2}r^2\theta = \frac{1}{2}(6)^2 \cdot 2 = 36 \text{ in.}^2$

54. First find the radius of the circle. Given $A = \frac{\pi}{3} \text{ cm}^2$ and $\theta = 30° = \frac{\pi}{6}$ radians :

$$\frac{\pi}{3} = \frac{1}{2}r^2 \cdot \frac{\pi}{6}$$
$$\frac{\pi}{3} = \frac{\pi}{12}r^2$$
$$r^2 = 4$$
$$r = 2 \text{ cm}$$

The arc length is therefore: $s = r\theta = 2 \cdot \frac{\pi}{6} = \frac{\pi}{3} \approx 1.05 \text{ cm}$

56. Given A = 25 in.^{2} and θ = 3 radians :

$$25 = \frac{1}{2}r^2 \cdot 3$$
$$50 = 3r^2$$
$$r^2 = \frac{50}{3}$$
$$r = \sqrt{\frac{50}{3}} = \frac{5\sqrt{6}}{3} \approx 4.08 \text{ in.}$$

58. First note that $\theta = 60° = \frac{\pi}{3}$ radians . Treat the area being cleaned as a large sector area (with radius 10 in.) minus a smaller sector area (with radius 1 in.). Therefore: $A = \frac{1}{2}(10)^2 \cdot \frac{\pi}{3} - \frac{1}{2}(1)^2 \cdot \frac{\pi}{3} = \frac{50\pi}{3} - \frac{\pi}{6} = \frac{33\pi}{2} \approx 51.8 \text{ in.}^2$

60. The radian measure formed by two adjacent spokes is: $\frac{2\pi}{16} = \frac{\pi}{8}$

The arc length is therefore: $s = r\theta = 350\left(\frac{\pi}{8}\right) = \frac{175\pi}{4} \approx 137 \text{ mm}$

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62. **a**. Finding the central angle:

$$s = r \cdot \theta$$
$$3.5 = 5 \cdot \theta$$
$$\theta = 0.7 \text{ radians}$$
$$\theta = 0.7 \cdot \frac{180^\circ}{\pi} \approx 40.1^\circ$$

b. The area that was removed is given by: $A = \frac{1}{2}(5)^2 \cdot 0.7 = 8.75 \text{ in.}^2$

Then the area that remains is given by: $A = \pi(5)^2 - 8.75 \text{ in.}^2 \approx 69.8 \text{ in.}^2$

64. Since $\theta = \frac{s}{r} = \frac{3{,}470}{384{,}000}$ is the central angle, the distance x the friend must stand is given by:

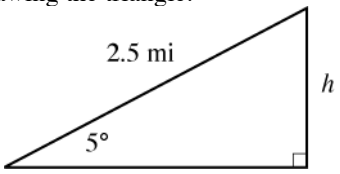
$$\frac{6 \text{ feet}}{x} = \frac{3{,}470}{384{,}000}$$
$$3{,}470x = 230{,}4000$$
$$x \approx 664 \text{ feet}$$

The friend must stand approximately 664 feet away.

66. The smallest apparent diameter is: $\theta = \frac{s}{r} = \frac{3{,}470}{406{,}630} \cdot \frac{180^\circ}{\pi} \approx 0.49^\circ$

The largest apparent diameter is: $\theta = \frac{s}{r} = \frac{3{,}470}{356{,}340} \cdot \frac{180^\circ}{\pi} \approx 0.56^\circ$

68. Let h represent the height of the hill. Drawing the triangle:

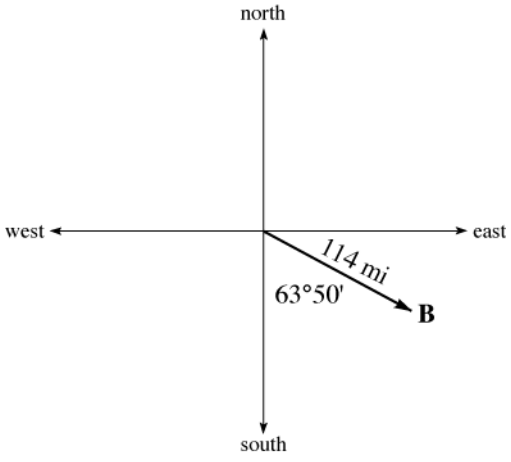


Therefore:

$$\sin 5^\circ = \frac{h}{2.5}$$
$$h = 2.5 \sin 5^\circ \approx 0.218$$

The hill is approximately 0.218 miles high.

70. Drawing the figure:



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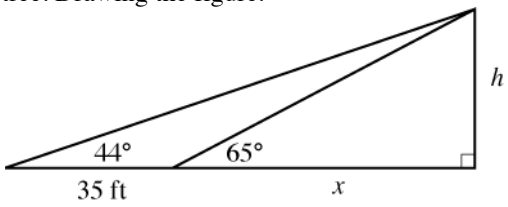
The components south and east are given by:

$$\mathbf{B}_S = 114\cos 63°50' = 114\cos\left(63\frac{5}{6}\right)^\circ \approx 50.3 \text{ miles}$$

$$\mathbf{B}_E = 114\sin 63°50' = 114\sin\left(63\frac{5}{6}\right)^\circ \approx 102 \text{ miles}$$

72. Let h represent the height of the tree. Drawing the figure:



Using the smaller triangle:

$$\tan 65° = \frac{h}{x}$$
$$x\tan 65° = h$$
$$x = \frac{h}{\tan 65°} = h\cot 65°$$

Using the larger triangle:

$$\tan 44° = \frac{h}{35+x}$$
$$(35+x)\tan 44° = h$$
$$35 + x = \frac{h}{\tan 44°} = h\cot 44°$$
$$x = h\cot 44° - 35$$

Setting these two expressions equal:

$$h\cot 65° = h\cot 44° - 35$$
$$35 = h\cot 44° - h\cot 65°$$
$$35 = h(\cot 44° - \cot 65°)$$
$$h = \frac{35}{\cot 44° - \cot 65°} \approx 61.5 \text{ ft}$$

The tree is approximately 61.5 feet tall.

74. Note that $\theta = 40° = 40 \bullet \frac{\pi}{180} = \frac{2\pi}{9}$ radians . Finding the sector area: $A = \frac{1}{2}r^2\theta = \frac{1}{2}(9)^2 \bullet \frac{2\pi}{9} = 9\pi \text{ ft}^2$.

The correct answer is b.

76. Note that $\theta = 110° = 110 \bullet \frac{\pi}{180} = \frac{11\pi}{18}$ radians .

Finding the sector area: $A = \frac{1}{2}r^2\theta = \frac{1}{2}(12)^2 \bullet \frac{11\pi}{18} = 44\pi \text{ ft}^2 \approx 138 \text{ ft}^2$. The correct answer is d.

ODD SOLUTIONS

1. radius, angle

3. sector

5.

$s = r\theta$	Formula for arc length
$= 3(2)$	Substitute given values
$= 6\ in$	Simplify

7.

$s = r\theta$	Formula for arc length
$= 1.5(1.5)$	Substitute given values
$= 2.25\ ft$	Simplify

9.

$s = r\theta$	Formula for arc length
$= 12\left(\frac{\pi}{6}\right)$	Substitute given values
$= 2\pi \approx 6.28$ cm	Simplify and round to 3 significant digits

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11. Remember to change θ to radians by multiplying by $\frac{\pi}{180}$:

$s = r\theta$ Formula for arc length

$= 4\left[(30)\left(\frac{\pi}{180}\right)\right]$ Substitute given values (θ in radians)

$= \frac{2\pi}{3} \approx 2.09$ mm Simplify and round to 3 significant digits

13. Remember to change θ to radians by multiplying by $\frac{\pi}{180}$:

$s = r\theta$ Formula for arc length

$= 5\left[(315)\left(\frac{\pi}{180}\right)\right]$ Substitute given values (θ in radians)

$= \frac{35\pi}{4} \approx 27.5$ *in* Simplify and round to 3 significant digits

15. First, we find θ: $\frac{\theta}{2\pi} = \frac{40}{60}$ One complete rotation is 60 minutes or 2π radians

$\theta = \frac{40}{60} \cdot 2\pi$ Multiply both sides by 2π

$= \frac{4\pi}{3}$ Simplify

The radius is 1.2 cm. Therefore, $s = r\theta$

$= 1.2\left(\frac{4\pi}{3}\right)$

$= 5.03$ cm

17. First, we find θ: $\frac{\theta}{2\pi} = \frac{1}{6}$ One complete rotation is 6 hours or 2π radians

$\theta = \frac{2\pi}{6}$ Multiply both sides by 2π

$= \frac{\pi}{3}$ Simplify

Also, the radius is 200 + 4,000 or 4,200 miles.

Therefore, $s = r\theta = 4{,}200\left(\frac{\pi}{3}\right) = 1{,}400\pi$ miles = 4,400 miles

19. Remember to change θ to radians by multiplying by $\frac{\pi}{180}$:

$s = r\theta$

$= 4\left[20\left(\frac{\pi}{180}\right)\right]$

$= \frac{4\pi}{9}$ *ft* $= 1.40$ *ft*

21. We are given that the diameter is 14 ft. Therefore, $r = \frac{1}{2}(14) = 7$ ft.

$s = r\theta$

$= 7\left[270\left(\frac{\pi}{180}\right)\right]$

$= \frac{21\pi}{2}$ *ft* $= 33.0$ *ft*

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23. We are given that the diameter is 320 mm. Therefore, $r = \frac{1}{2}(320) = 160$ mm.

$\theta = \frac{s}{r}$ $\qquad$ $\theta = 1.92\left(\frac{180}{\pi}\right) = 110^\circ$

$= \frac{307}{160} = 1.92$ radians

25. We convert 0.5° to radians by multiplying by $\frac{\pi}{180}$.

$s = r\theta$

$= 240{,}000\left[(0.5)\left(\frac{\pi}{180}\right)\right]$

$= \frac{2000\pi}{3}\ mi = 2{,}100\ mi$

27. $s = r\theta$

$= 125\left[30\left(\frac{\pi}{180}\right)\right]$

$= \frac{125\pi}{6}\ ft$

$= 65.4\ ft$

29. $s = r\theta$

$= 125\left[(220)\left(\frac{\pi}{180}\right)\right]$

$= \frac{1{,}375\pi}{9}\ ft$

$= 480\ ft$

31. $r = \frac{1}{2}(197) = 98.5$ ft

a. $s = r\theta$

$= 98.5\left[(60)\left(\frac{\pi}{180}\right)\right]$

$= 103\ ft$

b. $s = r\theta$

$= 98.5\left[(210)\left(\frac{\pi}{180}\right)\right]$

$= 361\ ft$

c. $s = r\theta$

$= 98.5\left[(285)\left(\frac{\pi}{180}\right)\right]$

$= 490\ ft$

33. $r = \frac{s}{\theta}$ Formula for arc length

$= \frac{3}{6}$ Substitute known values

$= 0.5\ ft$ Simplify

35. $r = \frac{s}{\theta}$ Formula for arc length

$= \frac{4.2}{1.4}$ Substitute known values

$= 3$ in Simplify

37. $r = \frac{s}{\theta}$ Formula for arc length

$= \frac{\pi}{\pi/4}$ Substitute known values

$= 4\ cm$ Simplify

39. Remember to convert θ to radians:

$r = \frac{s}{\theta}$ Formula for arc length

$= \frac{\frac{\pi}{2}}{90\left(\frac{\pi}{180}\right)}$ Substitute known values

$= 1\ m$ Simplify

41. Remember to convert θ to radians:

$r = \frac{s}{\theta}$ Formula for arc length

$= \frac{5}{150\left(\frac{\pi}{180}\right)}$ Substitute known values

$= \frac{6}{\pi}\ or\ 1.91\ km$ Simplify

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43.

$A = \frac{1}{2} r^2 \theta$ Formula for area of a sector

$= \frac{1}{2}(3)^2(2)$ Substitute known values

$= 9\ cm^2$ Simplify

45.

$A = \frac{1}{2} r^2 \theta$ Formula for area of a sector

$= \frac{1}{2}(4)^2(2.4)$ Substitute known values

$= 19.2\ in^2$ Simplify

47.

$A = \frac{1}{2} r^2 \theta$ Formula for area of a sector

$= \frac{1}{2}(3)^2\left(\frac{2\pi}{5}\right)$ Substitute known values

$= \frac{9\pi}{5} = 5.65\ m^2$ Simplify

49.

$A = \frac{1}{2} r^2 \theta$ Formula for area of a sector

$= \frac{1}{2}(5)^2\left(15 \cdot \frac{\pi}{180}\right)$ Substitute known values

$= \frac{25\pi}{24}\ or\ 3.27\ m^2$ Simplify

51.

$r = \frac{s}{\theta} = \frac{3\ ft}{\pi / 4\ rad} = \frac{12}{\pi}\ ft$

$A = \frac{1}{2} r^2 \theta$ Formula for area of a sector

$= \frac{1}{2}\left(\frac{12}{\pi}\right)^2\left(\frac{\pi}{4}\right)$ Substitute known values

$= \frac{18}{\pi} = 5.73\ ft^2$ Simplify

53.

$A = \frac{1}{2} r^2 \theta$ Formula for area of a sector

$r^2 = \frac{2A}{\theta}$ Solve for r^2

$= \frac{2\left(\frac{\pi}{3}\right)}{30\left(\frac{\pi}{180}\right)}$ Substitute known values

$r^2 = 4$ Simplify right side

$r = 2$ cm Take square root of both sides (r must be positive.)

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55. $\theta = 45^\circ = \frac{\pi}{4}$ *radians*

$A = \frac{1}{2}r^2\theta$ Formula for area of a sector

$r^2 = \frac{2A}{\theta}$ Solve for r^2

$= \frac{2\left(\frac{2\pi}{3}\right)}{\frac{\pi}{4}}$ Substitute known values

$= \frac{16}{3}$ Simplify right side

$r = \frac{4}{\sqrt{3}} = 2.31$ in Take square root of both sides. (r must be positive)

57. $A = \frac{1}{2}r^2\theta$ Formula for area of a sector

$= \frac{1}{2}(60)^2\left(90 \cdot \frac{\pi}{180}\right)$ Substitute known values

$= 900\pi$ *or* $2{,}830\ ft^2$ Simplify

59. $r = \frac{700}{2} = 350\ mm$

$\theta = \frac{2\pi}{22} = \frac{\pi}{11}$

$s = r\theta$

$= 350\left(\frac{\pi}{11}\right) = \frac{350\pi}{11}$ or 100 *mm*

61. $\theta = 360^\circ - 55^\circ = 305^\circ$

a. $s = r\theta$

$= 4.00\left[(305)\left(\frac{\pi}{180}\right)\right]$

$= \frac{61\pi}{9} = 21.3mm$

We have to add the two radii to this giving us $21.3 + 8 = 29.3$ mm.

b. $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2}(4)^2\left(\frac{65\pi}{36}\right)$

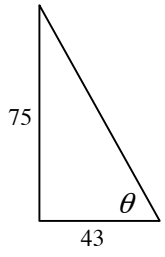
$= 42.6\ mm^2$

67. $\tan\theta = \frac{75}{43}$

$= 1.7442$

$\theta = \tan^{-1}(1.7442)$

$= 60.2^\circ$



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69. $\tan 12^\circ = \dfrac{x}{5.2}$

$x = 5.2 \tan 12^\circ$

$= 5.2(0.2126)$

$= 1.11$

$\tan 13^\circ = \dfrac{y}{5.2}$

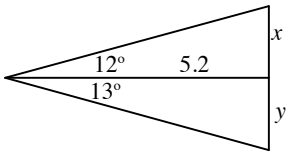
$y = 5.2 \tan 13^\circ$

$= 5.2(0.2309)$

$= 1.20$

The vertical dimension of the mirror is

$x + y = 1.11 + 1.20 = 2.31$ *ft*

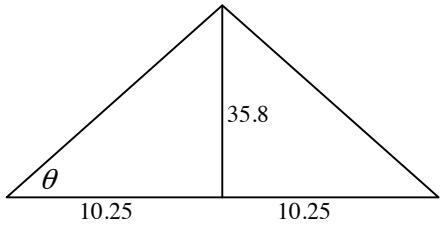


71. $\tan\theta = \dfrac{35.8}{10.25}$

$= 3.4927$

$\theta = \tan^{-1}(3.4927)$

$= 74.0^\circ$



3.5 Velocities

EVEN SOLUTIONS

2. For a point moving with uniform circular motion, the distance traveled per unit time by the point is called linear velocity, and the amount of rotation per unit time is called angular velocity.

4. The formula relating linear and angular velocity is $v = r\omega$, where the angular velocity must be measured in radians per unit time.

6. The linear velocity is given by: $v = \dfrac{s}{t} = \dfrac{10 \text{ ft}}{5 \text{ min}} = 2 \text{ ft/min}$

8. The linear velocity is given by: $v = \dfrac{s}{t} = \dfrac{6 \text{ cm}}{2 \text{ sec}} = 3 \text{ cm/sec}$

10. The linear velocity is given by: $v = \dfrac{s}{t} = \dfrac{220 \text{ mi}}{4 \text{ hr}} = 55 \text{ mi/hr}$

12. The distance is given by: $s = vt = 10\frac{\text{ft}}{\text{sec}} \cdot 5 \text{ sec} = 50 \text{ feet}$

14. The distance is given by: $s = vt = 55\frac{\text{mi}}{\text{hr}} \cdot \dfrac{1}{4} \text{ hr} = 13.75 \text{ miles}$

16. The distance is given by: $s = vt = 75\frac{\text{mi}}{\text{hr}} \cdot 15 \text{ sec} \cdot \dfrac{1 \text{ hr}}{3600 \text{ sec}} = 0.3125 \text{ miles}$

18. The angular velocity is given by: $\omega = \dfrac{\theta}{t} = \dfrac{48 \text{ radians}}{3 \text{ min}} = 16 \text{ radians/min}$

20. The angular velocity is given by: $\omega = \dfrac{\theta}{t} = \dfrac{\pi/2 \text{ radians}}{5\pi \text{ sec}} = 0.1 \text{ radians/sec}$

22. The angular velocity is given by: $\omega = \dfrac{\theta}{t} = \dfrac{24\pi \text{ radians}}{2.4 \text{ hr}} = 10\pi \approx 31.4 \text{ radians/hr}$

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24. The angular velocity is given by $\omega = \frac{\theta}{t} = \frac{2\pi \text{ radians}}{4 \text{ sec}} = \frac{\pi}{2}$ radians/sec . In t seconds, the angle θ is given by $\theta = \frac{\pi}{2}t$.

Therefore:

$$\cos\left(\frac{\pi}{2}t\right) = \frac{100}{l}$$

$$l\cos\left(\frac{\pi}{2}t\right) = 100$$

$$l = \frac{100}{\cos\left(\frac{\pi}{2}t\right)} = 100\sec\left(\frac{\pi}{2}t\right)$$

Substituting values for t:

$t = 0.5$: $l = 100\sec\frac{\pi}{4} = 100\sqrt{2} \approx 141$ ft

$t = 1.0$: $l = 100\sec\frac{\pi}{2}$, which is undefined

$t = 1.5$: $l = 100\sec\frac{3\pi}{4} = -100\sqrt{2} \approx -141$ ft

The negative value for t = 1.5 indicates the light is pointed out to sea, rather than towards the wall.

26. First find $v = r\omega = 4 \cdot 5 = 20\frac{\text{inches}}{\text{sec}}$. Thus $s = vt = 20\frac{\text{inches}}{\text{sec}} \cdot 2 \text{ sec} = 40$ inches .

28. First find $v = r\omega = 8 \cdot \frac{4\pi}{3} = \frac{32\pi}{3}\frac{\text{m}}{\text{sec}}$. Thus $s = vt = \frac{32\pi}{3}\frac{\text{m}}{\text{sec}} \cdot 15 \text{ sec} = 160\pi \text{ m} \approx 503 \text{ m}$.

30. First find $v = r\omega = 5 \cdot 15 = 75\frac{\text{ft}}{\text{sec}}$. Thus $s = vt = 75\frac{\text{ft}}{\text{sec}} \cdot 0.5 \text{ min} \cdot \frac{60 \text{ sec}}{1 \text{ min}} = 2{,}250$ ft .

32. Finding the angular velocity: $\omega = 120\frac{\text{rev}}{\text{min}} \cdot 2\pi\frac{\text{radians}}{\text{rev}} = 240\pi\frac{\text{radians}}{\text{min}} \approx 754\frac{\text{radians}}{\text{min}}$

34. Finding the angular velocity: $\omega = 6\frac{2}{3}\frac{\text{rev}}{\text{min}} \cdot 2\pi\frac{\text{radians}}{\text{rev}} = \frac{40\pi}{3}\frac{\text{radians}}{\text{min}} \approx 41.9\frac{\text{radians}}{\text{min}}$

36. Finding the angular velocity: $\omega = 8.5\frac{\text{rev}}{\text{min}} \cdot 2\pi\frac{\text{radians}}{\text{rev}} = 17\pi\frac{\text{radians}}{\text{min}} \approx 53.4\frac{\text{radians}}{\text{min}}$

38. Finding the linear velocity: $v = r\omega = 12 \cdot 4 = 48\frac{\text{inches}}{\text{sec}}$

40. Finding the angular velocity: $\omega = \frac{v}{r} = \frac{15}{3}\frac{\text{radians}}{\text{sec}} = 5\frac{\text{radians}}{\text{sec}}$

42. The angular velocity is $\omega = 20\frac{\text{rev}}{\text{min}} \cdot 2\pi\frac{\text{radians}}{\text{rev}} = 40\pi\frac{\text{radians}}{\text{min}}$. Thus the linear velocity is given by

$v = r\omega = 10 \cdot 40\pi = 400\pi\frac{\text{feet}}{\text{min}} \approx 1{,}257\frac{\text{feet}}{\text{min}}$.

44. The angular velocity is $\omega = \frac{1}{24}\frac{\text{rev}}{\text{hr}} \cdot 2\pi\frac{\text{radians}}{\text{rev}} = \frac{\pi}{12}\frac{\text{radians}}{\text{hr}}$. The linear velocity is given by

$v = r\omega = 4{,}000 \cdot \frac{\pi}{12} = \frac{1{,}000\pi}{3} \approx 1{,}050$ mph .

46. The linear velocity is the blade is $v = 1{,}100\frac{\text{ft}}{\text{sec}}$. Since r = 10 in., $\omega = \frac{v}{r} = \frac{1{,}100\frac{\text{ft}}{\text{sec}}}{\frac{5}{6}\text{ ft}} = 1{,}320\frac{\text{radians}}{\text{sec}}$.

Now convert to rpm: $\omega = 1{,}320\frac{\text{radians}}{\text{sec}} \cdot \frac{1 \text{ rev}}{2\pi \text{ radians}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} = \frac{39{,}600}{\pi}$ rpm $\approx 12{,}605$ rpm

48. Given 18 rpm, the angular velocity is $\omega = 18\frac{\text{rev}}{\text{min}} \cdot 2\pi\frac{\text{radians}}{\text{rev}} = 36\pi\frac{\text{radians}}{\text{min}}$. The linear velocity would be

$v = r\omega = 6.5 \text{ ft} \cdot 36\pi\frac{\text{radians}}{\text{min}} = 234\pi\frac{\text{ft}}{\text{min}}$. Converting to mph: $234\pi\frac{\text{ft}}{\text{min}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \approx 8.35$ mph .

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50. First convert the linear velocity to feet per minute: $10\frac{\text{mi}}{\text{hr}} \bullet \frac{5{,}280 \text{ ft}}{1 \text{ mi}} \bullet \frac{1 \text{ hr}}{60 \text{ min}} = 880\frac{\text{ft}}{\text{min}}$

Thus the angular velocity is: $\omega = \frac{v}{r} = \frac{880\frac{\text{ft}}{\text{min}}}{6 \text{ ft}} \approx 146.67\frac{\text{radians}}{\text{min}}$

Now converting to rpm: $146.67\frac{\text{radians}}{\text{min}} \bullet \frac{1 \text{ rev}}{2\pi \text{ radians}} \approx 23.3 \text{ rpm}$

52. The angular velocity is: $\omega = 10\frac{\text{rev}}{\text{min}} \bullet 2\pi\frac{\text{radians}}{\text{rev}} = 20\pi\frac{\text{radians}}{\text{min}}$.

Converting units: $\omega = 20\pi\frac{\text{radians}}{\text{min}} \bullet \frac{1 \text{ min}}{60 \text{ sec}} = \frac{\pi}{3}\frac{\text{radians}}{\text{sec}}$

Therefore the radius is given by: $r = \frac{v}{\omega} = \frac{2.0\frac{\text{meters}}{\text{sec}}}{\frac{\pi}{3}\frac{\text{radians}}{\text{sec}}} \approx 1.91 \text{ meters}$.

The diameter of the pulley should be 3.82 meters.

54. Given 1.5 rpm, the angular velocity is $\omega = 1.5\frac{\text{rev}}{\text{min}} \bullet 2\pi\frac{\text{radians}}{\text{rev}} = 3\pi\frac{\text{radians}}{\text{min}}$. The linear velocity would be $v = r\omega = 82.5 \text{ ft} \bullet 3\pi\frac{\text{radians}}{\text{min}} = 247.5\pi\frac{\text{ft}}{\text{min}}$. Converting to mph: $247.5\pi\frac{\text{ft}}{\text{min}} \bullet \frac{1 \text{ mi}}{5{,}280 \text{ ft}} \bullet \frac{60 \text{ min}}{1 \text{ hr}} \approx 8.8 \text{ mph}$

The 10 mph figure is too large.

56. The wheel rotates at $\omega = 1.5\frac{\text{rev}}{\text{min}} \bullet 2\pi\frac{\text{radians}}{\text{rev}} = 3\pi\frac{\text{radians}}{\text{min}}$, so in t minutes it has rotated $3\pi t$ radians. The radius of the wheel is 82.5 feet, so in t minutes the rider has moved vertically by $82.5\cos(3\pi t)$ feet. Since the center of the wheel is at a height of 82.5 + 9 = 91.5, the height of the rider is given by the formula $h = 91.5 - 82.5\cos(3\pi t)$.

58. In 1 hour the bicycle travels 16,000 m, so the linear velocity is $v = 16{,}000\frac{\text{m}}{\text{hr}}$. Thus the angular velocity is $\omega = \frac{v}{r} = \frac{16{,}000\frac{\text{m}}{\text{hr}}}{0.3 \text{ m}} \approx 53{,}333\frac{\text{radians}}{\text{hr}}$. Converting to rpm:

$$\omega = 53{,}333\frac{\text{radians}}{\text{hr}} \bullet \frac{1 \text{ rev}}{2\pi \text{ radians}} \approx 8{,}488\frac{\text{rev}}{\text{hr}} \bullet \frac{1 \text{ hr}}{60 \text{ min}} \approx 141 \text{ rpm}$$

60. Using $n_2 = 6, \omega_2 = 90 \text{ rpm}, n_1 = 54$: $\omega_1 = \frac{n_2}{n_1}\omega_2 = \frac{6}{54}(90 \text{ rpm}) = 10 \text{ rpm}$

62. Since $v_2 = v_1$:

$$r_2\omega_2 = r_1\omega_1$$
$$\omega_2 = \frac{r_1}{r_2}\omega_1$$
$$\omega_2 = \frac{n_1}{n_2}\omega_1$$

Since $v_3 = v_4$:

$$r_3\omega_3 = r_4\omega_4$$
$$\omega_3 = \frac{r_4}{r_3}\omega_4$$
$$\omega_3 = \frac{n_4}{n_3}\omega_4$$

Since $\omega_2 = \omega_3$:

$$\frac{n_4}{n_3}\omega_4 = \frac{n_1}{n_2}\omega_1$$
$$\omega_4 = \frac{n_1 n_3}{n_2 n_4}\omega_1$$

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64. First find the angular velocity of the chainring: $\omega = 80\frac{\text{rev}}{\text{min}} \cdot 2\pi\frac{\text{rad}}{\text{rev}} = 160\pi\frac{\text{rad}}{\text{min}}$

The linear velocity of the chain is given by: $v = r\omega = (105\text{ mm}) \cdot \left(160\pi\frac{\text{rad}}{\text{min}}\right) = 16{,}800\pi\frac{\text{mm}}{\text{min}}$

The angular velocity of the rear sprocket is: $\omega = \frac{v}{r} = \frac{16{,}800\pi\frac{\text{mm}}{\text{min}}}{20\text{ mm}} = 840\pi\frac{\text{rad}}{\text{min}}$

The linear velocity of the rear tire is: $v = r\omega = (350\text{ mm}) \cdot \left(840\pi\frac{\text{rad}}{\text{min}}\right) = 294{,}000\pi\frac{\text{mm}}{\text{min}}$

Converting to kilometers per hour: $v = 294{,}000\pi\frac{\text{mm}}{\text{min}} \cdot \frac{60\text{ min}}{1\text{ hr}} \cdot \frac{1\text{ km}}{1{,}000{,}000\text{ mm}} \approx 55.4\frac{\text{km}}{\text{hour}}$

66. First find the angular velocity of the chainring: $\omega = 95\frac{\text{rev}}{\text{min}} \cdot 2\pi\frac{\text{rad}}{\text{rev}} = 190\pi\frac{\text{rad}}{\text{min}}$

The linear velocity of the chain is given by: $v = r\omega = (75\text{ mm}) \cdot \left(190\pi\frac{\text{rad}}{\text{min}}\right) = 14{,}250\pi\frac{\text{mm}}{\text{min}}$

The linear velocity of the rear tire is: $24\frac{\text{km}}{\text{hour}} \cdot \frac{1{,}000{,}000\text{ mm}}{1\text{ km}} \cdot \frac{1\text{ hour}}{60\text{ min}} = 400{,}000\frac{\text{mm}}{\text{min}}$

The angular velocity of the rear tire is: $\omega = \frac{v}{r} = \frac{400{,}000\frac{\text{mm}}{\text{min}}}{350\text{ mm}} \approx 1{,}142.86\frac{\text{rad}}{\text{min}}$

Now let R represent the radius of the sprocket. Since $v = r\omega$:

$$14{,}250\pi = R \cdot 1{,}142.86$$

$$R = \frac{14{,}250\pi\frac{\text{mm}}{\text{min}}}{1{,}142.86\frac{\text{rad}}{\text{min}}} \approx 39.17\text{ mm}$$

The diameter of the sprocket would need to be 78.3 mm.

68. The linear velocity of the rear tire is: $40\frac{\text{km}}{\text{hour}} \cdot \frac{1{,}000{,}000\text{ mm}}{1\text{ km}} \cdot \frac{1\text{ hour}}{60\text{ min}} = 666{,}666.7\frac{\text{mm}}{\text{min}}$

Thus the angular velocity of the rear tire is: $\omega = \frac{v}{r} = \frac{666{,}666.7\frac{\text{mm}}{\text{min}}}{350\text{ mm}} \approx 1{,}904.76\frac{\text{rad}}{\text{min}}$

The linear velocity of the chain is: $v = r\omega = (20\text{ mm}) \cdot \left(1{,}904.76\frac{\text{rad}}{\text{min}}\right) \approx 38{,}095.24\frac{\text{mm}}{\text{min}}$

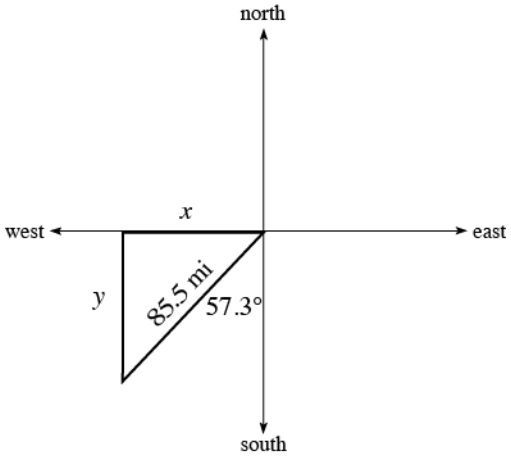
The angular velocity of the chainring is: $\omega = \frac{v}{r} = \frac{38{,}095.24\frac{\text{mm}}{\text{min}}}{105\text{ mm}} \approx 362.81\frac{\text{rad}}{\text{min}}$

Converting to rpm: $\omega = 362.81\frac{\text{rad}}{\text{min}} \cdot \frac{1\text{ rev}}{2\pi\text{ rad}} \approx 57.7\text{ rpm}$

70. The magnitudes are given by:

horizontal: $x = 68\cos 37° \approx 54.3$ vertical: $y = 68\sin 37° \approx 40.9$

72. Drawing the figure:



The south and west directions are given by:

$y = 85.5\cos 57.3° \approx 46.2$ mi south $x = 85.5\sin 57.3° \approx 71.9$ mi west

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74. The linear velocity is given by: $v = \frac{s}{t} = \frac{45 \text{ m}}{15 \text{ sec}} = 3$ m/sec . The correct answer is b.

76. The angular velocity is $\omega = 30 \frac{\text{rev}}{\text{sec}} \cdot 2\pi \frac{\text{radians}}{\text{rev}} = 60\pi \frac{\text{radians}}{\text{sec}}$. Thus the linear velocity is given by $v = r\omega = 6 \cdot 60\pi = 360\pi \frac{\text{cm}}{\text{sec}}$. The correct answer is d.

ODD SOLUTIONS

1. uniform circular

3. proportional, twice

5.
$v = \frac{s}{t}$ Formula for linear velocity
$= \frac{3}{2}$ Substitute known values
$= 1.5 \ ft / \min$ Simplify

7.
$v = \frac{s}{t}$ Formula for linear velocity
$= \frac{12}{4}$ Substitute known values
$= 3 \ cm / \sec$ Simplify

9.
$v = \frac{s}{t}$ Formula for linear velocity
$= \frac{30}{2}$ Substitute known values
$= 15 \ mi / hr$ Simplify

11.
$s = vt$ Formula for linear velocity
$= 20(4)$ Substitute known values
$= 80 \ ft$ Simplify

13.
$s = vt$ Formula for linear velocity
$= 45\left(\frac{1}{2}\right)$ Substitute known values
$= 22.5$ mi Simplify

15. The time must be in hours
$t = \frac{20}{60} = \frac{1}{3}$ hr
$s = vt$
$= 21\left(\frac{1}{3}\right) = 7$ mi

17.
$\omega = \frac{\theta}{t}$ Formula for angular velocity
$= \frac{24}{6}$ Substitute known values
$= 4$ rad/min

19.
$\omega = \frac{\theta}{t}$ Formula for angular velocity
$= \frac{8\pi}{3\pi}$ Substitute known values
$= \frac{8}{3}$ rad/sec Simplify
$= 2.67$ rad/sec Rounded to 3 significant digits

21.
$\omega = \frac{\theta}{t}$ Formula for angular velocity
$= \frac{45\pi}{1.2}$ Substitute known values
$= 37.5\pi = 118$ rad/hr

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23. We know that $\tan\theta = \dfrac{d}{100}$.

Therefore, $d = 100\tan\theta$.

Next, we must find θ. We know that $\omega = \dfrac{\theta}{t}$. Therefore, $\theta = \omega t$. We also know that

$\omega = \dfrac{2\pi \text{ radians}}{4 \text{ seconds}} = \dfrac{\pi}{2} \text{ rad/sec}$

We have $\theta = \dfrac{\pi}{2}t$.

Substituting θ, we get: $d = 100\tan\theta$

$$= 100\tan\frac{\pi}{2}t$$

When $t = \dfrac{1}{2}$, then $d = 100\tan\left[\dfrac{\pi}{2}\left(\dfrac{1}{2}\right)\right]$

$$= 100\tan\frac{\pi}{4} = 100(1) = 100 \text{ feet}$$

When $t = \dfrac{3}{2}$, then $d = 100\tan\left[\dfrac{\pi}{2}\left(\dfrac{3}{2}\right)\right]$

$$= 100\tan\frac{3\pi}{4} = 100(-1) = -100 \text{ feet}$$

When $t = 1$, then $d = 100\tan\left[\dfrac{\pi}{2}(1)\right] = 100\tan\dfrac{\pi}{2}$

d is undefined because $\tan\dfrac{\pi}{2}$ is undefined

When $t = 1, \theta = \dfrac{\pi}{2}$ and the light rays are parallel to the wall.

25. First, we find θ using the formula for angular velocity. Then we apply the formula for arc length.

$\theta = \omega t$	Formula for angular velocity
$= 4(5)$	Substitute known values
$= 20$ rad	Simplify
$s = r\theta$	Formula for arc length
$= 2(20)$	Substitute values
$= 40$ in	Simplify

27. First, we find θ, using the formula for angular velocity. Then we apply the formula for arc length.

$\theta = \omega t$	Formula for angular velocity	$s = r\theta$	Formula for arc length
$= \dfrac{3\pi}{2}(30)$	Substitute known values	$= 4(45\pi)$	Substitute known values
$= 45\pi$	Simplify	$= 180\pi$	Simplify
		$= 565$ m	(3 significant digits)

29. Time must be in seconds: $t = 2(60) = 120$ seconds.

$\theta = \omega t$	Formula for angular velocity	$s = r\theta$	Formula for arc length
$= 10(120)$	Substitute known values	$= 6(1{,}200)$	Substitute known values
$= 1{,}200$ rad	Simplify	$= 7{,}200$ ft	Simplify

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In problems 31 through 35, we convert revolutions per minute to radians per minute by multiplying by 2π.

31. $\omega = 10(2\pi)$

$= 20\pi \ rad/\min$

$= 62.8 \ rad/\min$

33. $\omega = 33\frac{1}{3}(2\pi)$

$= \frac{200\pi}{3} \ rad/\min$

$= 209 \ rad/\min$

35. $\omega = 5.8(2\pi)$

$= 11.6\pi \ rad/\min$

$= 36.4 \ rad/\min$

Use the relationship between angular velocity and linear velocity, $v = r\omega$, for problems 37 – 41.

37. $v = r\omega$

$= 2(5)$

$= 10 \ in/\sec$

39. $\omega = \frac{v}{r}$

$= \frac{3}{6}$

$= 0.5 \ rad/\sec$

41. $v = r\omega$

$= 4\left[10(2\pi)\right]$

$= 80\pi \ ft/\min$

$= 251 \ ft/\min$

43. Angular velocity, ω, is $\frac{1}{24}$ revolutions per hour. To convert this to radians per hour, we multiply by 2π.

$\omega = \frac{1}{24}(2\pi)$

$= \frac{\pi}{12} \ rad/hr = 0.262 \ rad/hr$

45. We are given that the radius is 3 inches and the angular velocity is 600 rev/min.
First, we will convert the angular velocity to rad/min:

$$\omega = 600 \cdot 2\pi = 1200\pi \text{ rad/min}$$

Then we find the linear velocity:

$$v = r\omega$$
$$= 3(1200\pi) = 3{,}600\pi \text{ in/min}$$

We want the answer in feet, so we have to divide by 12:

$$v = \frac{3600\pi}{12}$$
$$= 300\pi \ ft/\min = 942 \ ft/\min$$

47. Angular velocity, ω, is 19 revolutions per minute. We want to convert ω to radians per hour:

$\omega = 19 \frac{rev}{\min} \cdot 2\pi \frac{rad}{rev} \cdot \frac{60 \min}{1 \ hr}$

$= 2280\pi$ rad/hr

Next we want to find the radius and convert it to miles:

$r = \frac{1}{2}(14 \ ft) \cdot \frac{1 \ mi}{5{,}280 \ ft}$

$= \frac{7}{5{,}280} mi$

Now we are ready to compute the linear velocity:

$v = r\omega$

$= \frac{7}{5{,}280} mi \cdot 2{,}280\pi \frac{rad}{hr}$

$= \frac{133\pi}{44} \cdot \frac{mi}{hr} = 9.50 \ mi/hr$

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49. Linear velocity is 12 miles per hour. We want to find angular velocity in revolutions per minute:
In problem 47, we found the radius and converted it to miles:

$$r = \frac{7}{5280} \text{ mi}$$

Next, we find the angular velocity in radians per hour:

$$\omega = \frac{v}{r}$$
$$= \frac{12\ mi/hr}{\frac{7}{5{,}280}\ mi}$$
$$= \frac{63{,}360}{7}\frac{rad}{hr}$$

Now we convert ω to revolutions per minute:

$$\omega = \frac{63{,}360}{7}\frac{rad}{hr} \cdot \frac{1hr}{60 \text{ min}} \cdot \frac{1\ rev}{2\pi\ rad}$$
$$= 24.0\ rpm$$

51. Angular velocity, ω, is 9 revolutions per minute. We want to convert ω to radians per second:

$$\omega = 9\frac{rev}{\text{min}} \cdot \frac{2\pi\ rad}{1\ rev} \cdot \frac{1 \text{ min}}{60 \text{ sec}}$$
$$= \frac{3\pi}{10}\frac{rad}{\text{sec}}$$

Next, we want to find the radius in feet:

$$r = \frac{1}{2}(12\ ft) = 6 \text{ ft}$$

Now we can compute the linear velocity, in feet per second:

$$v = r\omega$$
$$= 6\ ft\left(\frac{3\pi}{10}\right)\frac{rad}{\text{sec}}$$
$$= \frac{9\pi}{5}\frac{ft}{\text{sec}}$$
$$= 5.65\ ft/\text{sec}$$

53. Angular velocity, ω, is $\frac{1}{15}$ revolutions per minute. To convert to radians per minute, we multiply by 2π.

$$\omega = \frac{1}{15}(2\pi)$$
$$= \frac{2\pi}{15} \text{ radians per minute}$$

$v = r\omega$ Relationship between angular and linear velocities

$= \left(\frac{197}{2}\right)\left(\frac{2\pi}{15}\right)$ Substitute known values

$$= \frac{197\pi}{15}\ ft/\text{min}$$

To convert to miles, we divide by 5,280 ft and to convert to hours, we multiply by 60 min.

$$\frac{197\pi}{15} \cdot \frac{1}{5{,}280} \cdot \frac{60}{1} = 0.47 \text{ mi/hr}$$

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55. The radius is 98.5 and the distance to the ground in 12.0 feet.

Based on our work in Example 6 (b): $h = (98.5 + 12.0) - 98.5\cos\theta$ and $\theta = \frac{2\pi}{15}t$ or

$$h = 110.5 - 98.5\cos\left(\frac{2\pi}{15}t\right)$$

57.

$v = \frac{s}{t}$ Formula for linear velocity

$= \frac{16\ km}{1\ hr}$ Substitute known values

$= 16$ km per hour Simplify

$= 16{,}000$ meters / hour Change to meters per hour by multiplying by 1,000

$\omega = \frac{v}{r}$ Relationship between angular velocity and linear velocity

$= \frac{16{,}000\ m/hr}{0.3\ m}$ Substitute known values and change radius to meters

$= 53{,}300$ radians / hour Simplify

59. $\omega_1 = 2\ \frac{\text{rev}}{\text{sec}}$, $n_1 = 42$, and $n_2 = 7$

$$\omega_2 = \frac{n_1}{n_2}\omega_1$$

$$= \frac{42}{7}(2) = 12 \text{ rps}$$

61. $\omega_3 = \frac{n_2}{n_3}\omega_2$ and $\omega_2 = \frac{n_1}{n_2}\omega_1$

Substituting ω_2 into the first equation, we get: $\omega_3 = \frac{n_2}{n_3}\left(\frac{n_1}{n_2}\right)\omega_1$

$$\omega_3 = \frac{n_1}{n_3}\omega_2$$

63. The angular velocity, ω_1, of the chain ring is 90 revolutions per minute. We want to convert this to radians per hour:

$$\omega_1 = 90\ \frac{rev}{\text{min}} \cdot \frac{2\pi\ rad}{1\ rev} \cdot \frac{60\ \text{min}}{1\ hr}$$

$$= 10{,}800\pi\ \frac{rad}{hr}$$

Next, we want the radii in kilometers:

radius of the chain ring: $r_1 = \frac{1}{2}(150\ mm) \cdot \frac{1\ km}{1{,}000{,}000\ mm} = 0.000075\ km$

radius of the sprocket: $r_2 = \frac{1}{2}(95\ mm) \cdot \frac{1\ km}{1{,}000{,}000\ mm} = 0.0000475\ km$

radius of the wheel: $r_3 = \frac{1}{2}(700\ mm) \cdot \frac{1\ km}{1{,}000{,}000\ mm} = 0.000350\ km$

We know that $r_1\omega_1 = r_2\omega_2$ because the linear velocity of the chain ring equals the linear velocity of the sprocket.

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Substituting the given values, we can solve for ω_2:

$$0.000075(10{,}800\pi) = 0.0000475\omega_2$$

$$\frac{0.000075(10{,}800\pi)}{0.0000475} = \omega_2$$

$$\omega_2 = 53{,}572.42209$$

We know that the angular velocity of the sprocket, ω_2, is equal to the angular velocity of the wheel, ω_3. Now we can find the linear velocity of the wheel, v_3:

$$v_3 = r_3\omega_3$$

$$= 0.000350(53{,}572.42209) = 18.8 \text{ km/hr}$$

65. We use the same notation and formulas that we did in problem 63. In this problem:

$$r_1 = \frac{1}{2}(210\ mm) = 105\ mm$$

$$\omega_1 = 85\ \frac{rev}{\min} \cdot \frac{2\pi\ rad}{1\ rev} \cdot \frac{60\ \min}{1\ hr} = 10{,}200\pi\ \frac{rad}{hr}$$

$$r_3 = 350\ mm$$

$$v_3 = 45\ \frac{km}{hr} \cdot \frac{1{,}000{,}000\ mm}{1\ km} = 45{,}000{,}000\ \frac{mm}{hr}$$

We can find $\omega_2 = \omega_3$

$$\omega_2 = \omega_3 = \frac{v_3}{r_3} = \frac{45{,}000{,}000}{350} = 128{,}571.4286\ \frac{rad}{hr}$$

Now we can find r_2:

$$r_2\omega_2 = r_1\omega_1$$

$$r_2 = \frac{r_1\omega_1}{\omega_2}$$

$$= \frac{105(10{,}200\pi)}{128571.4286}$$

$$= 26.17\ mm$$

The diameter is $2(26.17\ mm) = 52.3\ mm$

67. We will use the same notation and formulas used in problems 63 and 65.

We are given: $r_1 = 75\ mm$

$$r_2 = 40\ mm$$

$$r_3 = 350\ mm$$

$$v_3 = 20\ \frac{km}{hr}$$

We must convert v_3 to millimeters per minute:

$$20\ \frac{km}{hr} \cdot \frac{1{,}000{,}000\ mm}{1\ km} \cdot \frac{1\ hr}{60\ \min} = \frac{1{,}000{,}000}{3}\ \frac{mm}{\min}$$

Next we will find ω_3 which is equal to ω_2:

$$\omega_3 = \omega_2 = \frac{v_3}{r_3} = \frac{\frac{1{,}000{,}000}{3}}{350} = \frac{20{,}000}{21}\ \frac{rad}{\min}$$

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Now we can solve for ω_1:

$$r_1\omega_1 = r_2\omega_2$$

$$\omega_1 = \frac{r_2\omega_2}{r_1}$$

$$= \frac{40\left(\frac{20{,}000}{21}\right)}{75} = 507.9365\ \frac{rad}{\text{min}}$$

We must convert the angular velocity, ω_1, to $\frac{rev}{\text{min}}$:

$$507.9365\ \frac{rad}{\text{min}} \cdot \frac{1\ rev}{2\pi\ rad} = 80.8\ rpm$$

71. $|V| = \sqrt{75^2 + 45^2} = 87.5$

73. $\theta = 90^\circ + 48.7^\circ = 138.7^\circ$ and $|V| = 2(285.5) = 571$

$|V_x| = |V|\ \cos\theta$ $\quad$ $|V_y| = |V|\ \sin\theta$

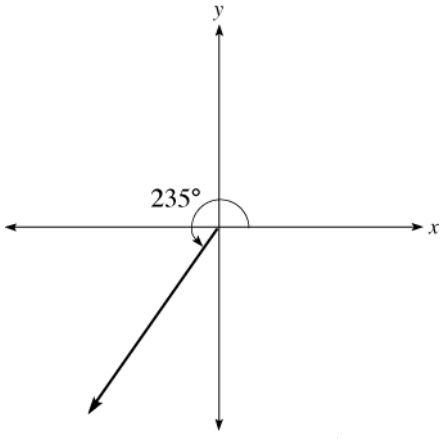
$= 571\cos 138.7^\circ$ $\quad$ $= 571\sin 138.7^\circ$

$= -429$ $\quad$ $= 377$

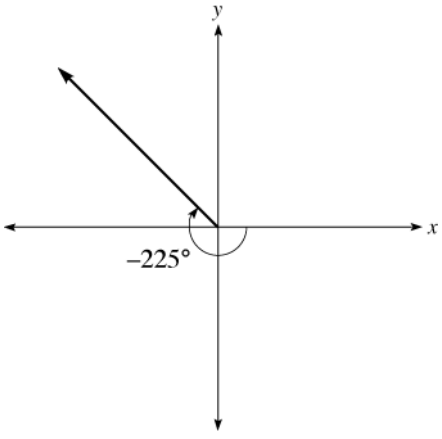
The ship has sailed 429 miles west and 377 miles north.

Chapter 3 Test

1. The reference angle for 235° is 235° – 180° = 55°:



2. The reference angle for –225° is –225° + 270° = 45°:



3. Calculating the value: $\cot 320° = \frac{1}{\tan 320°} \approx -1.1918$

4. Calculating the value: $\csc(-267.3°) = \frac{1}{\sin(-267.3°)} \approx 1.0011$

5. Calculating the value: $\sec 140°20' = \sec\left(140\frac{1}{3}\right)^\circ = \frac{1}{\cos\left(140\frac{1}{3}\right)^\circ} \approx -1.2991$

6. The reference angle is given by $\hat{\theta} = \sin^{-1}(0.4444) \approx 26.4°$. Since θ terminates in quadrant II, $\theta = 180° - 26.4° = 153.6°$.

7. Since $\cot\theta = 0.9659$, $\tan\theta = \frac{1}{0.9659}$. The reference angle is given by $\hat{\theta} = \tan^{-1}\left(\frac{1}{0.9659}\right) \approx 46.0°$. Since θ terminates in quadrant III, $\theta = 180° + 46.0° = 226.0°$.

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8. The reference angle for 135° is 45°. Since 135° terminates in quadrant II, its cosine will be negative. Therefore:

$$\cos 135° = -\cos 45° = -\frac{\sqrt{2}}{2}$$

9. The reference angle for 330° is 30°. Since 330° terminates in quadrant IV, its tangent will be negative. Therefore:

$$\tan 330° = -\tan 30° = -\frac{\sin 30°}{\cos 30°} = -\frac{1/2}{\sqrt{3}/2} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

10. Converting to radian measure: $200° = 200 \bullet \frac{\pi}{180} = \frac{10\pi}{9}$ radians

11. Converting to degree measure: $\frac{7\pi}{12}$ radians $= \frac{7\pi}{12} \bullet \frac{180°}{\pi} = 105°$

12. Computing the value: $\cos\left(-\frac{4\pi}{3}\right) = -\cos\frac{\pi}{3} = -\frac{1}{2}$

13. Computing the value: $\sec\frac{5\pi}{6} = \frac{1}{\cos\frac{5\pi}{6}} = -\frac{1}{\cos\frac{\pi}{6}} = -\frac{1}{\sqrt{3}/2} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

14. Since t is the radian measure: $\sin t = y = -\frac{3}{\sqrt{13}} = -\frac{3\sqrt{13}}{13}$, $\cos t = x = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$, $\tan t = \frac{y}{x} = -\frac{3}{2}$

15. The argument is $4x$.

16. Using a calculator: $\cos 45 \approx 0.5253$

17. Since $-1 \le \sin z \le 1$, the statement is not possible.

18. Substituting $x = \frac{\pi}{4}$: $2\cos\left(3 \bullet \frac{\pi}{4} - \frac{\pi}{2}\right) = 2\cos\left(\frac{3\pi}{4} - \frac{\pi}{2}\right) = 2\cos\frac{\pi}{4} = 2 \bullet \frac{\sqrt{2}}{2} = \sqrt{2}$

19. Note that $60° = \frac{\pi}{3}$ radians, so: $s = r\theta = 6 \bullet \frac{\pi}{3} = 2\pi \approx 6.28$ feet

20. Using $s = r\theta$:

$$\pi = r \bullet \frac{\pi}{5}$$
$$5\pi = r\pi$$
$$r = 5 \text{ cm}$$

21. Finding the sector area: $A = \frac{1}{2}r^2\theta = \frac{1}{2}(3)^2(2.4) = 10.8 \text{ cm}^2$

22. In 2 minutes, the string has twirled $\theta = \left(0.5 \frac{\text{rev}}{\text{min}}\right) \bullet (2 \text{ min}) = 1 \text{ rev} = 2\pi$ radians.

Thus the plane has traveled: $s = r\theta = (5 \text{ feet}) \bullet (2\pi \text{ radians}) = 10\pi \approx 31.4$ feet

23. Using $s = r\theta$ to find the central angle:

$$8 = r \bullet 4$$
$$r = 2 \text{ in.}$$

Now finding the sector area: $A = \frac{1}{2}r^2\theta = \frac{1}{2}(2)^2(4) = 8 \text{ in.}^2$

24. Since $v = r\omega$, $v = 5 \bullet 4 = 20 \frac{\text{in.}}{\text{sec}}$. Thus $s = vt = 20 \bullet 6 = 120$ inches.

25. The angular velocity is given by: $\omega = \frac{v}{r} = \frac{5 \text{ cm/sec}}{10 \text{ cm}} = \frac{1}{2} \frac{\text{radians}}{\text{sec}}$

26. First find $\omega = 40 \frac{\text{rev}}{\text{min}} \bullet 2\pi \frac{\text{radians}}{\text{rev}} = 80\pi \frac{\text{radians}}{\text{min}}$. Thus: $v = r\omega = 2 \text{ ft} \bullet 80\pi \frac{\text{radians}}{\text{min}} = 160\pi \frac{\text{feet}}{\text{min}}$

27. For the smaller pulley: $\omega = \frac{v}{r} = \frac{24 \text{ cm/sec}}{6 \text{ cm}} = 4 \frac{\text{radians}}{\text{sec}}$

For the larger pulley: $\omega = \frac{v}{r} = \frac{24 \text{ cm/sec}}{8 \text{ cm}} = 3 \frac{\text{radians}}{\text{sec}}$

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28. First find $\omega = 900\frac{\text{rev}}{\text{min}} \bullet 2\pi\frac{\text{radians}}{\text{rev}} = 1{,}800\pi\frac{\text{radians}}{\text{min}}$. Thus the linear velocity is given by:

$$v = r\omega = 3.5 \text{ ft} \bullet 1{,}800\pi\frac{\text{radians}}{\text{min}} = 6{,}300\pi\frac{\text{ft}}{\text{min}} \approx 19{,}792\frac{\text{ft}}{\text{min}} \approx 19{,}800\frac{\text{ft}}{\text{min}}$$

29. The linear velocity is: $11\frac{\text{mi}}{\text{hour}} \bullet \frac{5{,}280 \text{ ft}}{1 \text{ mi}} \bullet \frac{1 \text{ hour}}{60 \text{ min}} = 968\frac{\text{ft}}{\text{min}}$

Finding the angular velocity: $\omega = \frac{v}{r} = \frac{968\frac{\text{ft}}{\text{min}}}{7 \text{ ft}} \approx 138.29\frac{\text{radians}}{\text{min}}$

Converting to rpm: $\omega = 138.29\frac{\text{radians}}{\text{min}} \bullet \frac{1 \text{ rev}}{2\pi \text{ radians}} \approx 22.0 \text{ rpm}$

30. First find the angular velocity of the chainring: $\omega = 75\frac{\text{rev}}{\text{min}} \bullet 2\pi\frac{\text{rad}}{\text{rev}} = 150\pi\frac{\text{rad}}{\text{min}}$

The linear velocity of the chain is given by: $v = r\omega = (105 \text{ mm}) \bullet \left(150\pi\frac{\text{rad}}{\text{min}}\right) = 15{,}750\pi\frac{\text{mm}}{\text{min}}$

The angular velocity of the rear sprocket is: $\omega = \frac{v}{r} = \frac{15{,}750\pi\frac{\text{mm}}{\text{min}}}{25 \text{ mm}} = 630\pi\frac{\text{rad}}{\text{min}}$

The linear velocity of the rear tire is: $v = r\omega = (350 \text{ mm}) \bullet \left(630\pi\frac{\text{rad}}{\text{min}}\right) = 220{,}500\pi\frac{\text{mm}}{\text{min}}$

Converting to kilometers per hour: $v = 220{,}500\pi\frac{\text{mm}}{\text{min}} \bullet \frac{60 \text{ min}}{1 \text{ hr}} \bullet \frac{1 \text{ km}}{1{,}000{,}000 \text{ mm}} \approx 41.6\frac{\text{km}}{\text{hour}}$

Cumulative Test Chapters 1–3

1. If $y = 3$, then $x = 2y = 2 \bullet 3 = 6$ and $h = y\sqrt{3} = 3\sqrt{3}$. Since triangle *ACD* is isosceles, $s = 3\sqrt{3}$.
Finally $r = s\sqrt{2} = 3\sqrt{3} \bullet \sqrt{2} = 3\sqrt{6}$.

2. The exterior angle has a measure of $\frac{360°}{6} = 60°$, so the interior angle has a measure of $180° - 60° = 120°$.

3. Using the distance formula: $d = \sqrt{(a-0)^2 + (b-0)^2} = \sqrt{a^2 + b^2}$

4. The coterminal angles are $150° + 360°k$, for any integer k.

5. If $\sin\theta < 0$ and $\cos\theta > 0$, θ must lie in quadrant IV.

6. First find r: $r = \sqrt{(-6)^2 + (8)^2} = \sqrt{36 + 64} = \sqrt{100} = 10$. Using Definition I for the trigonometric functions:

$$\sin\theta = \frac{y}{r} = \frac{8}{10} = \frac{4}{5} \qquad \csc\theta = \frac{r}{y} = \frac{10}{8} = \frac{5}{4}$$

$$\cos\theta = \frac{x}{r} = -\frac{6}{10} = -\frac{3}{5} \qquad \sec\theta = \frac{r}{x} = \frac{10}{-6} = -\frac{5}{3}$$

$$\tan\theta = \frac{y}{x} = \frac{8}{-6} = -\frac{4}{3} \qquad \cot\theta = \frac{x}{y} = \frac{-6}{8} = -\frac{3}{4}$$

7. If $\tan\theta = \frac{12}{5}$ and θ terminates in quadrant III, $x < 0$ and $y < 0$, so choose $y = -12$ and $x = -5$. Finding r using $x^2 + y^2 = r^2$:

$$(-5)^2 + (-12)^2 = r^2$$
$$25 + 144 = r^2$$
$$r^2 = 169$$
$$r = 13$$

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Using $x = -5$, $y = -12$, and $r = 13$ in the definitions:

$$\sin\theta = \frac{y}{r} = -\frac{12}{13} \qquad \csc\theta = \frac{r}{y} = -\frac{13}{12} \qquad \cot\theta = \frac{y}{x} = \frac{5}{12}$$

$$\cos\theta = \frac{x}{r} = -\frac{5}{13} \qquad \sec\theta = \frac{r}{x} = -\frac{13}{5}$$

8. Using the reciprocal identity: $\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{1}{-4} = -\dfrac{1}{4}$

9. Multiplying: $(\sin\theta + 3)(\sin\theta - 7) = \sin^2\theta + 3\sin\theta - 7\sin\theta - 21 = \sin^2\theta - 4\sin\theta - 21$

10. Working from the left side: $\sin\theta(\csc\theta + \cot\theta) = \sin\theta\left(\dfrac{1}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta}\right) = \sin\theta \bullet \dfrac{1+\cos\theta}{\sin\theta} = 1 + \cos\theta$

11. Using the Pythagorean Theorem, first find c:

$$5^2 + 12^2 = c^2$$
$$25 + 144 = c^2$$
$$c^2 = 169$$
$$c = 13$$

Using $a = 5$, $b = 12$, and $c = 13$, find the three trigonometric functions of A:

$$\sin A = \frac{a}{c} = \frac{5}{13} \qquad \cos A = \frac{b}{c} = \frac{12}{13} \qquad \tan A = \frac{a}{b} = \frac{5}{12}$$

Now use the Cofunction Theorem to find the three trigonometric functions of B:

$$\sin B = \cos A = \frac{12}{13} \qquad \cos B = \sin A = \frac{5}{13} \qquad \tan B = \cot A = \frac{b}{a} = \frac{12}{5}$$

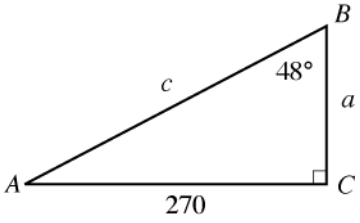
12. Using the Cofunction Theorem, $\csc 63° = \sec(90° - 63°) = \sec 27°$

13. Subtracting the angles: $25°15' - 15°32' = 24°75' - 15°32' = 9°43'$

14. Finding the angle θ: $\theta = \tan^{-1}(0.1608) \approx 9.1°$

15. **a**. two **b**. two **c**. four

16. Begin by drawing ΔABC and label missing information:



Note that $A = 90° - 48° = 42°$. Therefore:

$$\sin 48° = \frac{270}{c} \qquad \tan 48° = \frac{270}{a}$$
$$c\sin 48° = 270 \qquad a\tan 48° = 270$$
$$c = \frac{270}{\sin 48°} \approx 360 \qquad a = \frac{270}{\tan 48°} \approx 240$$

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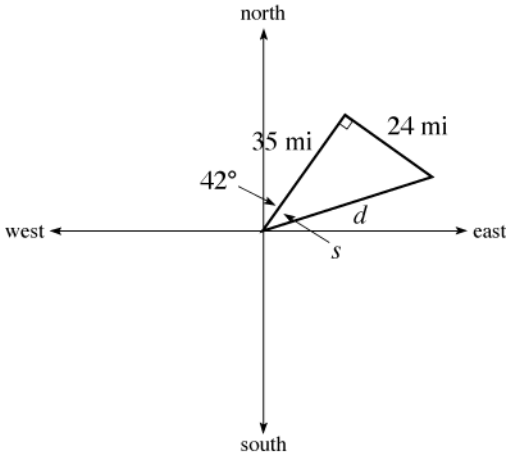
17. Construct the figure:



To find his distance from the starting point, use the Pythagorean Theorem:

$$d^2 = 35^2 + 24^2 = 1801$$

$$d = \sqrt{1801} \approx 42.4 \text{ mi}$$

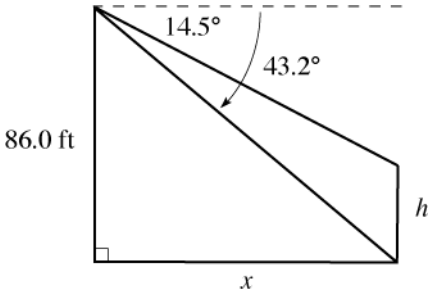
Now find angle s:

$$\tan s = \frac{24}{35}$$

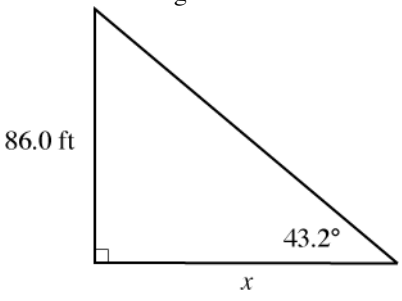
$$s = \tan^{-1}\left(\frac{24}{35}\right) \approx 34.4°$$

His bearing is N 76.4° E.

18. Construct the figure:



First consider the triangle:



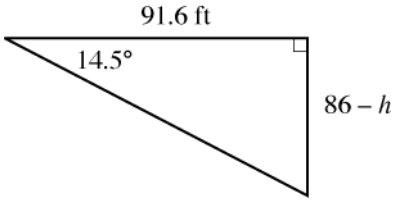
Therefore:

$$\tan 43.2° = \frac{86.0}{x}$$

$$x \tan 43.2° = 86.0$$

$$x = \frac{86.0}{\tan 43.2°} \approx 91.6 \text{ ft}$$

Now consider the triangle:



Therefore:

$$\tan 14.5° = \frac{86 - h}{91.6}$$

$$86 - h = 91.6 \tan 14.5°$$

$$h = 86 - 91.6 \tan 14.5° \approx 62.3 \text{ ft}$$

The building next door is approximately 62.3 feet tall.

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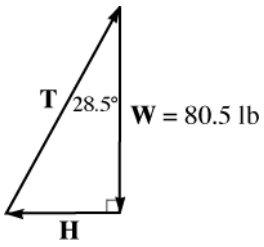
19. Computing the magnitudes of $\mathbf{V}_x$ and $\mathbf{V}_y$:

$$|\mathbf{V}_x| = 5.0\cos 30° \approx 4.3 \qquad |\mathbf{V}_y| = 5.0\sin 30° = 2.5$$

20. The corresponding force diagram would be:



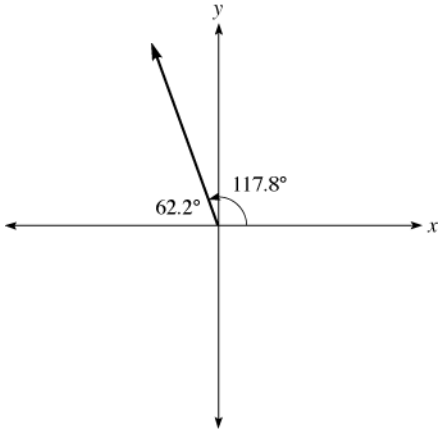
Therefore:

$$\cos 28.5° = \frac{80.5}{|\mathbf{T}|} \qquad \tan 28.5° = \frac{|\mathbf{H}|}{80.5}$$

$$|\mathbf{T}|\cos 28.5° = 80.5 \qquad |\mathbf{H}| = 80.5\tan 28.5° \approx 43.7 \text{ lb}$$

$$|\mathbf{T}| = \frac{80.5}{\cos 28.5°} \approx 91.6 \text{ lb}$$

21. The reference angle for $117.8°$ is $180° - 117.8° = 62.2°$:



22. The reference angle is given by $\hat{\theta} = \cos^{-1}(0.4772) \approx 61.5°$. Since θ terminates in quadrant IV, $\theta = 360° - 61.5° = 298.5°$.

23. Converting to radian measure: $-390° = -390 \bullet \frac{\pi}{180} = -\frac{13\pi}{6}$ radians

24. Since $\frac{5\pi}{3}$ terminates in quadrant IV, its sine will be negative. The reference angle is $2\pi - \frac{5\pi}{3} = \frac{\pi}{3}$, therefore:

$$\sin\frac{5\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

25. From the unit circle, $\cos\theta = -\frac{1}{2}$ when $\theta = \frac{2\pi}{3}$ and $\theta = \frac{4\pi}{3}$.

26. As t increases from $\frac{\pi}{2}$ to π, the tangent values (which are negative) are increasing towards 0.

27. Finding the arc length: $s = r\theta = 12 \bullet \frac{\pi}{6} = 2\pi \approx 6.28$ m

28. Note that $\theta = 60° = 60 \bullet \frac{\pi}{180} = \frac{\pi}{3}$ radians. Finding the sector area: $A = \frac{1}{2}r^2\theta = \frac{1}{2}(4)^2 \bullet \frac{\pi}{3} = \frac{8\pi}{3}$ in.^{2}

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29. In 20 seconds, the point has traveled $\theta = \left(\frac{3\pi}{4}\frac{\text{radians}}{\text{sec}}\right) \bullet (20 \text{ sec}) = 15\pi$ radians .

Thus the distance moved is: $s = r\theta = (8 \text{ feet}) \bullet (15\pi \text{ radians}) = 120\pi$ feet

30. First find $\omega = 2{,}941\frac{\text{rev}}{\text{min}} \bullet 2\pi\frac{\text{radians}}{\text{rev}} = 5{,}882\pi\frac{\text{radians}}{\text{min}}$. Thus the linear velocity is given by:

$$v = r\omega = 1.5 \text{ in} \bullet 5{,}882\pi\frac{\text{radians}}{\text{min}} = 8{,}823\pi\frac{\text{in}}{\text{min}}$$

Converting to miles per hour:

$$v = 8{,}823\pi\frac{\text{in}}{\text{min}} \bullet 60\frac{\text{min}}{\text{hr}} \bullet \frac{1}{12}\frac{\text{ft}}{\text{in}} \bullet \frac{1}{5{,}280}\frac{\text{mi}}{\text{ft}} \approx 26.2 \text{ mph}$$

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Chapter 4
Graphing and Inverse Functions

4.1 Basic Graphs

EVEN SOLUTIONS

2. For a periodic function f, the period p is the smallest positive number for which $f(x+p)=f(x)$ for all x in the domain of f.

4. If c is a domain value for a function f and $f(c)=0$, then $x=c$ is called a zero of the function. If c is a real number, then $x=c$ will appear as an x-intercept for the graph of f.

6. The graph of an even function is symmetric about the y-axis and the graph of an odd function is symmetric about the origin.

8. The tangent, cotangent, secant, and cosecant functions all have asymptotes.

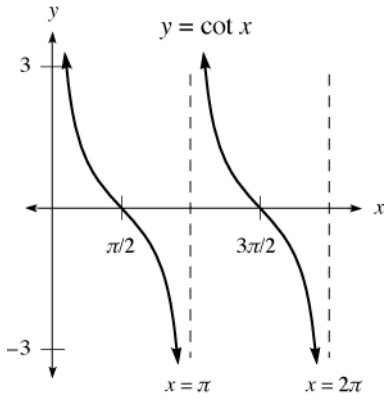
10. The tangent and cotangent functions have a period of π.

12. The cosine and secant functions are even.

14. Making a table of values:

x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
$y=\cot x$	undefined	1	0	-1	undefined	1	0	-1	undefined

Sketching the graph:



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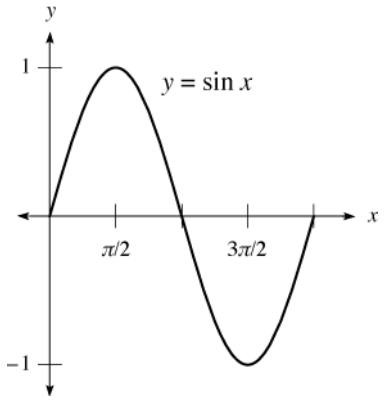
16. Making a table of values:

x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
$y = \sin x$	0	$1/\sqrt{2} \approx 0.7$	1	$1/\sqrt{2} \approx 0.7$	0	$-1/\sqrt{2} \approx -0.7$	-1	$-1/\sqrt{2} \approx -0.7$	0

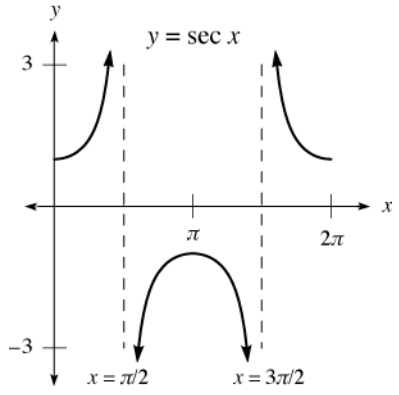
Sketching the graph:



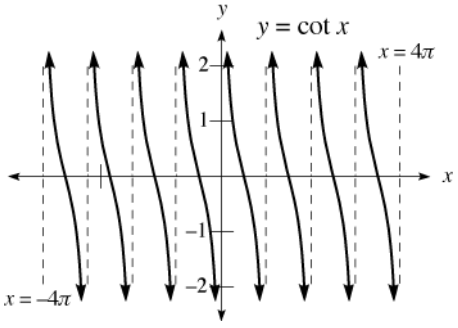
18. Making a table of values:

x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
$y = \sec x$	1	$\sqrt{2} \approx 1.4$	undefined	$-\sqrt{2} \approx -1.4$	-1	$-\sqrt{2} \approx -1.4$	undefined	$\sqrt{2} \approx 1.4$	1

Sketching the graph:



20. Sketching $y = \cot x$ from $x = -4\pi$ to $x = 4\pi$:



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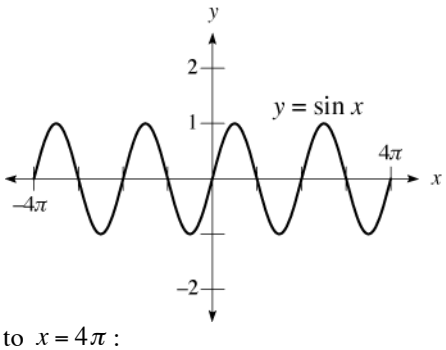
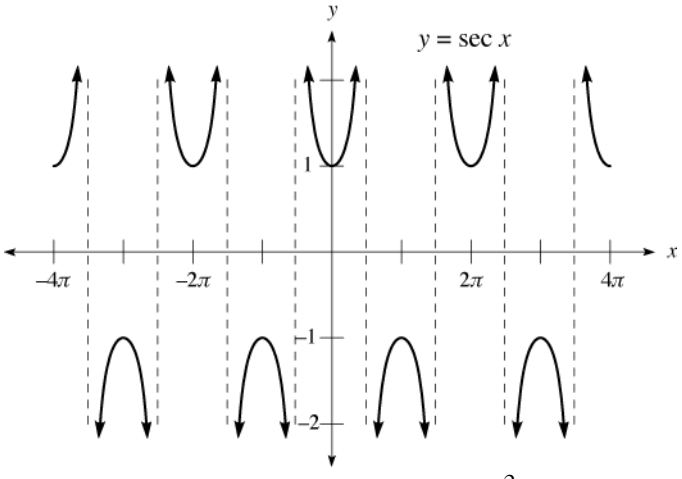
22. Sketching $y = \sin x$ from $x = -4\pi$ to $x = 4\pi$:



24. Sketching $y = \sec x$ from $x = -4\pi$ to $x = 4\pi$:



26. $\cos x = 0$ when x is any odd multiple of $\frac{\pi}{2}$. So $\cos x = 0$ when $x = \frac{\pi}{2}, \frac{3\pi}{2}$.

28. $\cos x = 1$ when x is any multiple of 2π. So $\cos x = 1$ when $x = 0, 2\pi$.

30. $\cot x = 0$ whenever $\cos x = 0$, which occurs when x is any odd multiple of $\frac{\pi}{2}$. So $\cot x = 0$ when $x = \frac{\pi}{2}, \frac{3\pi}{2}$.

32. $\csc x = 1$ whenever $\sin x = 1$, which occurs when x is $\frac{\pi}{2}$ plus any multiple of 2π. So $\csc x = 1$ when $x = \frac{\pi}{2}$.

34. The function $\cot x$ is undefined when $\sin x = 0$, which occurs when $x = 0, \pi, 2\pi$.

36. The function $\sec x$ is undefined when $\cos x = 0$, which occurs when $x = \frac{\pi}{2}, \frac{3\pi}{2}$.

38. Since cosine is an even function: $\cos(-135°) = \cos 135° = -\frac{\sqrt{2}}{2}$

40. Since cosine is an even function: $\cos\left(-\frac{5\pi}{3}\right) = \cos\frac{5\pi}{3} = \frac{1}{2}$

42. Since sine is an odd function: $\sin(-120°) = -\sin 120° = -\frac{\sqrt{3}}{2}$

44. Since sine is an odd function: $\sin\left(-\frac{7\pi}{6}\right) = -\sin\frac{7\pi}{6} = -\left(-\frac{1}{2}\right) = \frac{1}{2}$

46. Since cosine is an even function: $\cos(-\theta) = \cos\theta = -\frac{1}{4}$

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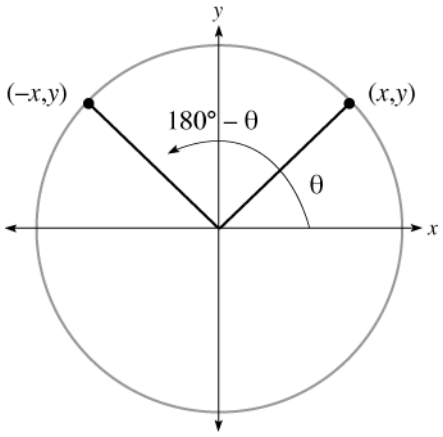
48. Drawing the figure:



Since $\cos(180° - \theta) = -x$ while $\cos\theta = x$, $\cos(180° - \theta) = -\cos\theta$.

50. Since sine is an odd function and cosine is an even function: $\cot(-\theta) = \dfrac{\cos(-\theta)}{\sin(-\theta)} = \dfrac{\cos\theta}{-\sin\theta} = -\dfrac{\cos\theta}{\sin\theta} = -\cot\theta$

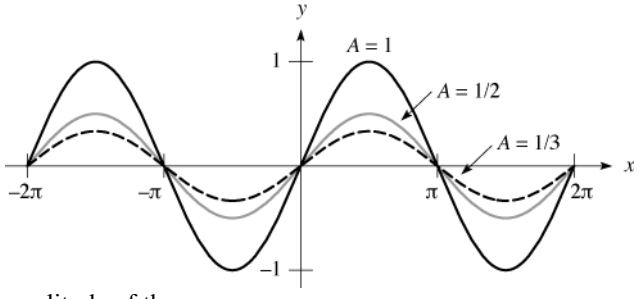
Thus cotangent is an odd function.

52. Working from the left side: $\cos(-\theta)\tan\theta = \cos\theta \cdot \dfrac{\sin\theta}{\cos\theta} = \sin\theta$

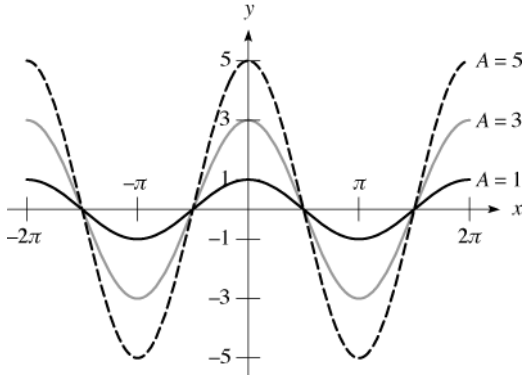
54. Working from the left side: $\cos(-\theta)\csc(-\theta)\tan(-\theta) = \cos\theta \cdot \dfrac{1}{\sin(-\theta)} \cdot \dfrac{\sin(-\theta)}{\cos(-\theta)} = \cos\theta \cdot \dfrac{1}{-\sin\theta} \cdot \dfrac{-\sin\theta}{\cos\theta} = 1$

56. Working from the left side: $\sec\theta - \cos(-\theta) = \dfrac{1}{\cos\theta} - \cos\theta \cdot \dfrac{\cos\theta}{\cos\theta} = \dfrac{1 - \cos^2\theta}{\cos\theta} = \dfrac{\sin^2\theta}{\cos\theta}$

58. The value of *A* effects the amplitude of the curve:



60. The value of *A* effects the amplitude of the curve:



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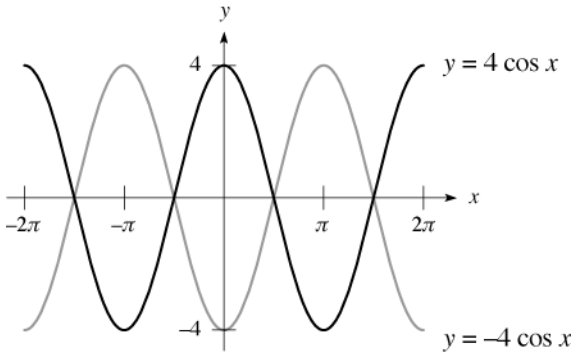
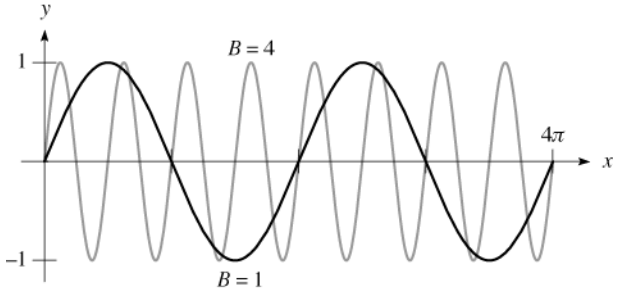
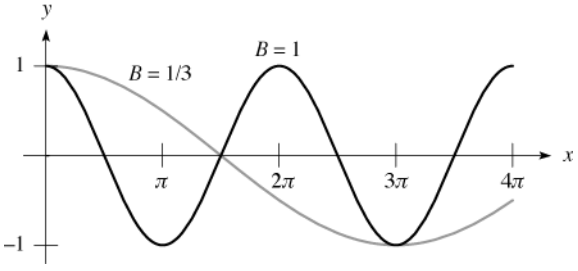
62. The negative sign reflects the curve across the x-axis:



64. The value of B effects how many cycles are completed in the given interval. The period is one-fourth as long:



66. The value of B effects how many cycles are completed in the given interval. The period is tripled.



68. Working from the left side: $\sin\theta\tan\theta+\cos\theta=\sin\theta\bullet\dfrac{\sin\theta}{\cos\theta}+\cos\theta\bullet\dfrac{\cos\theta}{\cos\theta}=\dfrac{\sin^2\theta+\cos^2\theta}{\cos\theta}=\dfrac{1}{\cos\theta}=\sec\theta$

70. Working from the left side:

$$(\sin\theta+\cos\theta)^2=(\sin\theta+\cos\theta)(\sin\theta+\cos\theta)=\sin^2\theta+\sin\theta\cos\theta+\sin\theta\cos\theta+\cos^2\theta=1+2\sin\theta\cos\theta$$

72. Converting to degrees: $\dfrac{3\pi}{4}$ radians $=\dfrac{3\pi}{4}\bullet\dfrac{180°}{\pi}=135°$

74. Converting to degrees: $\dfrac{3\pi}{2}$ radians $=\dfrac{3\pi}{2}\bullet\dfrac{180°}{\pi}=270°$

76. The values of $\sec x$ are undefined when $\cos x=0$, which occur when $x=\dfrac{\pi}{2},\dfrac{3\pi}{2}$. The correct answer is c.

78. Since cosine is an even function, $\cos(-\theta)=\cos\theta$. Since cosecant and tangent are odd functions, $\csc(-\theta)=-\csc\theta$ and $\tan(-\theta)=-\tan\theta$. The correct answer is a.

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ODD SOLUTIONS

1. y- coordinate, arc length
3. difference, greatest, least
5. equal, opposite
7. sine and cosine
9. secant and cosecant
11. sine, cosine, secant, and cosecant

For problems 13 through 23, see textbook answer section for graphs.

25. Refer to the graph in problem 16. Find the x-values corresponding to $y = 0$. You will notice that $x = 0$, $x = \pi$ and $x = 2\pi$.

27. Refer to the graphs in problem 16. Find the x-values corresponding to $y = 1$. You will notice that $x = \frac{\pi}{2}$.

29. Refer to the graphs in problem 17. Find the x-values corresponding to $y = 0$. You will notice that $x = 0$, $x = \pi$, and $x = 2\pi$.

31. Refer to the graphs in problem 18. Find the x-values corresponding to $y = 1$. You will notice that $x = 0$ and $x = 2\pi$.

33. Refer to the graphs in problem 17. Find the x-values where there is a vertical asymptote and the y-value is undefined. You will that $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$.

35. Refer to the graphs in problem 15. Find the x-values where there is a vertical asymptote and the y-value is undefined. You will that $x = 0$, $x = \pi$, and $x = 2\pi$.

37. $\cos(-60^\circ) = \cos 60^\circ$

$= \frac{1}{2}$

39. $\cos\left(-\frac{5\pi}{6}\right) = \cos\left(\frac{5\pi}{6}\right)$

$= -\frac{\sqrt{3}}{2}$

41. $\sin(-30^\circ) = -\sin 30^\circ$

$= -\frac{1}{2}$

43. $\sin\left(-\frac{3\pi}{4}\right) = -\sin\left(\frac{3\pi}{4}\right)$

$= -\frac{\sqrt{2}}{2}$

45. $\sin(-\theta) = -\sin\theta$

$= -\left(-\frac{1}{3}\right) = \frac{1}{3}$

49. $\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)}$

$= \frac{-\sin(\theta)}{\cos(\theta)}$

$= -\tan(\theta)$

Therefore, the tangent is an odd function.

51. $\sin(-\theta)\cot(-\theta) = (-\sin\theta)(-\cot\theta)$

$= \sin\theta\cot\theta$

$= \sin\theta\,\frac{\cos\theta}{\sin\theta}$

$= \cos\theta$

53. $\sin(-\theta)\sec(-\theta)\cot(-\theta) = \frac{\sin(-\theta)}{1} \cdot \frac{1}{\cos(-\theta)} \cdot \frac{\cos(-\theta)}{\sin(-\theta)} = 1$

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55. $\csc\theta + \sin(-\theta) = \dfrac{1}{\sin\theta} + (-\sin\theta)$ Sine is an odd function

$= \dfrac{1}{\sin\theta} - \dfrac{\sin^2\theta}{\sin\theta}$ LCD is $\sin\theta$

$= \dfrac{\cos^2\theta}{\sin\theta}$ Pythagorean identity

67. $\cos\theta\tan\theta = \cos\theta \cdot \dfrac{\sin\theta}{\cos\theta}$ Ratio identity

$= \dfrac{\cos\theta\sin\theta}{\cos\theta}$ Multiplication of fractions

$= \sin\theta$ Division of common factor

69. $(1+\sin\theta)(1-\sin\theta) = 1 - \sin^2\theta$ Multiplication

$= \cos^2\theta$ Pythagorean identity

71. $\dfrac{2\pi}{3} = \dfrac{2\pi}{3} \cdot \left(\dfrac{180}{\pi}\right)^\circ = 120^\circ$

73. $\dfrac{11\pi}{6} = \dfrac{11\pi}{6} \cdot \left(\dfrac{180}{\pi}\right)^\circ = 330^\circ$

4.2 Amplitude, Reflection, and Period

EVEN SOLUTIONS

2. If A is negative, then the graph of $y = A\sin x$ will be reflected about the x-axis.

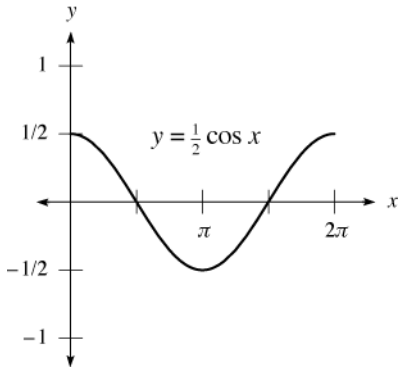
4. If $0 < B < 1$, then the period of $y = \sin Bx$ will be greater than 2π, and if $B > 1$ the period will be less than 2π.

6. To graph $y = \cos(-Bx)$, first write the function as $y = \cos Bx$ because cosine is an even function.

8. Making a table of values:

x	0	$\pi/2$	π	$3\pi/2$	2π
$y = \frac{1}{2}\cos x$	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$\frac{1}{2}$

Sketching the graph:



The amplitude is $A = \dfrac{1}{2}\left|\dfrac{1}{2} - \left(-\dfrac{1}{2}\right)\right| = \dfrac{1}{2}(1) = \dfrac{1}{2}$.

10. The amplitude is 0.5.

12. The amplitude is $\dfrac{5}{2}$.

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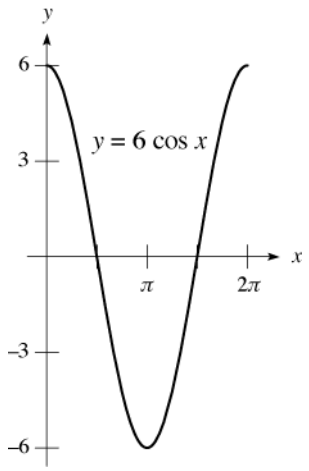
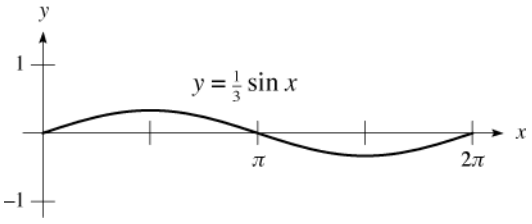
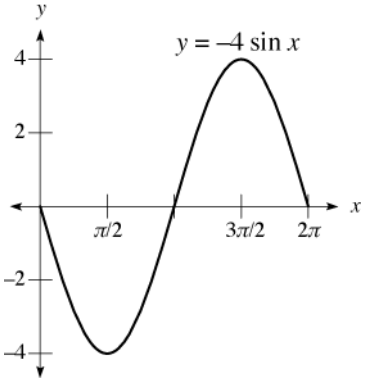
14. The amplitude is 6:



16. The amplitude is $\frac{1}{3}$:



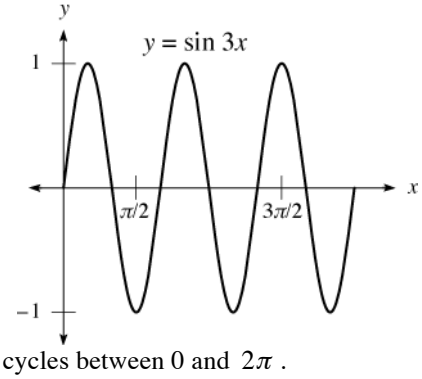
18. The amplitude is 4 and the curve is reflected across the x-axis:



20. Making a table of values:

x	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π	$7\pi/6$	$4\pi/3$	$3\pi/2$	$5\pi/3$	$11\pi/6$	2π
$y = \sin 3x$	0	1	0	−1	0	1	0	−1	0	1	0	−1	0

Sketching the graph:



The graph goes through 3 complete cycles between 0 and 2π.

22. The period is $\frac{2\pi}{5}$.

24. The period is $\frac{2\pi}{1/3} = 6\pi$.

26. The period is $\frac{2\pi}{\pi/2} = 4$.

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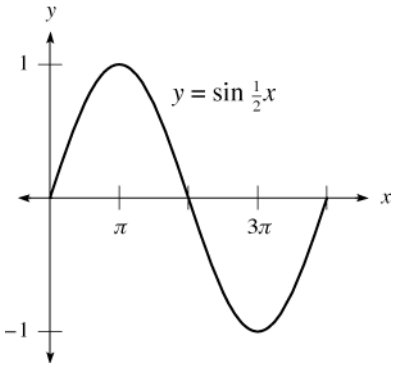
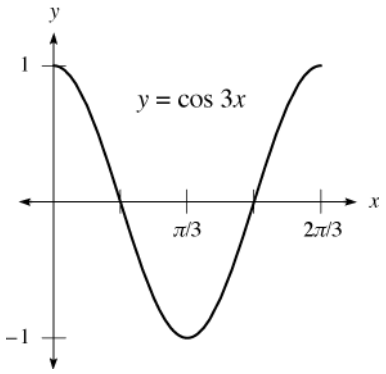
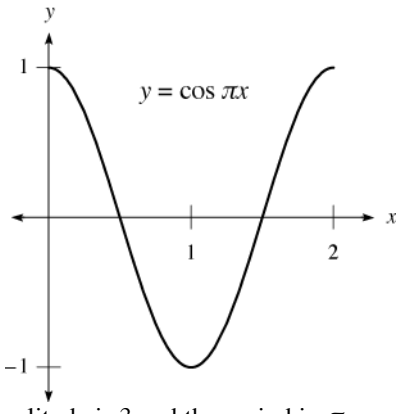
28. The period is $\dfrac{2\pi}{1/2} = 4\pi$. Sketching the graph:



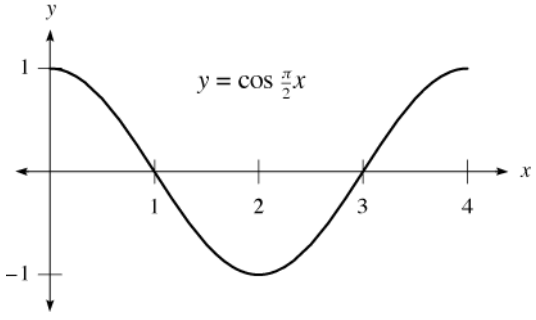
30. The period is $\dfrac{2\pi}{3}$. Sketching the graph:



32. The period is $\dfrac{2\pi}{\pi} = 2$. Sketching the graph:



34. The period is $\dfrac{2\pi}{\pi/2} = 4$. Sketching the graph:



36. The amplitude is 3 and the period is π .

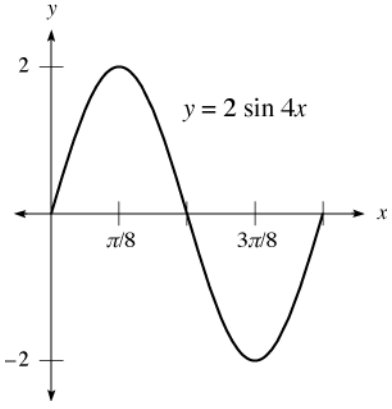
38. The amplitude is 2 and the period is 2π .

40. The amplitude is 3 and the period is $\dfrac{3\pi}{4} - \left(-\dfrac{\pi}{4}\right) = \dfrac{3\pi}{4} + \dfrac{\pi}{4} = \pi$.

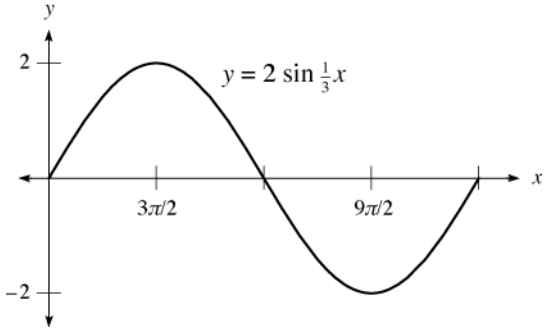
42. The amplitude is $\dfrac{1}{2}$ and the period is $\dfrac{2\pi}{4} = \dfrac{\pi}{2}$.

44. The amplitude is 5 and the period is $\dfrac{2\pi}{\pi/5} = 10$.

46. The amplitude is 2 and the period is $\dfrac{2\pi}{4} = \dfrac{\pi}{2}$:



48. The amplitude is 2 and the period is $\dfrac{2\pi}{1/3} = 6\pi$:



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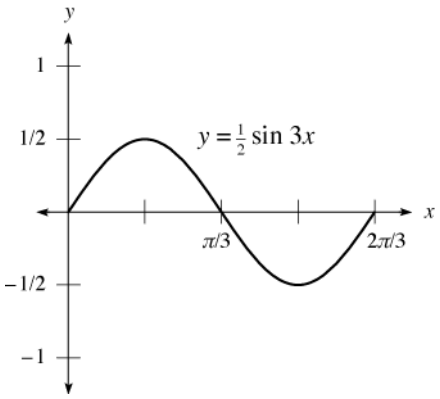
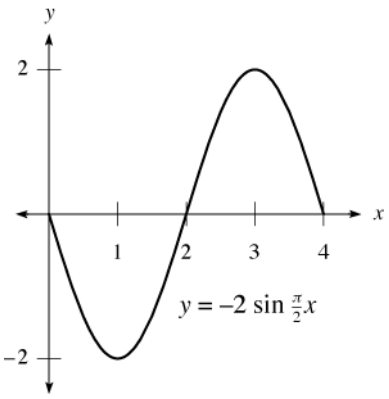
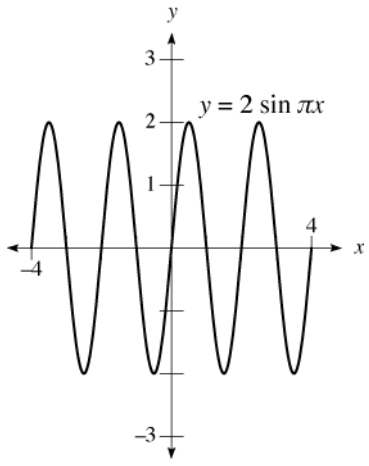
50. The amplitude is $\frac{1}{2}$ and the period is $\frac{2\pi}{3}$:



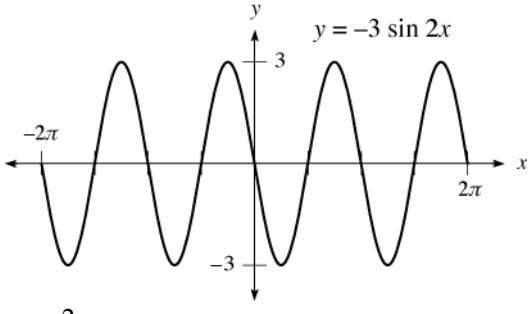
52. The amplitude is 2 and the period is $\frac{2\pi}{\pi/2} = 4$:



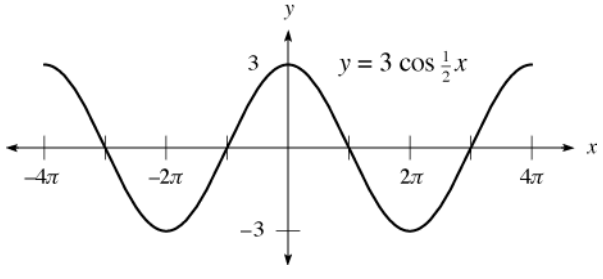
54. The amplitude is 2 and the period is $\frac{2\pi}{\pi} = 2$:



56. The amplitude is 3 and the period is $\frac{2\pi}{2} = \pi$. Note the reflection across the x-axis:



58. The amplitude is 3 and the period is $\frac{2\pi}{1/2} = 4\pi$. Sketching the graph:



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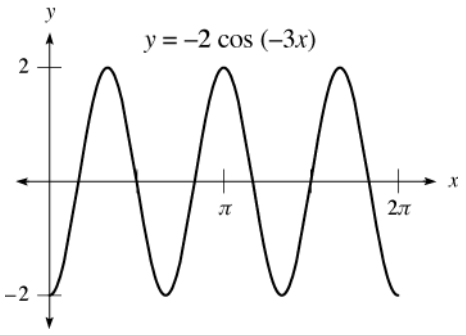
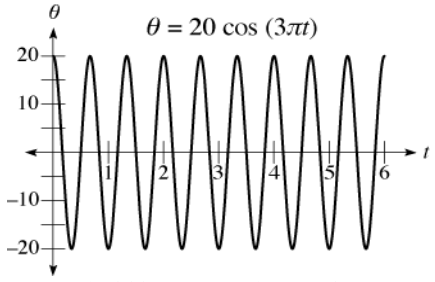
60. Note this graph is the same as $y = -2\cos 3x$, since cosine is an even function. The amplitude is 2 and the period is $\frac{2\pi}{3}$. Also note the reflection across the x-axis:



62. The maximum velocity is the amplitude, which is $2.5\,\frac{\text{meters}}{\text{sec}}$. The time it takes to move from lowest to highest position is $\frac{1}{2}$ of the period, which is $\frac{1}{2}\left(\frac{2\pi}{4\pi}\right) = 0.25$ second.

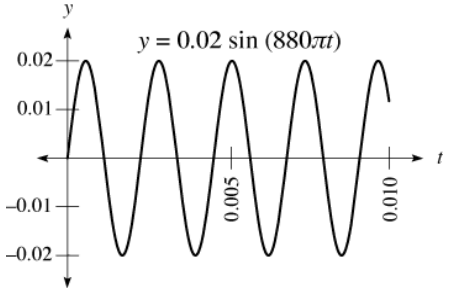
64. **a**. Graphing the function:



b. The maximum angular displacement is 20°, the amplitude of the curve.

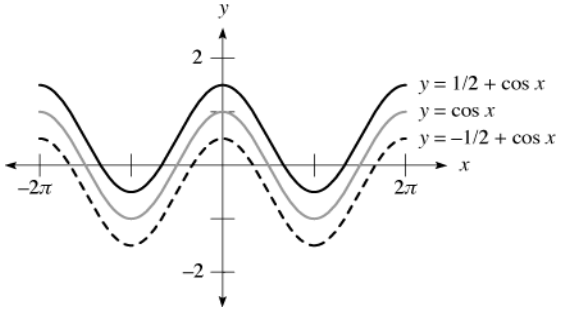
c. The pendulum completes one oscillation in $\frac{2\pi}{3\pi} = \frac{2}{3}$ second.

66. Sketching the graph:

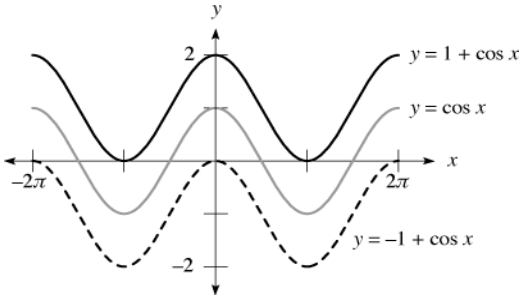


68. Since the period is $\frac{2\pi}{500\pi} = \frac{1}{250}$, the frequency is 250 Hz.

70. The graph is shifted $\frac{1}{2}$ unit upward or downward:



72. The graph is shifted 1 unit upward or downward:



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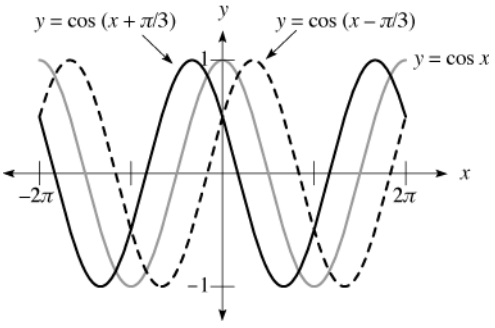
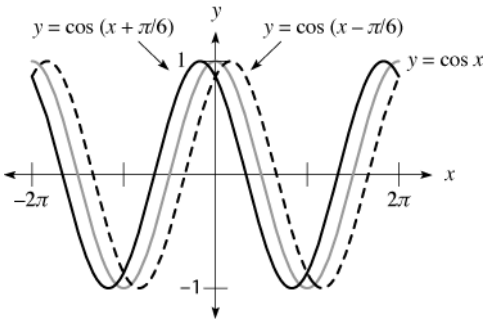
74. The graph is shifted $\frac{\pi}{3}$ units left or right:



76. The graph is shifted $\frac{\pi}{6}$ units left or right:



78. Evaluating when $y = \frac{\pi}{6}$: $\sin\left(\frac{\pi}{6} - \frac{\pi}{2}\right) = \sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

80. Evaluating when $x = \frac{\pi}{2}$: $\cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right) = \cos\frac{2\pi}{3} = -\frac{1}{2}$

82. Evaluating when $x = \frac{\pi}{2}$ and $y = \frac{\pi}{6}$: $\cos\left(\frac{\pi}{2} - \frac{\pi}{6}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$

84. Evaluating when $x = \frac{\pi}{2}$ and $y = \frac{\pi}{6}$: $\cos\frac{\pi}{2} - \cos\frac{\pi}{6} = 0 - \frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}$

86. Converting to radians: $70° = 70° \cdot \frac{\pi}{180°} = \frac{7\pi}{18}$ radians

88. Converting to radians: $330° = 330° \cdot \frac{\pi}{180°} = \frac{11\pi}{6}$ radians

90. The period is $\frac{2\pi}{2} = \pi$. The correct answer is d.

92. The period is $\frac{2\pi}{2\pi} = 1$. Since this is an inverted cosine curve, its lowest position occurs when $t = 0$ and its highest position occurs when $t = \frac{1}{2}$. So the time is $\frac{1}{2}$ second. The correct answer is b.

ODD SOLUTIONS

1. $|A|$, $[-|A|, |A|]$ **3.** $\frac{2\pi}{B}$ **5.** $-\sin Bx$, odd

7.

x	0	$\pi/2$	π	$3\pi/2$	2π
$y = 2\sin x$	0	2	0	−2	0

The amplitude of this curve is 2.

For Problems 9 and 11, see answer section of textbook.

13.

x	0	$\pi/2$	π	$3\pi/2$	2π
$y = 6\sin x$	0	6	0	−6	0

The amplitude of this curve is 6.

15.

x	0	$\pi/2$	π	$3\pi/2$	2π
$y = \frac{1}{2}\cos x$	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$\frac{1}{2}$

The amplitude of this curve is $\frac{1}{2}$.

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17.

x	0	$\pi/2$	π	$3\pi/2$	2π
$y = -3\cos x$	-3	0	3	0	-3

The amplitude of this curve is 3.

19.

x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
$y = \sin 2x$	0	1	0	-1	0	1	0	-1	0

There are two complete cycles of the graph.

21. $y = \cos 4x$

$\text{Period} = \dfrac{2\pi}{4} = \dfrac{\pi}{2}$

23. $y = \sin\dfrac{1}{6}x$

$\text{Period} = \dfrac{2\pi}{1/6} = 12\pi$

25. $y = \cos 2\pi x$

$\text{Period} = \dfrac{2\pi}{2\pi} = 1$

27. $y = \sin 2x$

$\text{Period} = \dfrac{2\pi}{2} = \pi$

29. $y = \cos\dfrac{1}{3}x$

$\text{Period} = \dfrac{2\pi}{1/3} = 6\pi$

31. $y = \sin \pi x$

$\text{Period} = \dfrac{2\pi}{\pi} = 2$

33. $y = \sin\dfrac{\pi}{2}x$

$\text{Period} = \dfrac{2\pi}{\pi/2} = 4$

35. The amplitude is 4 because the greatest y-value is 4 and the least y-value is -4. The period is 4π because the graph repeats itself every 4π units, or $f(x+4\pi) = f(x)$.

37. The amplitude is 2 because the greatest y-value is 2 and the least y-value is -2. The period is 2 because the graph repeats itself every 2 units, or $f(x+2) = f(x)$.

39. The amplitude is 5 because the greatest y-value is 5 and the least y-value is -5. The period is 3 because one complete curve goes from $-\dfrac{3}{4}$ to $\dfrac{9}{4}$ or $\dfrac{9}{4} - \left(-\dfrac{3}{4}\right) = \dfrac{9}{4} + \dfrac{3}{4} = \dfrac{12}{4} = 3$.

41. $y = \dfrac{1}{3}\sin 3x$ $\quad \text{Amplitude} = \dfrac{1}{3}$ $\quad \text{Period} = \dfrac{2\pi}{3}$

43. $y = -10\cos\dfrac{x}{10}$ $\quad \text{Amplitude} = 10$ $\quad \text{Period} = \dfrac{2\pi}{1/10} = 20\pi$

45. $y = 4\sin 2x$ $\quad \text{Amplitude} = 4$ $\quad \text{Period} = \dfrac{2\pi}{2} = \pi$

47. $y = 3\sin\dfrac{1}{2}x$ $\quad \text{Amplitude} = 3$ $\quad \text{Period} = \dfrac{2\pi}{1/2} = 4\pi$

49. $y = \dfrac{1}{2}\cos 3x$ $\quad \text{Amplitude} = \dfrac{1}{2}$ $\quad \text{Period} = \dfrac{2\pi}{3}$

51. $y = -\dfrac{1}{2}\sin\dfrac{\pi}{2}x$ $\quad \text{Amplitude} = \left|-\dfrac{1}{2}\right| = \dfrac{1}{2}$ $\quad \text{Period} = \dfrac{2\pi}{\pi/2} = 4$

53. $y = 3\cos \pi x$ $\quad \text{Amplitude} = 3$ $\quad \text{Period} = \dfrac{2\pi}{\pi} = 2$

55. $y = 3\sin 2x$ $\quad \text{Amplitude} = 3$ $\quad \text{Period} = \dfrac{2\pi}{\pi} = 2$

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57. $y = -3\cos\frac{1}{2}x$ Amplitude $= |-3| = 3$

Period $= \frac{2\pi}{1/2} = 4\pi$

This graph is reflected about the x-axis.

59. $y = -2\sin(-3x)$ Amplitude $= 2$

$= -(-2\sin 3x)$ Period $= \frac{2\pi}{3}$

$= 2\sin 3x$

Note: $\sin(-x) = -\sin x$ because sine is an odd function.

61. $I = 20\sin(120\pi t)$

Amplitude = 20 The maximum value of I is 20 amperes.

Period $= \frac{2\pi}{120\pi} = \frac{1}{60}$ One complete cycle takes $\frac{1}{60}$ second.

63. $d = -3.5\cos(2\pi t)$

(b) Amplitude $= |-3.5| = 3.5$ Farthest distance is 3.5 centimeters.

(c) Period $= \frac{2\pi}{2\pi} = 1$ It takes 1 second to complete one oscillation.

67. $V = 230\sin(100\pi t)$

Period $= \frac{2\pi}{100\pi} = \frac{1}{50}$ Frequency $= \frac{1}{1/50} = 50$ Hz

77. $\sin\left(x + \frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2} + \frac{\pi}{2}\right)$

$= \sin\pi = 0$

79. $\cos\left(y - \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6} - \frac{\pi}{6}\right)$

$= \cos 0 = 1$

81. $\sin(x + y) = \sin\left(\frac{\pi}{2} + \frac{\pi}{6}\right)$

$= \sin\left(\frac{2\pi}{3}\right)$

$= \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

83. $\sin x + \sin y = \sin\frac{\pi}{2} + \sin\frac{\pi}{6}$

$= 1 + \frac{1}{2}$

$= \frac{3}{2}$

85. $150^\circ = 150\left(\frac{\pi}{180}\right) = \frac{5\pi}{6}$

87. $225^\circ = 225\left(\frac{\pi}{180}\right) = \frac{5\pi}{4}$

4.3 Vertical and Horizontal Translations

EVEN SOLUTIONS

2. The value h in $y = \sin(x - h)$ represents a horizontal translation. If h is positive, the graph will be shifted h units to the right, and if h is negative, the graph will be shifted h units to the left.

4. The graph of $y = \sin(Bx + C)$ will have a horizontal shift of $-\frac{C}{B}$.

6. The vertical translation is 0.5 units up.

8. The vertical translation is 2 units down.

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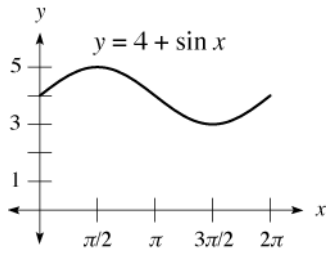
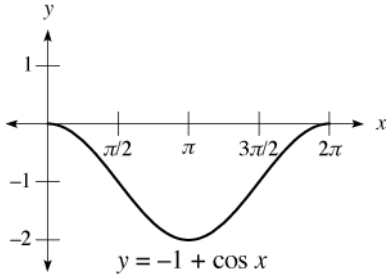
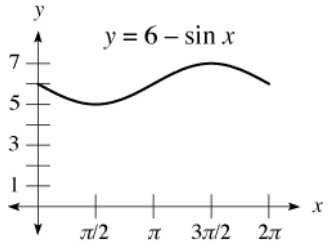
10. The vertical translation is 4 units up:



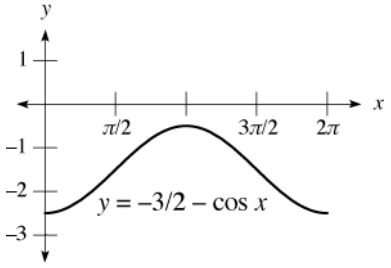
12. The vertical translation is 1 unit down:



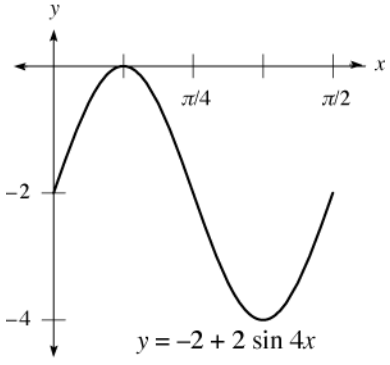
14. The vertical translation is 6 units up:



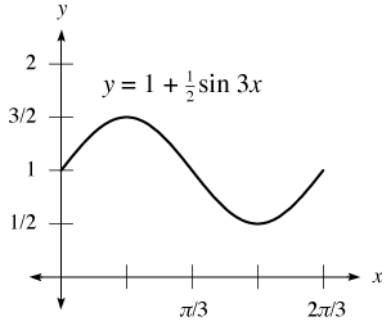
16. The vertical translation is $\frac{3}{2}$ units down:



18. The amplitude is 2, the period is $\frac{2\pi}{4} = \frac{\pi}{2}$, and the vertical translation is 2 units down:



20. The amplitude is $\frac{1}{2}$, the period is $\frac{2\pi}{3}$, and the vertical translation is 1 unit up:



22. The horizontal shift is 2 units to the right.

24. The horizontal shift is $\frac{\pi}{4}$ units to the left.

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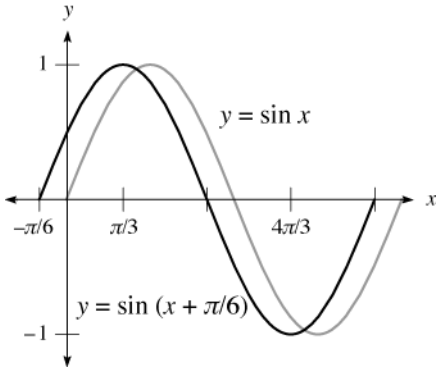
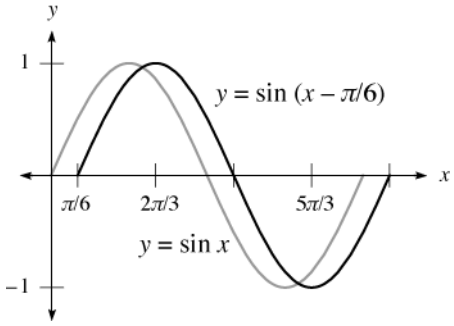
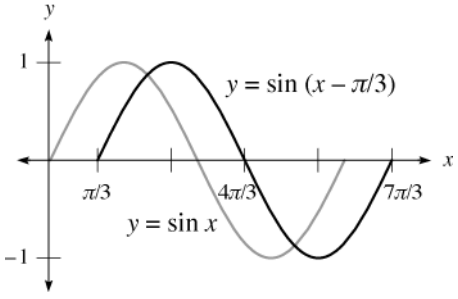
26. The horizontal shift is $\frac{\pi}{6}$ to the left:



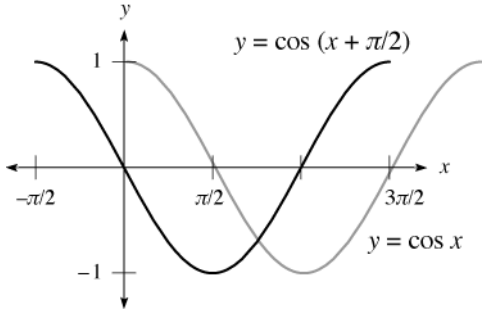
28. The horizontal shift is $\frac{\pi}{6}$ to the right.



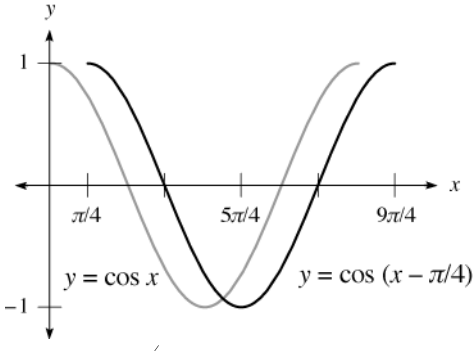
30. The horizontal shift is $\frac{\pi}{3}$ to the right:



32. The horizontal shift is $\frac{\pi}{2}$ to the left:



34. The horizontal shift is $\frac{\pi}{4}$ to the right:



36. The period is $\frac{2\pi}{\pi} = 2$, the horizontal shift is $\frac{\pi/4}{\pi} = \frac{1}{4}$, and the phase is $-\frac{\pi}{4}$.

38. The period is $\frac{2\pi}{2} = \pi$, the horizontal shift is $-\frac{\pi}{2}$, and the phase is π.

40. The period is $\frac{2\pi}{3}$, the horizontal shift is $\frac{\pi/2}{3} = \frac{\pi}{6}$, and the phase is $-\frac{\pi}{2}$.

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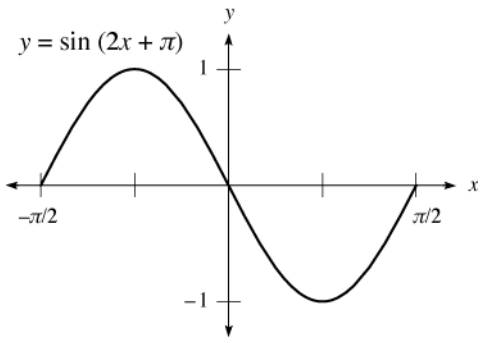
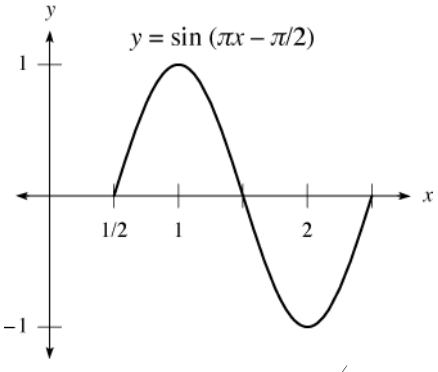
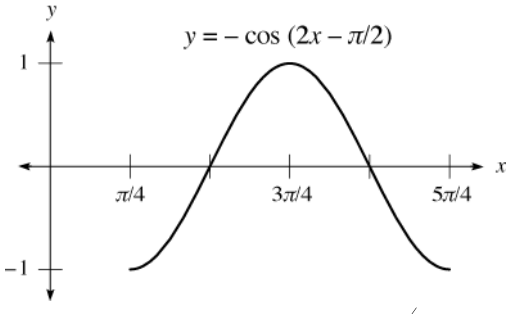
42. The amplitude is 1, the period is $\frac{2\pi}{2} = \pi$, the horizontal shift is $-\frac{\pi}{2}$, and the phase is π:



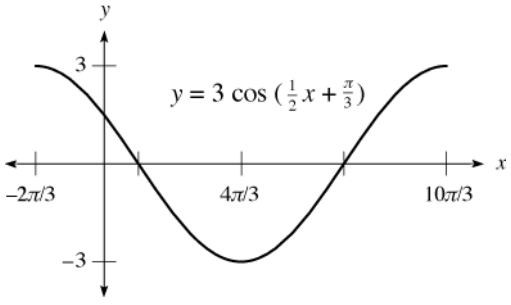
44. The amplitude is 1, the period is $\frac{2\pi}{\pi} = 2$, the horizontal shift is $\frac{\pi/2}{\pi} = \frac{1}{2}$, and the phase is $-\frac{\pi}{2}$:



46. The amplitude is 1, the period is $\frac{2\pi}{2} = \pi$, the horizontal shift is $\frac{\pi/2}{2} = \frac{\pi}{4}$, and the phase is $-\frac{\pi}{2}$. Note this is a reflection of $y = \cos\left(2x - \frac{\pi}{2}\right)$ across the x-axis:



48. The amplitude is 3, the period is $\frac{2\pi}{1/2} = 4\pi$, the horizontal shift is $\frac{-\pi/3}{1/2} = -\frac{2\pi}{3}$, and the phase is $\frac{\pi}{3}$:



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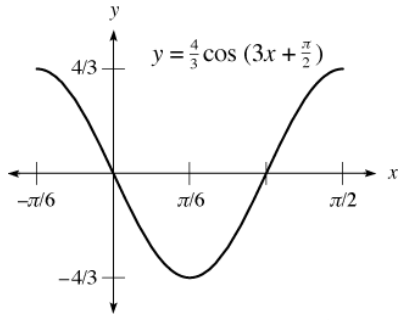
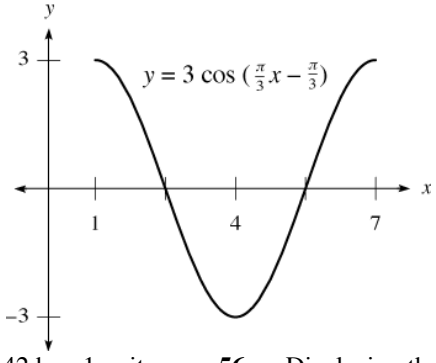
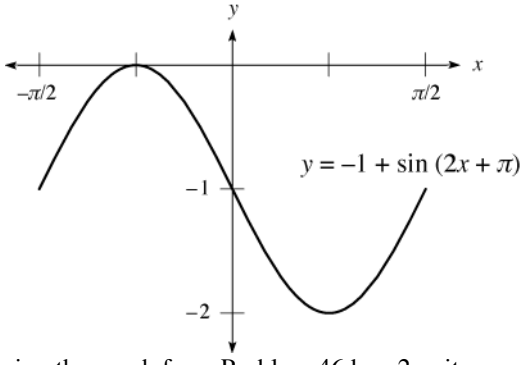
50. The amplitude is $\frac{4}{3}$, the period is $\frac{2\pi}{3}$, the horizontal shift is $\frac{-\pi/2}{3} = -\frac{\pi}{6}$, and the phase is $\frac{\pi}{2}$:



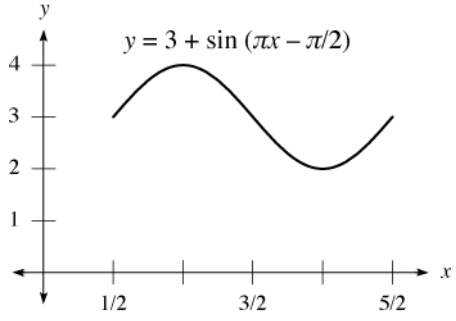
52. The amplitude is 3, the period is $\frac{2\pi}{\pi/3} = 6$, the horizontal shift is $\frac{\pi/3}{\pi/3} = 1$, and the phase is $-\frac{\pi}{3}$:



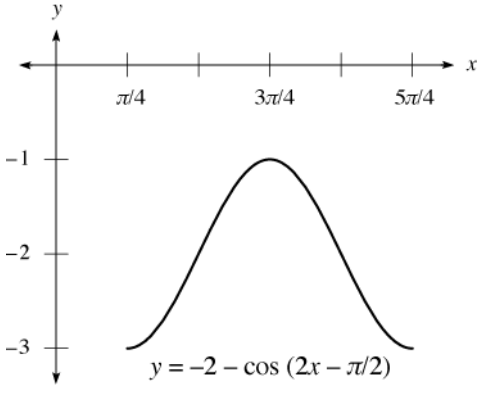
54. Displacing the graph from Problem 42 by –1 units:



56. Displacing the graph from Problem 44 by 3 units:



58. Displacing the graph from Problem 46 by –2 units:



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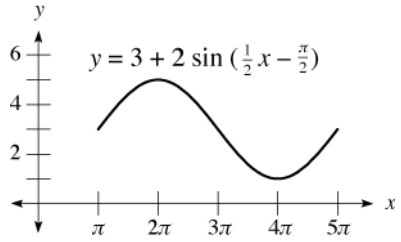
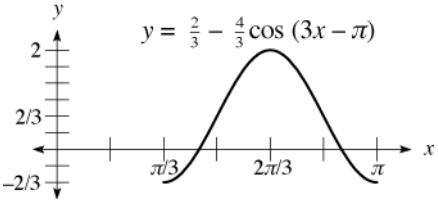
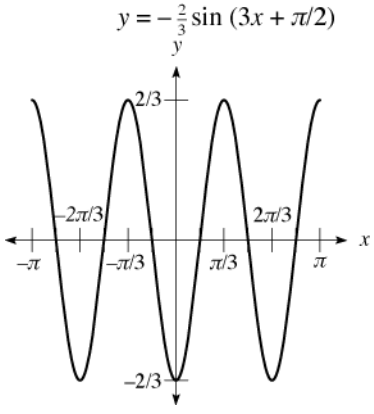
60. The amplitude is 2, the period is $\frac{2\pi}{1/2} = 4\pi$, the vertical translation is 3, the horizontal shift is $\frac{\pi/2}{1/2} = \pi$, and the phase is $-\frac{\pi}{2}$:



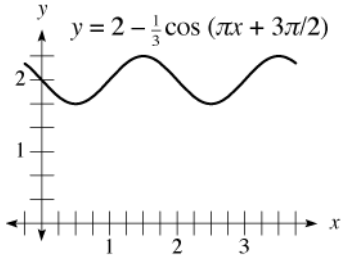
62. The amplitude is $\frac{4}{3}$, the period is $\frac{2\pi}{3}$, the vertical translation is $\frac{2}{3}$, the horizontal shift is $\frac{\pi}{3}$, and the phase is $-\pi$. Note that this curve also has an *x*-axis reflection:



64. The amplitude is $\frac{2}{3}$, the period is $\frac{2\pi}{3}$, there is no vertical translation, and the horizontal shift is $\frac{-\pi/2}{3} = -\frac{\pi}{6}$. Note that this curve also has an *x*-axis reflection:



66. The amplitude is $\frac{1}{3}$, the period is $\frac{2\pi}{\pi} = 2$, the vertical translation is 2, and the horizontal shift is $\frac{-3\pi/2}{\pi} = -\frac{3}{2}$. Note that this curve also has an *x*-axis reflection:



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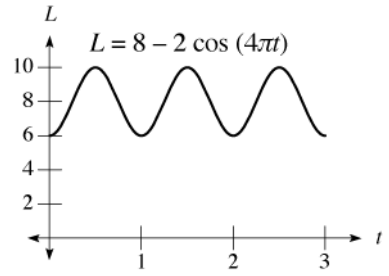
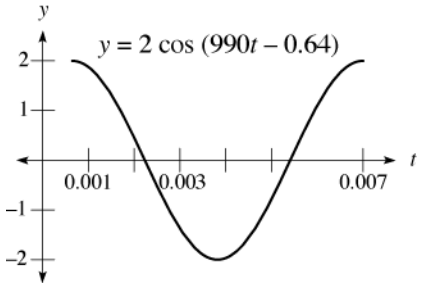
68. **a**. Sketching the graph:



b. At equilibrium, its length is 8 inches.
c. It is shshortest at a length of 6 inches.
d. It is longest at a length of 10 inches.

70. **a**. Sketching the graph:



b. The phase angle is –0.64.

72. First convert 60° to radians: $60° = 60 \bullet \frac{\pi}{180} = \frac{\pi}{3}$ radians

The arc length is given by: $s = r\theta = 24 \bullet \frac{\pi}{3} = 8\pi$ cm

74. First convert 135° to radians: $135° = 135 \bullet \frac{\pi}{180} = \frac{3\pi}{4}$ radians . Since $s = r\theta$:

$$60 = r \bullet \frac{3\pi}{4}$$
$$240 = 3\pi r$$
$$r = \frac{80}{\pi} \text{ m}$$

The radius of the circle is $\frac{80}{\pi}$ meters.

76. The vertical translation is 1. The correct answer is c.

78. The phase is $\frac{\pi}{2}$. The correct answer is c.

ODD SOLUTIONS

1. vertical, upward, downward

3. phase

5. Up 5 units

7. Down $\frac{1}{4}$ unit

For problems 9 through 15, see textbook answer section for graphs.

17. $y = 4 + 4\sin 2x$

Amplitude = 4

$\text{Period} = \frac{2\pi}{2} = \pi$

Vertical translation = 4

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19. $y = -1 + \frac{1}{2}\cos 3x$

$\text{Amplitude} = \frac{1}{2}$

$\text{Period} = \frac{2\pi}{3}$

$\text{Vertical translation} = -1$

For problems 21 through 33, see textbook answer section for graphs and answers.

35. $y = \cos\left(\frac{\pi}{3}x + \frac{\pi}{3}\right)$

$\text{Period} = \frac{2\pi}{\pi/3} = 6$

$\text{Horizontal shift} = -\frac{\pi/3}{\pi/3} = -1$

$\text{Phase} = \frac{\pi}{3}$

37. $y = \sin(6x - \pi)$

$\text{Period} = \frac{2\pi}{6} = \frac{\pi}{3}$

$\text{Horizontal shift} = \frac{\pi}{6}$

$\text{Phase} = -\pi$

39. $y = 3 - \sin\left(\frac{1}{2}x + \frac{\pi}{6}\right)$

$\text{Period} = \frac{2\pi}{1/2} = 4\pi$

$\text{Horizontal shift} = -\frac{\pi/6}{1/2} = -\frac{\pi}{3}$

$\text{Phase} = \frac{\pi}{6}$

41. $\text{Amplitude} = 1$ $\quad \text{Period} = \frac{2\pi}{2} = \pi$ $\quad \text{Horizontal Shift} = \frac{\pi}{2}$

One cycle: $0 \le 2x - \pi \le 2\pi$ $\quad \text{Phase} = -\pi$

$\pi \le 2x \le 3\pi$ $\quad \text{Spacing} = \frac{1}{4}(\pi) = \frac{\pi}{4}$

$\frac{\pi}{2} \le x \le \frac{3\pi}{2}$

The 5 points we use on the x-axis are $\frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}$.

The 2 points we use on the y-axis are -1 and 1.

43. $\text{Amplitude} = 1$ $\quad \text{Period} = \frac{2\pi}{\pi} = 2$ $\quad \text{Horizontal Shift} = \frac{-\pi/2}{\pi} = -\frac{1}{2}$

One cycle: $0 \le \pi x + \frac{\pi}{2} \le 2\pi$ $\quad \text{Phase} = \frac{\pi}{2}$

$-\frac{\pi}{2} \le \pi x \le \frac{3\pi}{2}$ $\quad \text{Spacing} = \frac{1}{4}(2) = \frac{1}{2}$

$-\frac{1}{2} \le x \le \frac{3}{2}$

The 5 points we use on the x-axis are $-\frac{1}{2}, 0, \frac{1}{2}, 1, \frac{3}{2}$.

The 2 points we use on the y-axis are -1 and 1.

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45. Amplitude $= |-1| = 1$ Period $= \dfrac{2\pi}{2} = \pi$

One cycle: $0 \le 2x + \dfrac{\pi}{2} \le 2\pi$ Horizontal Shift $= \dfrac{-\pi/2}{2} = -\dfrac{\pi}{4}$

$-\dfrac{\pi}{2} \le 2x \le \dfrac{3\pi}{2}$ Phase $= \dfrac{\pi}{2}$

$-\dfrac{\pi}{4} \le x \le \dfrac{3\pi}{4}$ Spacing $= \dfrac{1}{4}(\pi) = \dfrac{\pi}{4}$

The 5 points we use on the x-axis are $-\dfrac{\pi}{4}, 0, \dfrac{\pi}{4}, \dfrac{\pi}{2}, \dfrac{3\pi}{4}$.

The 2 points we use on the y-axis are -1 and 1. The graph is reflected about the x-axis.

47. Amplitude $= 2$ Period $= \dfrac{2\pi}{1/2} = 4\pi$

One cycle: $0 \le \dfrac{1}{2}x + \dfrac{\pi}{2} \le 2\pi$ Horizontal Shift $= \dfrac{-\pi/2}{1/2} = -\pi$

$-\dfrac{\pi}{2} \le \dfrac{1}{2}x \le \dfrac{3\pi}{2}$ Phase $= \dfrac{\pi}{2}$

$-\pi \le x \le 3\pi$ Spacing $= \dfrac{1}{4}(4\pi) = \pi$

The 5 points we use on the x-axis are $-\pi, 0, \pi, 2\pi, 3\pi$.

The 2 points we use on the y-axis are -2 and 2.

49. Amplitude $= \dfrac{1}{2}$ Period $= \dfrac{2\pi}{3}$ Horizontal Shift $= \dfrac{\pi/2}{3} = \dfrac{\pi}{6}$

One cycle: $0 \le 3x - \dfrac{\pi}{2} \le 2\pi$ Phase $= -\dfrac{\pi}{2}$

$\dfrac{\pi}{2} \le 3x \le \dfrac{5\pi}{2}$ Spacing $= \dfrac{1}{4}\left(\dfrac{2\pi}{3}\right) = \dfrac{\pi}{6}$

$\dfrac{\pi}{6} \le x \le \dfrac{5\pi}{6}$

The 5 points we use on the x-axis are $\dfrac{\pi}{6}, \dfrac{\pi}{3}, \dfrac{\pi}{2}, \dfrac{2\pi}{3}, \dfrac{5\pi}{6}$.

The 2 points we use on the y-axis are $-\dfrac{1}{2}$ and $\dfrac{1}{2}$.

51. Amplitude $= 3$ Period $= \dfrac{2\pi}{\pi/3} = 6$ Horizontal Shift $= \dfrac{\pi/3}{\pi/3} = 1$

One cycle: $0 \le \dfrac{\pi}{3}x - \dfrac{\pi}{3} \le 2\pi$ Phase $= -\dfrac{\pi}{3}$

$\dfrac{\pi}{3} \le \dfrac{\pi}{3}x \le \dfrac{7\pi}{3}$ Spacing $= \dfrac{1}{4}(6) = \dfrac{3}{2}$

$1 \le x \le 7$

The 5 points we use on the x-axis are $1, \dfrac{5}{2}, 4, \dfrac{11}{2}, 7$.

The 2 points we use on the y-axis are -3 and 3.

53. The graph in problem 41 is shifted up 1 unit.

55. The graph in problem 43 is shifted down 3 unit.

57. The graph in problem 45 is shifted up 2 units.

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59. Amplitude $= 3$ Period $= \dfrac{2\pi}{1/2} = 4\pi$ Horizontal Shift $= \dfrac{\pi/3}{1/2} = \dfrac{2\pi}{3}$

One cycle: $0 \le \dfrac{1}{2}x - \dfrac{\pi}{3} \le 2\pi$

$\dfrac{\pi}{3} \le \dfrac{1}{2}x \le \dfrac{7\pi}{3}$

$\dfrac{2\pi}{3} \le x \le \dfrac{14\pi}{3}$

Phase $= -\dfrac{\pi}{3}$

Spacing $= \dfrac{1}{4}(4\pi) = \pi$

Vertical Translation $= -2$

The 5 points we use on the x-axis are $\dfrac{2\pi}{3}, \dfrac{5\pi}{3}, \dfrac{8\pi}{3}, \dfrac{11\pi}{3}, \dfrac{14\pi}{3}$.

The 2 points we use on the y-axis are $-2-3=-5$ and $-2+3=1$.

61. Amplitude $= \dfrac{1}{2}$ Period $= \dfrac{2\pi}{3}$ Horizontal Shift $= \dfrac{-\pi}{3} = -\dfrac{\pi}{3}$

One cycle: $0 \le 3x + \pi \le 2\pi$

$-\pi \le 3x \le \pi$

$-\dfrac{\pi}{3} \le x \le \dfrac{\pi}{3}$

Spacing $= \dfrac{1}{4}\left(\dfrac{2\pi}{3}\right) = \dfrac{\pi}{6}$

Phase $= \pi$

Vertical Translation $= \dfrac{3}{2}$

The 5 points we use on the x-axis are $-\dfrac{\pi}{3}, -\dfrac{\pi}{6}, 0, \dfrac{\pi}{6}, \dfrac{\pi}{3}$.

The 2 points we use on the y-axis are $\dfrac{3}{2}-\dfrac{1}{2}=1$ and $\dfrac{3}{2}+\dfrac{1}{2}=2$.

This graph is also reflected about the x-axis.

63. Amplitude $= 4$ Period $= \dfrac{2\pi}{2} = \pi$ Horizontal Shift $= \dfrac{\frac{\pi}{2}}{2} = \dfrac{\pi}{4}$

One cycle: $0 \le 2x - \dfrac{\pi}{2} \le 2\pi$

$\dfrac{\pi}{2} \le 2x \le \dfrac{5\pi}{2}$

$\dfrac{\pi}{4} \le x \le \dfrac{5\pi}{4}$

Spacing $= \dfrac{1}{4}(\pi) = \dfrac{\pi}{4}$

No Vertical Translation

For one complete cycle, the points we use on the x-axis are $\dfrac{\pi}{4}, \dfrac{\pi}{2}, \dfrac{3\pi}{4}, \pi, \dfrac{5\pi}{4}$.

The points we use on the y-axis are 4 and -4.

We must extend our graph from $-\dfrac{\pi}{4}$ *to* $\dfrac{3\pi}{2}$.

65. Amplitude $= 3$ Period $= \dfrac{2\pi}{\pi} = 2$ Horizontal Shift $= \dfrac{\pi/4}{\pi} = \dfrac{1}{4}$

One cycle: $0 \le \pi x - \dfrac{\pi}{4} \le 2\pi$

$\dfrac{\pi}{4} \le \pi x \le \dfrac{9\pi}{4}$

$\dfrac{1}{4} \le x \le \dfrac{9}{4}$

Spacing $= \dfrac{1}{4}(2) = \dfrac{1}{2}$

Vertical Translation $= \dfrac{5}{2}$

The 5 points we use on the x-axis are $\dfrac{1}{4}, \dfrac{3}{4}, \dfrac{5}{4}, \dfrac{7}{4}, \dfrac{9}{4}$.

The 2 points we use on the y-axis are $\dfrac{5}{2}-3=-\dfrac{1}{2}$ and $\dfrac{5}{2}+3=\dfrac{11}{2}$.

The graph is reflected about the x-axis.

We must extend our graph from -2 to 2.

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67. Amplitude = $|-3.5| = 3.5$ Vertical Translation = 15

(b) At equilibrium, the length of the spring is 15 cm.

(c) The spring is the shortest at $15 - 3.5 = 11.5$ cm.

(d) The spring is the longest at $15 + 3.5 = 18.5$ cm.

71. $s = r\theta$

$= 10\left(\frac{\pi}{6}\right)$

$= \frac{5\pi}{3}\ cm$

73. $r = \frac{s}{\theta}$

$= \frac{4}{6}$

$= \frac{2}{3}\ ft\ or\ 8\ in$

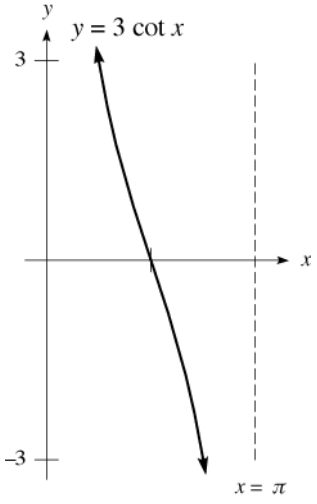
4.4 The Other Trigonometric Functions

EVEN SOLUTIONS

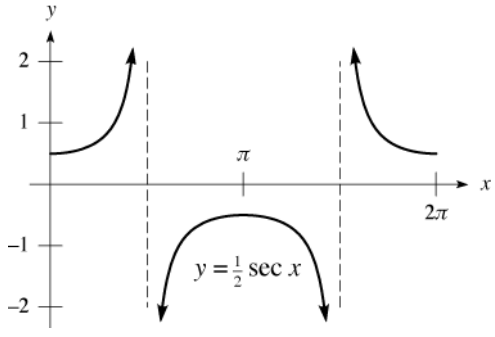
2. The number 3 in $y = 3\sec x$ will expand the graph of $y = \sec x$ vertically by a factor of 3. The number $\frac{1}{3}$ in $y = \frac{1}{3}\sec x$ will contract the graph of $y = \sec x$ vertically by a factor of $\frac{1}{3}$.

4. To graph $y = k + A\csc(Bx + C)$, first sketch the graph of the corresponding sine function to use as a guide.

6. Graphing one cycle:



8. Graphing one cycle:



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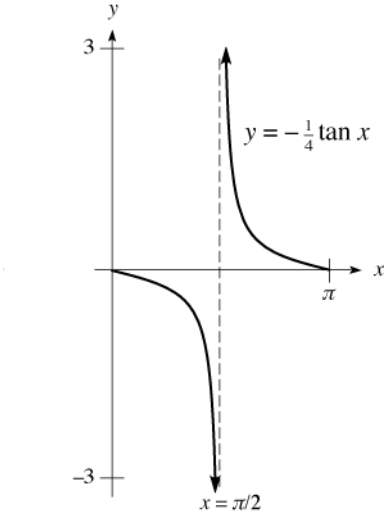
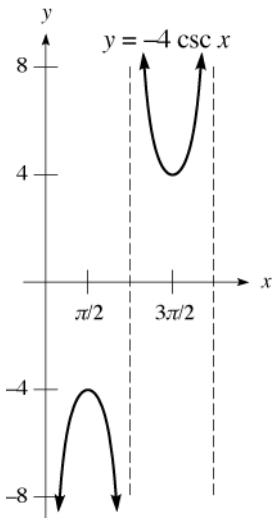
10. Graphing one cycle:



12. Graphing one cycle:



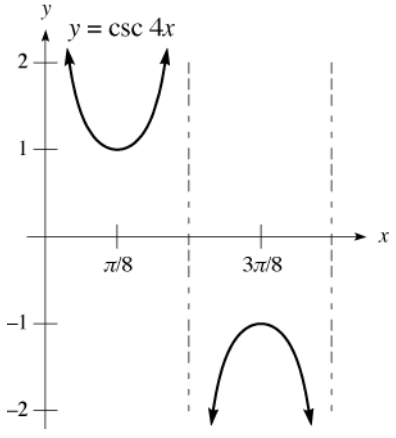
14. The period is $\frac{2\pi}{2} = \pi$.

16. The period is $\frac{2\pi}{1/6} = 12\pi$.

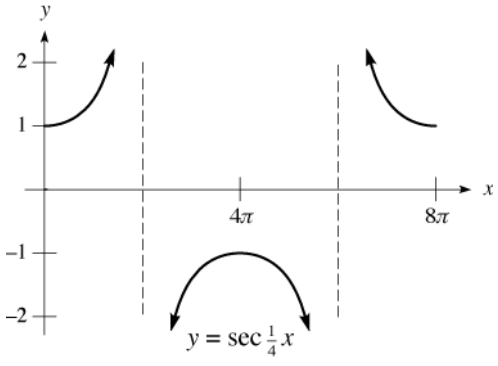
18. The period is $\frac{\pi}{3}$.

20. The period is $\frac{\pi}{\pi/4} = 4$.

22. The period is $\frac{2\pi}{4} = \frac{\pi}{2}$. Sketching the graph:



24. The period is $\frac{2\pi}{1/4} = 8\pi$. Sketching the graph:



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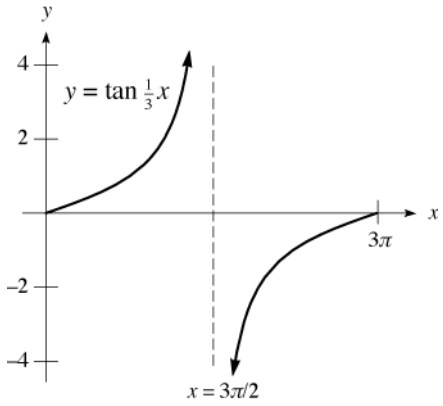
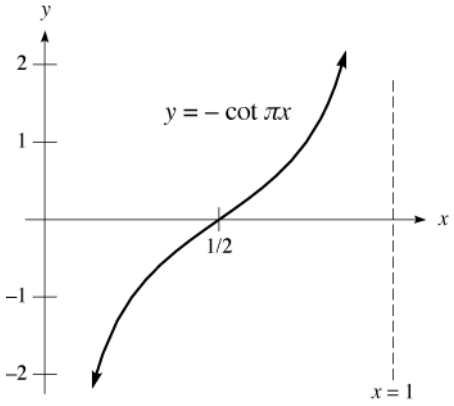
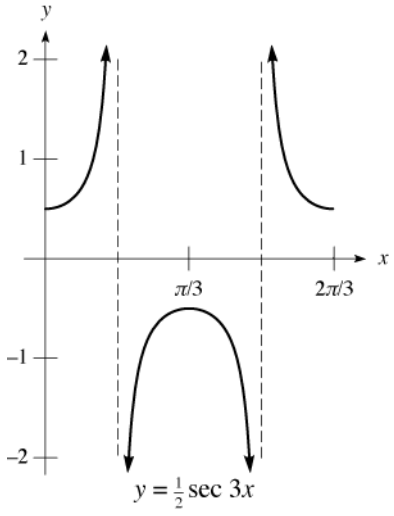
26. The period is $\dfrac{\pi}{1/3} = 3\pi$. Sketching the graph:



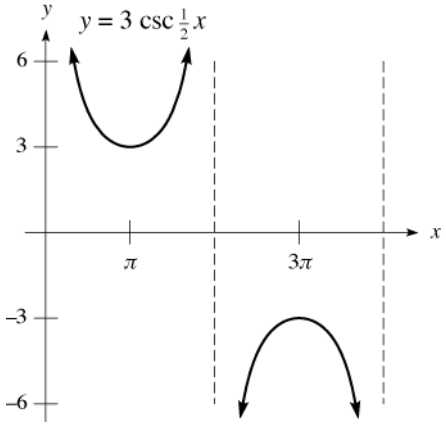
28. The period is $\dfrac{\pi}{\pi} = 1$. Sketching the graph:



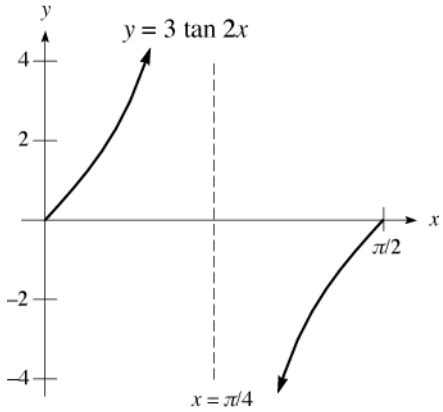
30. The period is $\dfrac{2\pi}{3}$:



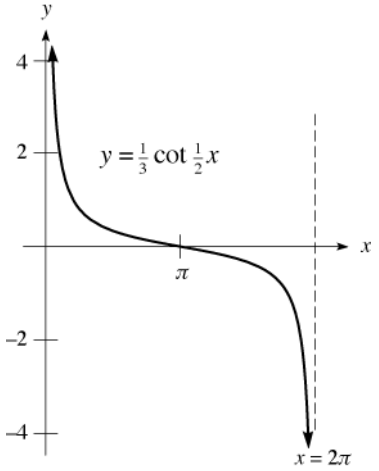
32. The period is $\dfrac{2\pi}{1/2} = 4\pi$:



34. The period is $\dfrac{\pi}{2}$:



36. The period is $\dfrac{\pi}{1/2} = 2\pi$:



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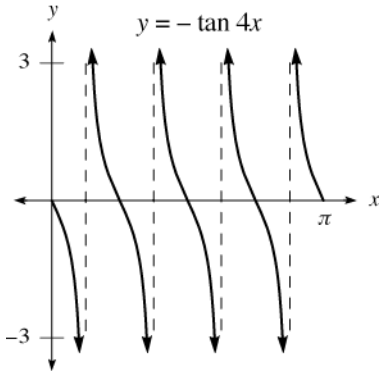
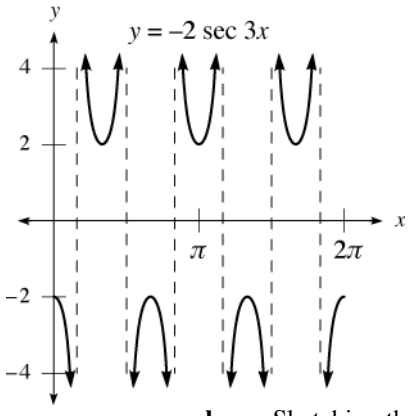
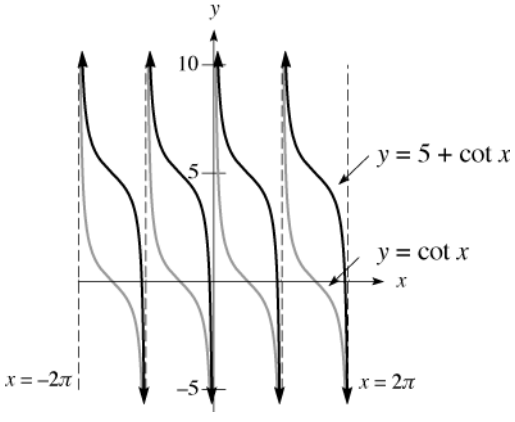
38. The period is $\frac{\pi}{4}$. Note this is a reflection of $y = \tan 4x$ across the x-axis. Sketching the graph:



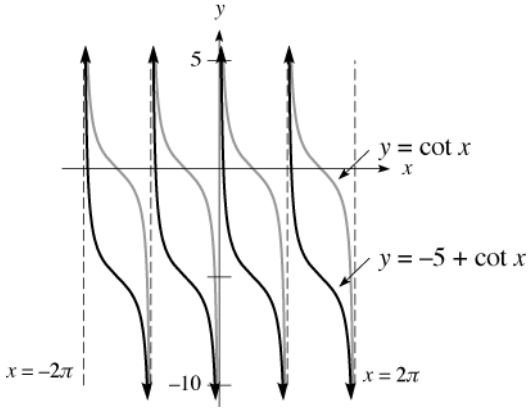
40. The period is $\frac{2\pi}{3}$. Sketching the graph for $0 \le x \le 2\pi$:



42. **a**. Sketching the graphs:



b. Sketching the graphs:



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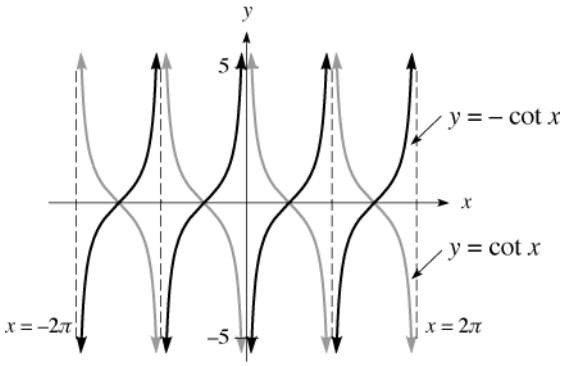
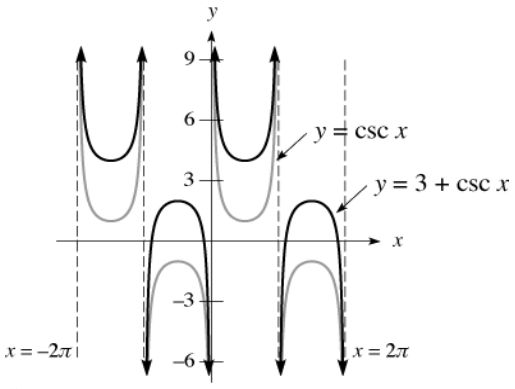
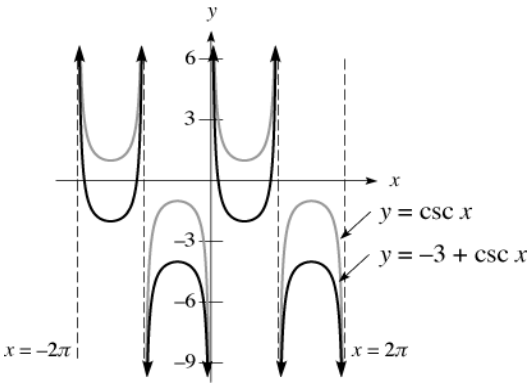
c. Sketching the graphs:



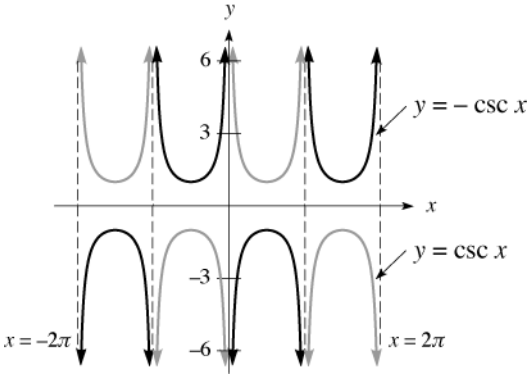
44. **a**. Sketching the graphs:



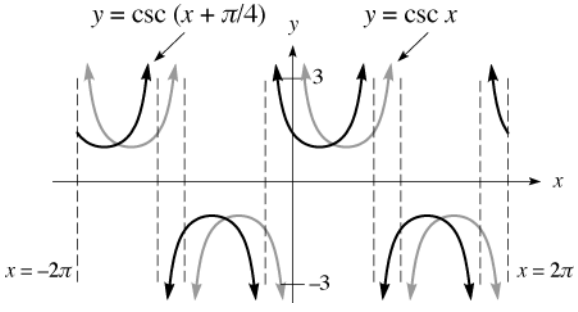
b. Sketching the graphs:



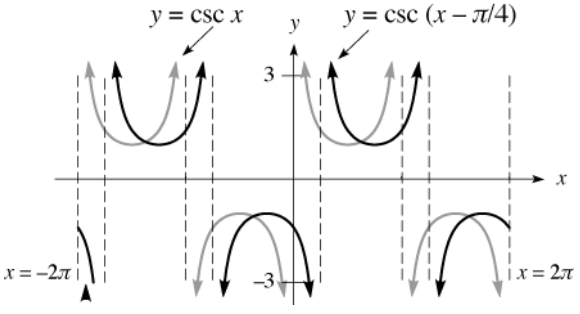
c. Sketching the graphs:



46. **a**. The graph is shifted $\frac{\pi}{4}$ to the left:



b. The graph is shifted $\frac{\pi}{4}$ to the right:



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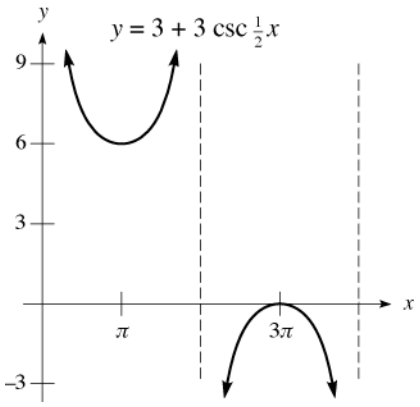
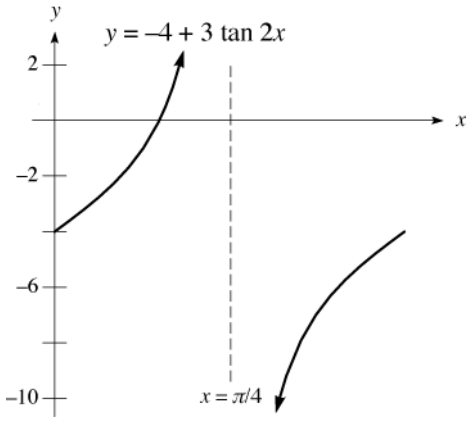
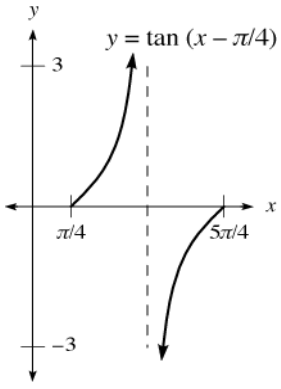
48. Displacing the graph from Problem 24 by 3 units:



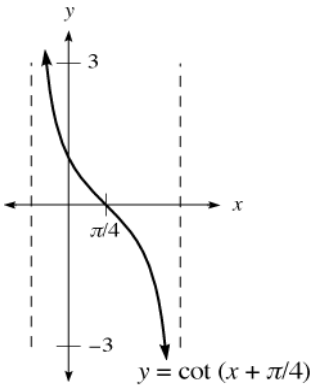
50. Displacing the graph from Problem 26 by –4 units:



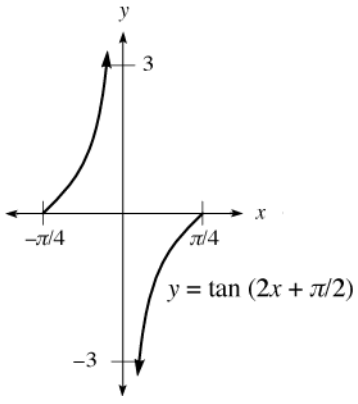
52. The period is π and the horizontal shift is $\frac{\pi}{4}$:



54. The period is π and the horizontal shift is $-\frac{\pi}{4}$:



56. The period is $\frac{\pi}{2}$ and the horizontal shift is $\frac{-\pi/2}{2} = -\frac{\pi}{4}$:



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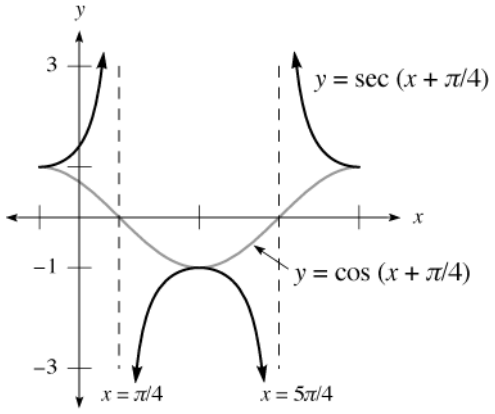
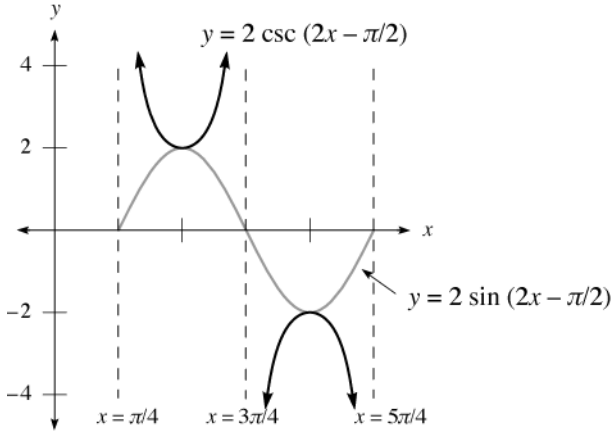
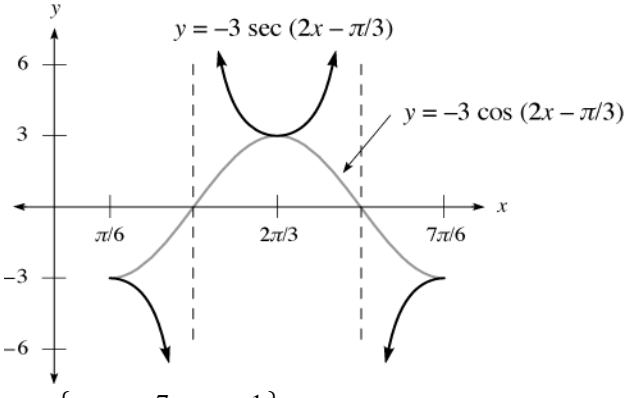
58. Sketching the graph by first graphing $y = \cos\left(x + \frac{\pi}{4}\right)$:



60. Sketching the graph by first graphing $y = 2\sin\left(2x - \frac{\pi}{2}\right)$:



62. Sketching the graph by first graphing $y = -3\cos\left(2x - \frac{\pi}{3}\right)$:



64. The period is $\frac{2\pi}{3}$, the range is $\left\{y \mid y \le -\frac{7}{4} \text{ or } y \ge \frac{1}{4}\right\}$, the vertical translation is down $\frac{3}{4}$, and the horizontal shift is $\frac{-\pi/2}{3} = -\frac{\pi}{6}$.

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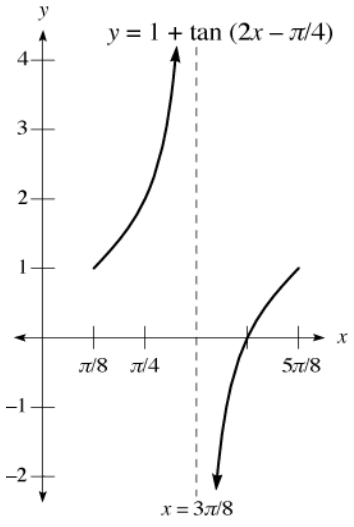
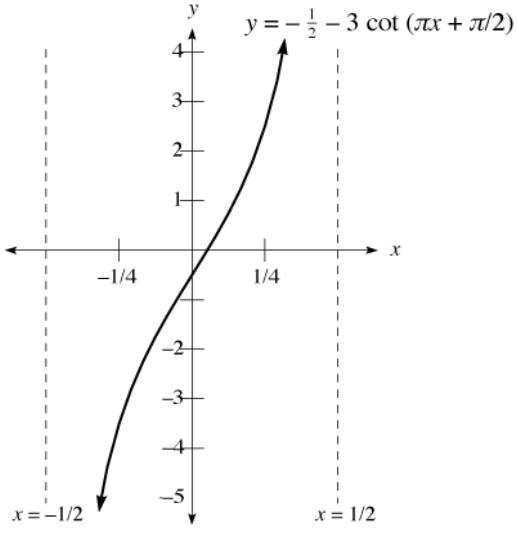
66. The period is $\frac{2\pi}{2\pi} = 1$, the range is $\left\{ y \mid y \le -\frac{1}{2} \text{ or } y \ge \frac{11}{2} \right\}$, the vertical translation is up $\frac{5}{2}$, and the horizontal shift is $\frac{\pi/4}{2\pi} = \frac{1}{8}$.

68. The period is $\frac{\pi}{\pi/3} = 3$, the range is $(-\infty, \infty)$, the vertical translation is up 4, and the horizontal shift is $\frac{-\pi/3}{\pi/3} = -1$.

70. The period is $\frac{\pi}{2}$, the vertical translation is 1, and the horizontal shift is $\frac{\pi/4}{2} = \frac{\pi}{8}$:



72. The period is $\frac{\pi}{\pi} = 1$, the vertical translation is $-\frac{1}{2}$, and the horizontal shift is $\frac{-\pi/2}{\pi} = -\frac{1}{2}$. Note that this curve is reflected across the x-axis:



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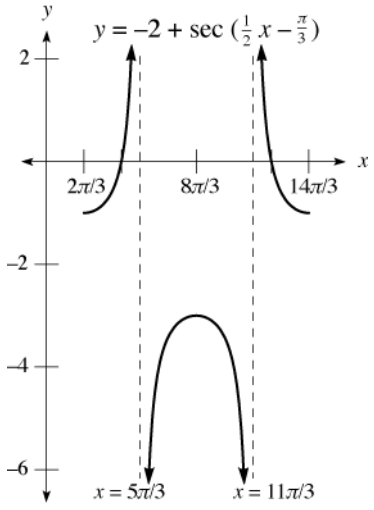
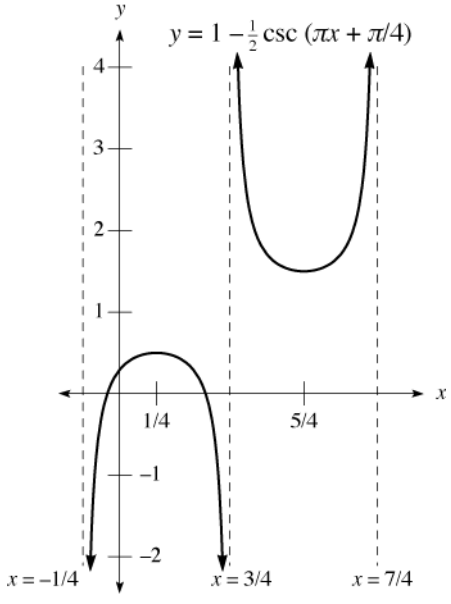
74. The period is $\dfrac{2\pi}{1/2} = 4\pi$, the vertical translation is –2, and the horizontal shift is $\dfrac{\pi/3}{1/2} = \dfrac{2\pi}{3}$:



76. The period is $\dfrac{2\pi}{\pi} = 2$, the vertical translation is 1, and the horizontal shift is $\dfrac{-\pi/4}{\pi} = -\dfrac{1}{4}$. Note that this curve is reflected across the *x*-axis:



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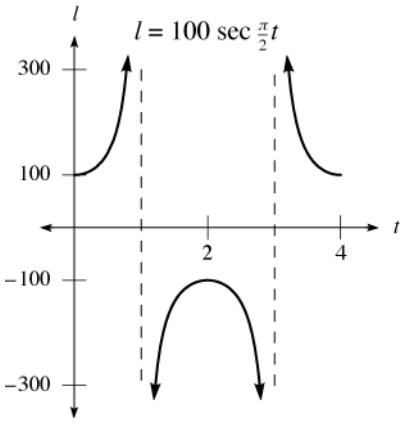
78. Sketching the graph:



80. The argument is $90° - \theta$.

82. The argument is $\frac{x}{2} + \frac{\pi}{4}$.

84. The value is 0.9848.

86. The value is 0.1335.

88. The value is 3.5629.

90. This is a cosecant graph shifted vertically by 1 unit and translated horizontally by $\frac{\pi}{4}$. The correct answer is b.

92. The period is $\frac{2\pi}{\pi/2} = 4$. The correct answer is a.

ODD SOLUTIONS

1. expanded, contracted, multiply

3. $\frac{\pi}{B}$

For problems 5 through 11, see textbook answer section for graphs.

13. Period $= \frac{2\pi}{2} = \pi$

15. Period $= \frac{2\pi}{\pi/3} = 6$

17. Period $= \frac{\pi}{3\pi} = \frac{1}{3}$

19. Period $= \frac{\pi}{1/2} = 2\pi$

21. $y = \csc 3x$

Period $= \frac{2\pi}{3}$

23. $y = \sec \frac{\pi}{4} x$

Period $= \frac{2\pi}{\pi/4} = 8$

25. $y = -\tan \frac{1}{2} x$

Period $= \frac{\pi}{1/2} = 2\pi$

Graph is reflected about the *x*-axis

27. $y = \cot 4x$

Period $= \frac{\pi}{4}$

29. $y = \frac{1}{2}\csc 3x$ Period $= \frac{2\pi}{3}$

31. $y = 3\sec \frac{1}{2} x$ Period $= \frac{2\pi}{1/2} = 4\pi$

33. $y = 2\tan 3x$ Period $= \frac{\pi}{3}$

Graph is stretched by a factor of 2

35. $y = \frac{1}{2}\cot \frac{\pi}{2} x$ Period $= \frac{\pi}{\pi/2} = 2$

Graph is contracted by a factor of $\frac{1}{2}$

37. $y = -\cot 2x$ Period $= \frac{\pi}{2}$

Graph is reflected about the *x*-axis

39. $y = -2\csc 3x$ Period $= \frac{2\pi}{3}$

Graph is reflected about the *x*-axis and is stretched by a factor of 2.

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For problems 41 through 45, see textbook answer section for graphs.

47. $y = -2 + 3\sec\frac{1}{2}x$ Period $= \frac{2\pi}{1/2} = 4\pi$

Vertical translation $= -2$

The graph in problem 31 is shifted down 2 units.

49. $y = 3 + \frac{1}{2}\cot\frac{\pi}{2}x$ Period $= \frac{\pi}{\pi/2} = 2$

Vertical translation $= 3$

The graph in problem 35 is shifted up 3 units.

51. Period $= \pi$ Horizontal shift $= -\frac{\pi}{4}$

For one complete cycle, the vertical asymptote will occur at $x = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$.

The x-intercepts will be at $x = -\frac{\pi}{4}$ and $x = \frac{3\pi}{4}$.

53. Period $= \pi$ Horizontal shift $= \frac{\pi}{4}$

For one complete cycle, the vertical asymptotes will occur at $x = 0 + \frac{\pi}{4} = \frac{\pi}{4}$ and at $x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$.

The x-intercept will be at $x = \frac{3\pi}{4}$.

55. Period $= \frac{\pi}{2}$ Horizontal shift $= \frac{\pi/2}{2} = \frac{\pi}{4}$

One cycle: $0 < 2x - \frac{\pi}{2} < \pi$

$$\frac{\pi}{2} < 2x < \frac{3\pi}{4}$$

$$\frac{\pi}{4} < x < \frac{3\pi}{4}$$

The x-intercepts will be at $x = \frac{\pi}{4}$ and $x = \frac{3\pi}{4}$. The vertical asymptote will occur in the middle of this interval at $x = \frac{\pi}{2}$.

57. First, we will sketch the reciprocal function $y = \sin\left(x + \frac{\pi}{4}\right)$

Amplitude $= 1$ Period $= 2\pi$ Horizontal shift $= -\frac{\pi}{4}$

The 5 points on the x-axis are $-\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$. The 2 points on the y-axis are 1 and -1.

We sketch this sine curve using a dotted line. The asymptotes will occur where $y = 0$, that is at $-\frac{\pi}{4}, \frac{3\pi}{4}, and\ \frac{7\pi}{4}$.

Using the asymptotes and the sine curve, we sketch the graph of $y = \csc\left(x + \frac{\pi}{4}\right)$.

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59. First, we will sketch the reciprocal function $y = 2\cos\left(2x - \frac{\pi}{2}\right)$ using a dotted line.

Amplitude $= 2$ Period $= \frac{2\pi}{2} = \pi$ Horizontal shift $= \frac{\pi/2}{2} = \frac{\pi}{4}$

The 5 points on the *x*-axis are $\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}$. The 2 points on the *y*-axis are -2 *and* 2.

The asymptotes will occur at $\frac{\pi}{2}$ *and* π. Using the asymptotes and the cosine curve, we sketch the graph of $y = 2\sec\left(2x - \frac{\pi}{2}\right)$.

61. First, we will sketch the reciprocal function $y = -3\sin\left(2x + \frac{\pi}{3}\right)$

Amplitude $= 3$ Period $= \frac{2\pi}{2} = \pi$ Horizontal shift $= \frac{-\pi/3}{2} = -\frac{\pi}{6}$

The 5 points on the *x*-axis are $-\frac{\pi}{6}, \frac{\pi}{12}, \frac{\pi}{3}, \frac{7\pi}{12}, \frac{5\pi}{6}$. The 2 points we use on the *y*-axis are 3 and -3. We sketch this sine curve which has been reflected about the x-axis using a dotted line. The asymptotes will occur where $y = 0$, that is at $-\frac{\pi}{6}, \frac{\pi}{3}$, *and* $\frac{5\pi}{6}$. Using the asymptotes and the sine curve, we sketch the graph of $y = -3\csc\left(2x + \frac{\pi}{3}\right)$.

63. Period $= \frac{2\pi}{2} = \pi$ Horizontal Shift $= \frac{\pi/3}{2} = \frac{\pi}{6}$ (to the right) Vertical Shift is up 3.

Normal Range for cosecant function is $y \le -1$ or $y \ge 1$. Adding 3, we get $y \le -1 + 3$ or $y \ge 1 + 3$.

Range is $y \le 2$ or $y \ge 4$.

65. Period $= \frac{2\pi}{\pi} = 2$ Horizontal Shift $= \frac{-3\pi/2}{\pi} = -\frac{3}{2}$ (to the left) Vertical Shift is down 2.

Normal Range for cosecant function is $y \le -1$ or $y \ge 1$. The factor of $\frac{1}{3}$ has to be multiplied, giving us $y \le -\frac{1}{3}$ or $y \ge \frac{1}{3}$. Subtracting 2, we get $y \le -\frac{1}{3} - 2$ or $y \ge \frac{1}{3} - 2$.

Range is $y \le -\frac{7}{3}$ or $y \ge -\frac{5}{3}$.

67. Period $= \frac{\pi}{1/2} = 2\pi$ Horizontal Shift $= \frac{\pi/6}{1/2} = \frac{\pi}{3}$ (to the right) Vertical Shift is down 1.

Normal Range for tangent function is $(-\infty, \infty)$. This will not change.

69. Period $= \frac{\pi}{1/2} = 2\pi$ Horizontal Shift $= \frac{-\pi/4}{1/2} = -\frac{\pi}{2}$ Vertical Translation $= -1$

One cycle: $0 < \frac{1}{2}x + \frac{\pi}{4} < \pi$

$$-\frac{\pi}{4} < \frac{1}{2}x < \frac{3\pi}{4}$$

$$-\frac{\pi}{2} < x < \frac{3\pi}{2}$$

Before the vertical translation, the *x*-intercepts will be at $x = -\frac{\pi}{2}$ and $x = \frac{3\pi}{2}$. The vertical asymptote will occur in the middle of this interval at $x = \frac{\pi}{2}$. Sketch this graph with a dotted line, reflect it about the *x*-axis, and then shift it down one unit.

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71. Period $= \dfrac{\pi}{\pi/2} = 2$ Horizontal shift $= \dfrac{3\pi/2}{\pi/2} = 3$ Vertical Translation $= \dfrac{3}{2}$

One cycle: $0 < \dfrac{\pi}{2}x - \dfrac{3\pi}{2} < \pi$

$$\frac{3\pi}{2} < \frac{\pi}{2}x < \frac{5\pi}{2}$$

$$3 < x < 5$$

The vertical asymptotes will be at $x = 3$ and $x = 5$. The x-intercept (before the vertical translation) will occur in the middle of this interval at $x = 4$. Graph is contracted by a factor of $\dfrac{1}{2}$, reflected about the x-axis, and shifted up $\dfrac{3}{2}$ units.

73. First, we will sketch the reciprocal function $y = \dfrac{1}{3} + \sin\left(3x - \dfrac{\pi}{2}\right)$.

Period $= \dfrac{2\pi}{3}$ Horizontal shift $= \dfrac{\pi/2}{3} = \dfrac{\pi}{6}$ Vertical Translation $= \dfrac{1}{3}$

The 5 points on the x-axis are $\dfrac{\pi}{6}, \dfrac{\pi}{3}, \dfrac{\pi}{2}, \dfrac{2\pi}{3}, \dfrac{5\pi}{6}$. The 2 points on the y-axis are $\dfrac{1}{3} + 1 = \dfrac{4}{3}$ *and* $\dfrac{1}{3} - 1 = -\dfrac{2}{3}$. We sketch this sine curve using a dotted line. The vertical asymptotes will occur at $x = \dfrac{\pi}{6}, x = \dfrac{\pi}{2}$, and $x = \dfrac{5\pi}{6}$. Using the asymptotes and the sine curve, we sketch the graph of $y = \dfrac{1}{3} + \csc\left(3x - \dfrac{\pi}{2}\right)$.

75. First, we will sketch the reciprocal function $y = -3 - 2\cos\left(\pi x + \dfrac{\pi}{3}\right)$

Amplitude $= 2$ Period $= \dfrac{2\pi}{\pi} = 2$ Horizontal shift $= \dfrac{-\pi/3}{\pi} = -\dfrac{1}{3}$ Vertical Translation $= -3$

The 5 points on the x-axis are $-\dfrac{1}{3}, \dfrac{1}{6}, \dfrac{2}{3}, \dfrac{7}{6}, \dfrac{5}{3}$. The 2 points we use on the y-axis are $-3 + 1 = -2$ and $-3 - 1 = -4$. This graph is also reflected about the x-axis. We sketch this cosine curve using a dotted line. The vertical asymptotes will occur at $x = \dfrac{1}{6}$ and $x = \dfrac{7}{6}$. Using the asymptotes and the cosine curve, we sketch the graph $y = -3 - 2\sec\left(\pi x + \dfrac{\pi}{3}\right)$.

77. $d = 100\tan\dfrac{\pi}{2}t$

t	0	1/2	1	3/2	2	5/2	3	7/2	4
d	0	100	N.D.	−100	0	100	N.D.	−100	0

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4.5 Finding an Equation from Its Graph

EVEN SOLUTIONS

2. Given a sine or cosine graph, the vertical distance between a maximum point and a minimum point is equal to twice the amplitude.

4. To find the horizontal shift for a cosine graph, find a maximum point and use the x-coordinate of that point. To find the horizontal shift for a reflected cosine graph, find a minimum point and use the x-coordinate of that point.

6. The slope is $\frac{1}{2}$ and the y-intercept is 1, so the equation of this line is $y = \frac{1}{2}x + 1$.

8. The slope is –2 and the y-intercept is 3, so the equation of this line is $y = -2x + 3$.

10. This is a cosine curve with amplitude = 1 and period = 2π. There is no horizontal translation, so the equation is $y = \cos x$.

12. This is a sine curve with amplitude = 2 and period = 2π. There is no horizontal translation, so the equation is $y = 2\sin x$.

14. This is a reflection of the curve from Problem 12 across the x-axis, so the equation is $y = -2\sin x$.

16. This is a cosine curve with amplitude = 1 and period = π, therefore:

$$\frac{2\pi}{B} = \pi$$
$$2\pi = \pi B$$
$$B = 2$$

There is no horizontal translation, so the equation is $y = \cos 2x$.

18. This is a cosine curve with amplitude = 1 and period = 4π, therefore:

$$\frac{2\pi}{B} = 4\pi$$
$$2\pi = 4\pi B$$
$$B = \frac{1}{2}$$

There is no horizontal translation, so the equation is $y = \cos\frac{1}{2}x$.

20. This is a sine curve with amplitude = 3 and period = π, therefore:

$$\frac{2\pi}{B} = \pi$$
$$2\pi = \pi B$$
$$B = 2$$

There is no horizontal translation, so the equation is $y = 3\sin 2x$.

22. This is a cosine curve with amplitude = 2 and period = 2, therefore:

$$\frac{2\pi}{B} = 2$$
$$2\pi = 2B$$
$$B = \pi$$

There is no horizontal translation, so the equation is $y = 2\cos \pi x$.

24. This is a reflection of the curve from Problem 22 across the x-axis, so the equation is $y = -2\cos \pi x$.

26. This is a vertical translation of the curve from Problem 24 of 3 units, so its equation is $y = -2\cos \pi x + 3$, or $y = 3 - 2\cos \pi x$.

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28. This is a sine curve with amplitude = 2 and period = π, therefore:

$$\frac{2\pi}{B} = \pi$$
$$B = 2$$

The horizontal translation is $\frac{\pi}{8}$, therefore:

$$-\frac{C}{B} = \frac{\pi}{8}$$
$$-\frac{C}{2} = \frac{\pi}{8}$$
$$C = -\frac{\pi}{4}$$

The equation is $y = 2\sin\left(2x - \frac{\pi}{4}\right)$.

30. This is a sine curve with amplitude = 3 and period = $\frac{2\pi}{3}$, therefore:

$$\frac{2\pi}{B} = \frac{2\pi}{3}$$
$$B = 3$$

The horizontal translation is $-\frac{\pi}{4}$, therefore:

$$-\frac{C}{B} = -\frac{\pi}{4}$$
$$-\frac{C}{3} = -\frac{\pi}{4}$$
$$C = \frac{3\pi}{4}$$

The curve is a reflection across the x-axis, so its equation is $y = -3\sin\left(3x + \frac{3\pi}{4}\right)$.

32. This is a cosine curve with amplitude = 2 and period = $\frac{2\pi}{3}$, therefore:

$$\frac{2\pi}{B} = \frac{2\pi}{3}$$
$$B = 3$$

The curve is a reflection across the x-axis with a vertical translation of –2 units, so its equation is $y = -2 - 2\cos(3x)$.

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34. This is a cosine curve with amplitude = 2 and period = $\frac{2\pi}{3}$, therefore:

$$\frac{2\pi}{B} = \frac{2\pi}{3}$$
$$B = 3$$

The horizontal translation is $-\frac{\pi}{6}$, therefore:

$$-\frac{C}{B} = -\frac{\pi}{6}$$
$$-\frac{C}{3} = -\frac{\pi}{6}$$
$$C = \frac{\pi}{2}$$

The curve is a reflection across the *x*-axis with a vertical translation of 3 units, so its equation is $y = 2 - 2\cos\left(3x + \frac{\pi}{2}\right)$.

36. The amplitude is 210. Since the frequency is the reciprocal of the period:

$$\frac{2\pi}{\omega} = \frac{1}{90}$$
$$\omega = 180\pi$$

So the voltage is $V = 210\sin(180\pi t)$.

38. The curve will be a cosine curve reflected across the *x*-axis. The highest point is 165 + 9 = 174, and the lowest point is 9, so the amplitude is $\frac{1}{2}(174 - 9) = 82.5$. There is no horizontal translation, so the curve will have the form $h = K - 82.5\cos(Bt)$. When $t = 0$, $h = 9$, so:

$$9 = K - 82.5$$
$$K = 91.5$$

Since the wheel revolves at $1.5\,\frac{\text{revolutions}}{\text{min}}$, it requires $\frac{2}{3}\text{min}$ to complete 1 revolution, so the period is $\frac{2}{3}$. Therefore:

$$\frac{2\pi}{B} = \frac{2}{3}$$
$$6\pi = 2B$$
$$B = 3\pi$$

Therefore, the equation is given by $h = 91.5 - 82.5\cos(3\pi t)$.

40. We will use a cosine function to model this data. The form is $T = k + A\cos\left(B\left(m - h\right)\right)$. First find the amplitude:

$$A = \frac{1}{2}\left(14.52 - 9.78\right) = 2.37$$

Now finding the vertical translation: $k = \frac{1}{2}\left(9.78 + 14.52\right) = 12.15$

The longest day occurs at $d = 180$ and the shortest occurs at $d = 360$, so the period is 2(360 – 180) = 360.
Finding *B*:

$$\frac{2\pi}{B} = 360$$
$$2\pi = 360B$$
$$B = \frac{\pi}{180}$$

Since the maximum occurs when $d = 180$, $h = 180$. Thus the curve is $T = 12.15 + 2.37\cos\left(\frac{\pi}{180}\left(d - 180\right)\right)$.

This can be written as $T = 12.15 + 2.37\cos\left(\frac{\pi}{180}d - \pi\right)$.

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42. We will use a sine function to model this data. The form is $T = k + A\sin\left(B\left(t-h\right)\right)$. First find the amplitude:

$$A = \frac{1}{2}\left(45-38\right) = 3.5$$

Now finding the vertical translation: $k = \frac{1}{2}\left(45+38\right) = 41.5$

Since the day is 24 hours:

$$\frac{2\pi}{B} = 24$$
$$2\pi = 24B$$
$$B = \frac{\pi}{12}$$

Since the lake reaches its average temperature at 10 AM, $h = 10$. Thus the curve is $T = 41.5 + 3.5\sin\left(\frac{\pi}{12}\left(t-10\right)\right)$.

This can be written as $T = 41.5 + 3.5\sin\left(\frac{\pi}{12}t - \frac{5\pi}{6}\right)$.

44. The reference angle is 180° – 168° = 12°.

46. The reference angle is –115° + 180° = 65°.

48. Since 490° has the same trigonometric values as 130°, the reference angle is 180° – 130° = 50°.

50. Since $\cos\theta = 0.4557$, the reference angle is given by $\hat{\theta} = \cos^{-1}(0.4557) \approx 62.9°$. Since θ terminates in quadrant IV, $\theta = 360° - 62.9° = 297.1°$.

52. Since $\sec\theta = -6.2832$, $\cos\theta = -\frac{1}{6.2832}$, so the reference angle is given by $\hat{\theta} = \cos^{-1}\left(\frac{1}{6.2832}\right) \approx 80.8°$. Since θ terminates in quadrant III, $\theta = 180° + 80.8° = 260.8°$.

54. Since $\tan\theta = 8.1506$, the reference angle is given by $\hat{\theta} = \tan^{-1}(8.1506) \approx 83.0°$. Since θ terminates in quadrant III, $\theta = 180° + 83.0° = 263.0°$.

56. This is a sine curve with amplitude 3, reflected across the *x*-axis, with a vertical translation of 2 and a period of 5. So:

$$\frac{2\pi}{B} = 5$$
$$2\pi = 5B$$
$$B = \frac{2\pi}{5}$$

Thus the equation is $y = 2 - 3\sin\left(\frac{2\pi}{5}x\right)$. The correct answer is d.

58. The form is $T = k - A\cos\left(B\left(t-h\right)\right)$. First find the amplitude:

$$A = \frac{1}{2}\left(80-40\right) = 20$$

The highest temperature occurs at $t = 15$ and the lowest occurs at $t = 3$, so the period is $2(15 - 3) = 24$.
Finding *B*:

$$\frac{2\pi}{B} = 24$$
$$2\pi = 24B$$
$$B = \frac{\pi}{12}$$

The correct answer is c.

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ODD SOLUTIONS

1. half, period **3.** average

5. Since the line crosses the y-axis at 1, we know that $b = 1$. The ratio of vertical change to horizontal change between any 2 points is $-\frac{1}{2}$. Therefore $m = -\frac{1}{2}$. The equation of the line must be $y = -\frac{1}{2}x + 1$.

7. Since the line crosses the y-axis at -3, we know that $b = -3$. The ratio of vertical change to horizontal change between any 2 points is $\frac{2}{1}$ *or* 2. Therefore $m = 2$. The equation of the line must be $y = 2x - 3$.

9. The graph is a sine curve with an amplitude of 1, period 2π, and no horizontal shift. The equation is $y = \sin x$.

11. The graph is a cosine curve with amplitude of 3, period 2π, and no horizontal shift. The equation is $y = 3\cos x$.

13. The graph is a cosine curve that has been reflected about the x-axis. The amplitude is 3, the period 2π, and no horizontal shift. The equation is $y = -3\cos x$.

15. The graph is a sine curve with an amplitude of 1, period $\frac{2\pi}{3}$, and no horizontal shift. To find B, we set $\frac{2\pi}{3}$ equal to $\frac{2\pi}{B}$:

$$\frac{2\pi}{3} = \frac{2\pi}{B}$$
$$B = 3$$

Therefore, the equation is $y = \sin 3x$.

17. The graph is a sine curve with amplitude of 1, period 6π, and no horizontal shift. To find B, we set 6π equal to $\frac{2\pi}{B}$:

$$6\pi = \frac{2\pi}{B}$$
$$6\pi B = 2\pi$$
$$B = \frac{1}{3}$$

Therefore, the equation is $y = \sin\frac{1}{3}x$.

19. The graph is a cosine curve with amplitude of 2, period of $\frac{2\pi}{3}$, and no horizontal shift. In problem 15, we found that $B = 3$. Therefore, the equation is $y = 2\cos 3x$.

21. The graph is a sine curve with amplitude of 4, period of 2, and no horizontal shift.

To find B, we use

$$2 = \frac{2\pi}{B}$$
$$2B = 2\pi$$
$$B = \pi$$

Therefore, the equation is $y = 4\sin \pi x$.

23. The graph is a sine curve that has been reflected about the x-axis. The amplitude is 4, the period of 2 and no horizontal shift. In problem 21, we found that $B = \pi$. Therefore, the equation is $y = -4\sin \pi x$.

25. The graph is the sine curve from problem 23 that has been moved up 2 units. Therefore, the equation is $y = 2 - 4\sin \pi x$.

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27. The graph is a cosine curve with amplitude of 3. The period is $\frac{5\pi}{8}-\left(-\frac{3\pi}{8}\right)$ or π and the horizontal shift is $-\frac{3\pi}{8}$. To find B, we use $\pi=\frac{2\pi}{B}$

$$B\pi=2\pi$$
$$B=2$$

To find h, we use $-\frac{3\pi}{8}=-\frac{h}{B}$

$$-\frac{3\pi}{8}=-\frac{h}{2}$$
$$h=\frac{3\pi}{4}$$

Therefore, the equation is $y=3\cos\left(2x+\frac{3\pi}{4}\right)$.

29. The graph is a cosine curve that has been reflected about the x-axis. The amplitude is 2. The period is $\frac{3\pi}{4}-\frac{\pi}{12}=\frac{8\pi}{12}=\frac{2\pi}{3}$. The horizontal shift is $\frac{\pi}{12}$.

To find B, we use $\frac{2\pi}{3}=\frac{2\pi}{B}$

$$B=3$$

To find h, we use $\frac{\pi}{12}=-\frac{h}{B}$

$$\frac{\pi}{12}=-\frac{h}{3}$$
$$h=-\frac{\pi}{4}$$

Therefore, the equation is $y=-2\cos\left(3x-\frac{\pi}{4}\right)$

31. The graph is the sine curve with amplitude of 3 and a period of π that has been reflected about the x-axis and shifted down 3 units. To find B, we use $\pi=\frac{2\pi}{B}$

$$B\pi=2\pi$$
$$B=2$$

Therefore the equation is $y=-3-3\sin(2x)$.

33. The graph is the sine curve that has been reflected about the x-axis. The amplitude is 3. The period is $\frac{5\pi}{4}-\left(\frac{\pi}{4}\right)=\frac{4\pi}{4}=\pi$. The horizontal shift is $\frac{\pi}{4}$ and the vertical translation is 3.

To find B, we use $\pi=\frac{2\pi}{B}$

$$B=2$$

To find h, we use $\frac{\pi}{4}=-\frac{h}{B}$

$$\frac{\pi}{4}=-\frac{h}{2}$$
$$h=-\frac{\pi}{2}$$

Therefore, the equation is $y=3-3\sin\left(2x-\frac{\pi}{2}\right)$.

35. Amplitude $=\frac{21-11}{2}=5$

Period $=\frac{1}{\text{frequency}}=\frac{1}{0.8}=\frac{5}{4}$

$$\frac{2\pi}{B}=\frac{5}{4}$$
$$5B=8\pi$$
$$B=\frac{8\pi}{5}$$

Therefore, the equation is $d=5\cos\left(\frac{8\pi}{5}t\right)$

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39. This is one possible solution. There are others. Consider the graph is a sine curve with amplitude 400 and period 4 that has been reflected about the *x*-axis and shifted up 1400:

To find B, we use $\frac{2\pi}{B} = 4$

$4B = 2\pi$

$B = \frac{\pi}{2}$

Therefore, the equation is $n = 1400 - 400\sin\left(\frac{\pi}{2}t\right)$.

41. This is one possible solution. There are others. Consider the graph is a cosine curve that has been reflected about the *x*-axis. It has been shifted up 60 units and to the right 3 units.

Amplitude $= \frac{80-40}{2} = 20$ To find B, we use $\frac{2\pi}{B} = 9$ To find h, we use $-\frac{h}{B} = 3$

$9B = 2\pi$ $-\frac{h}{2\pi/9} = 3$

$B = \frac{2\pi}{9}$ $h = -\frac{2\pi}{3}$

Therefore, the equation is $T = 60 - 20\cos\left(\frac{2\pi}{9}m - \frac{2\pi}{3}\right)$.

43. $\hat{\theta} = 360^\circ - 321^\circ$

$= 39^\circ$

45. $\hat{\theta} = 236^\circ - 180^\circ$

$= 56^\circ$

47. $\theta = -276^\circ + 360^\circ$

$= 84^\circ$

$\hat{\theta} = 84^\circ$

49. $\sin\hat{\theta} = 0.7455$

$\hat{\theta} = \sin^{-1}(0.7455)$

$\hat{\theta} = 48.2^\circ$

Therefore, $\theta = 180^\circ - 48.2^\circ$

$= 131.8^\circ$

51. $\csc\hat{\theta} = 2.3228$

$\sin\hat{\theta} = \frac{1}{2.3228}$

$\hat{\theta} = \sin^{-1}(0.4305)$

$\hat{\theta} = 25.5^\circ$

Therefore, $\theta = 180^\circ + 25.5^\circ$

$= 205.5^\circ$

53. $\cot\hat{\theta} = 0.2089$

$\tan\hat{\theta} = \frac{1}{0.2089}$

$\hat{\theta} = \tan^{-1}(4.7870)$

$\hat{\theta} = 78.2^\circ$

Therefore, $\theta = 360^\circ - 78.2^\circ$

$= 281.8^\circ$

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4.6 Graphing Combinations of Functions

EVEN SOLUTIONS

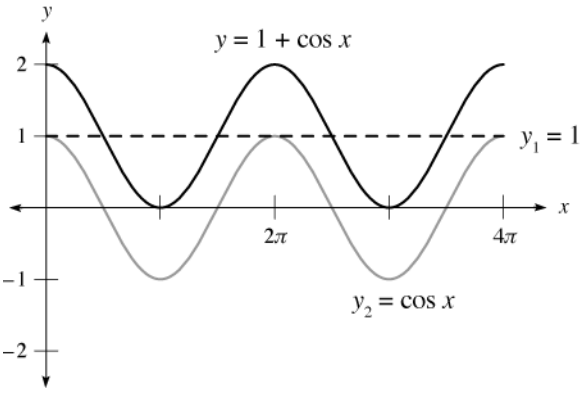
2. If y_2 is positive, then $y_1 + y_2$ will lie above y_1 by a distance equal to y_2.

4. A Fourier series can be used to represent certain waveforms as an infinite sum of sine and/or cosine functions.

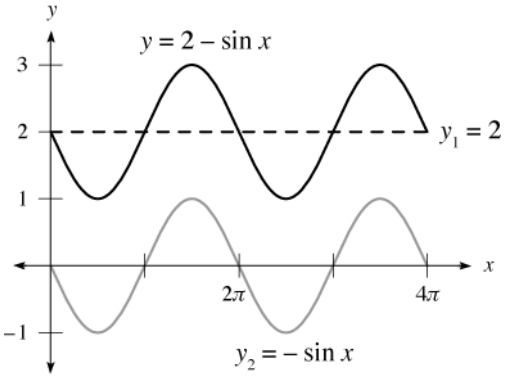
6. The corresponding point is $(0, 3-5) = (0, -2)$.

8. The corresponding point is $\left(\frac{\pi}{4}, -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) = \left(\frac{\pi}{4}, 0\right)$.

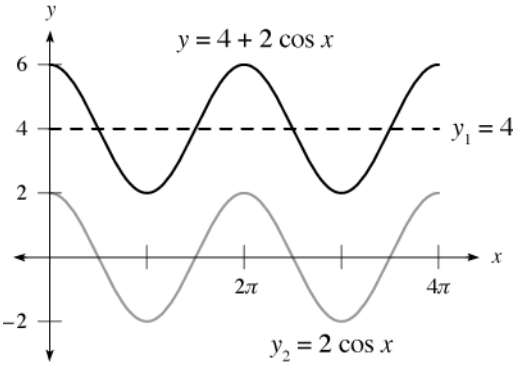
10. Graphing $y_1 = 1, y_2 = \cos x$, and $y = 1 + \cos x$:



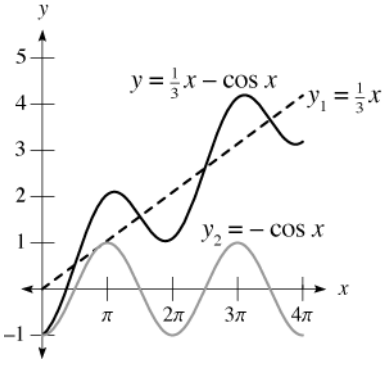
12. Graphing $y_1 = 2, y_2 = -\sin x$, and $y = 2 - \sin x$:



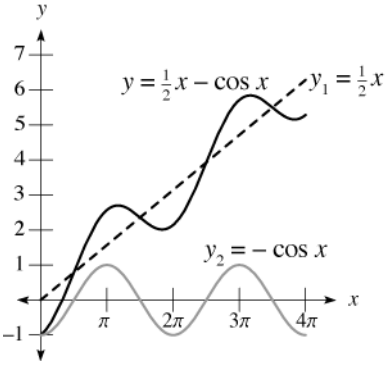
14. Graphing $y_1 = 4, y_2 = 2\cos x$, and $y = 4 + 2\cos x$:



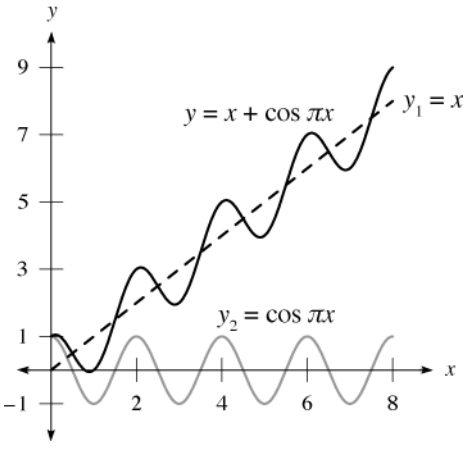
16. Graphing $y_1 = \frac{1}{3}x, y_2 = -\cos x$, and $y = \frac{1}{3}x - \cos x$:



18. Graphing $y_1 = \frac{1}{2}x, y_2 = -\cos x$, and $y = \frac{1}{2}x - \cos x$:



20. Graphing $y_1 = x, y_2 = \cos \pi x$, and $y = x + \cos \pi x$:



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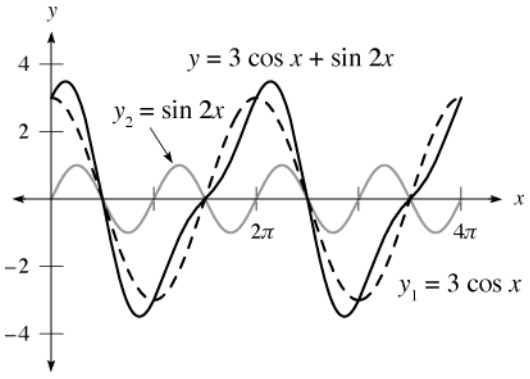
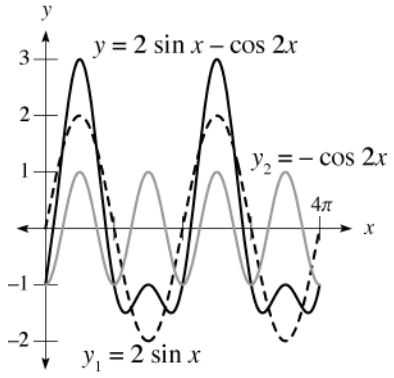
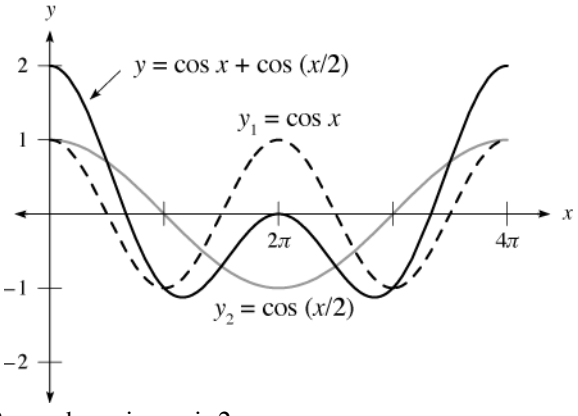
22. Graphing $y_1 = 3\cos x, y_2 = \sin 2x,$ and $y = 3\cos x + \sin 2x$:



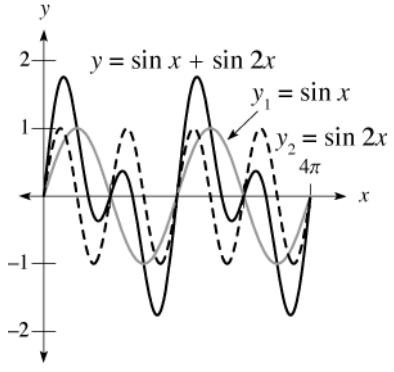
24. Graphing $y_1 = 2\sin x, y_2 = -\cos 2x,$ and $y = 2\sin x - \cos 2x$:



26. Graphing $y_1 = \cos x, y_2 = \cos\frac{x}{2},$ and $y = \cos x + \cos\frac{x}{2}$:



28. Graphing $y_1 = \sin x, y_2 = \sin 2x,$ and $y = \sin x + \sin 2x$:



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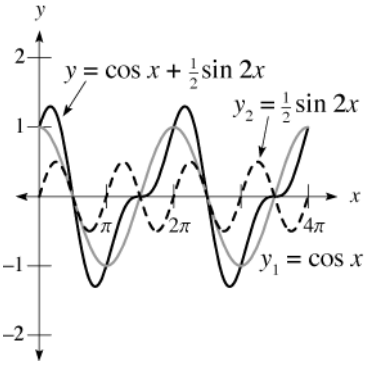
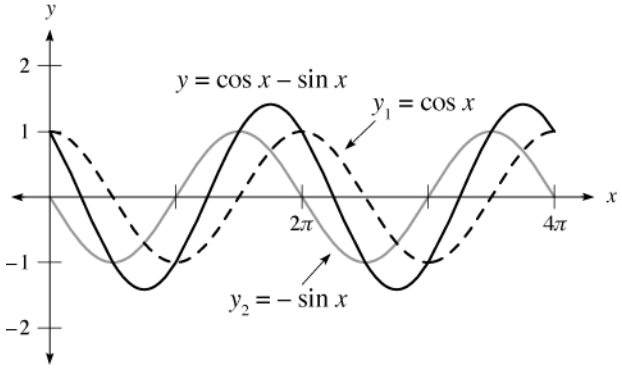
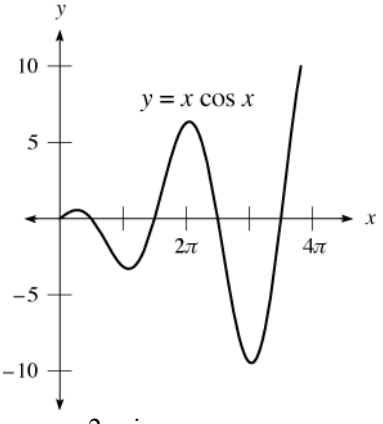
30. Graphing $y_1 = \cos x, y_2 = \frac{1}{2}\sin 2x,$ and $y = \cos x + \frac{1}{2}\sin 2x$:



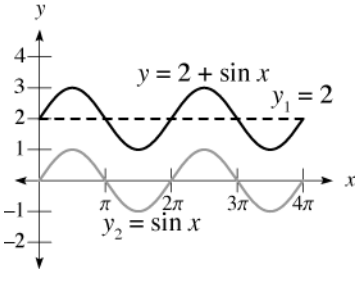
32. Graphing $y_1 = \cos x, y_2 = -\sin x,$ and $y = \cos x - \sin x$:



34. Sketching the graph of $y = x\cos x$:



36. Graphing $y_1 = 2$, $y_2 = \sin x$, and $y = y_1 + y_2 = 2 + \sin x$:



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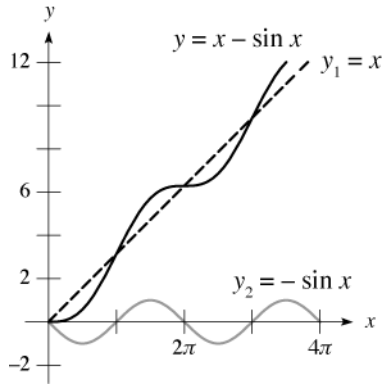
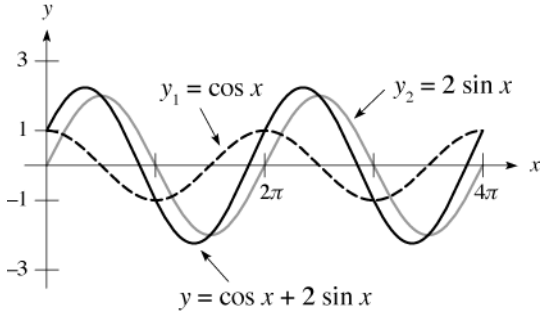
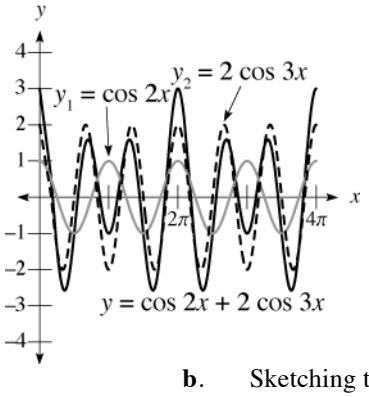
38. Graphing $y_1 = x$, $y_2 = -\sin x$, and $y = y_1 + y_2 = x - \sin x$:



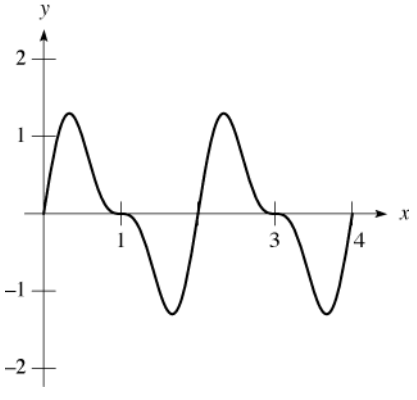
40. Graphing $y_1 = \cos x$, $y_2 = 2\sin x$, and $y = y_1 + y_2 = \cos x + 2\sin x$:



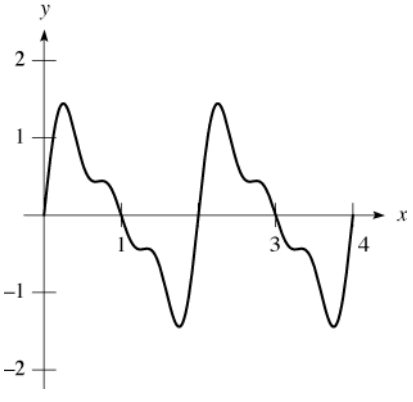
42. Graphing $y_1 = \cos 2x$, $y_2 = 2\cos 3x$, and $y = y_1 + y_2 = \cos 2x + 2\cos 3x$:



44. **a**. Sketching the graph:



b. Sketching the graph:



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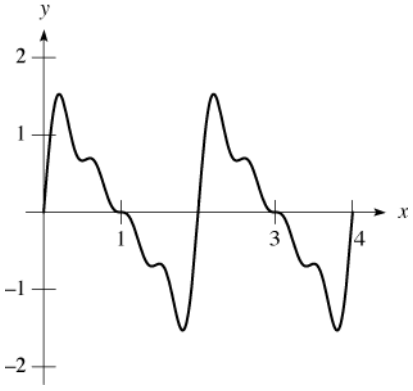
c. Sketching the graph:



46. The distance the point travels is given by: $s = vt = 15\frac{\text{m}}{\text{sec}} \cdot 60 \text{ sec} = 900 \text{ m}$

48. Converting to angular velocity: $10\frac{\text{rev}}{\text{min}} \cdot 2\pi \frac{\text{radians}}{\text{rev}} = 20\pi \frac{\text{radians}}{\text{min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = \frac{\pi}{3}\frac{\text{radians}}{\text{sec}}$

50. In 40 minutes the minute hand moves $\frac{2}{3}$ revolution $\cdot 2\pi \frac{\text{radians}}{\text{rev}} = \frac{4\pi}{3}$ radians .

Therefore: $s = r\theta = 10 \text{ cm} \cdot \frac{4\pi}{3} = \frac{40\pi}{3}$ cm

52. The correct graph is b.

ODD SOLUTIONS

1. y, y

3. below, y_2

5. For $x = 1$, $y = y_1 + y_2$

$= 1 + 2 = 3$

Therefore, the corresponding point is $(1,3)$.

7. For $x = \pi$, $y = y_1 + y_2$

$= -0.5 + (-1) = -1.5$

Therefore, the corresponding point is $(\pi, -1.5)$.

9. We let $y_1 = 1$ and $y_2 = \sin x$ and graph y_1, y_2, and $y = y_1 + y_2$ on the same coordinate system.

11. We let $y_1 = 2$ and $y_2 = -\cos x$ (which is a cosine curve reflected about the *x*-axis). Then we graph y_1, y_2, and $y = y_1 + y_2$ on the same coordinate system.

13. We let $y_1 = 4$ and $y_2 = 2\sin x$ (which is a sine curve with an amplitude of 2). Then we graph y_1, y_2, and $y = y_1 + y_2$ on the same coordinate system.

15. We let $y_1 = \frac{1}{2}x$ and $y_2 = -\sin x$ (which is a sine curve reflected about the *x*-axis). Then we graph y_1, y_2, and $y = y_1 + y_2$ on the same coordinate system.

17. We let $y_1 = \frac{1}{3}x$ and $y_2 = \cos x$ Then we graph y_1, y_2, and $y = y_1 + y_2$ on the same coordinate system.

19. We let $y_1 = x$ and $y_2 = \sin \pi x$ (which is a sine curve with a period of 2). Then we graph y_1, y_2, and $y = y_1 + y_2$ on the same coordinate system.

21. We let $y_1 = 3\sin x$ (which is a sine curve with an amplitude of 3) and $y_2 = \cos 2x$ (which is a cosine curve with a period of π). Then we graph y_1, y_2, and $y = y_1 + y_2$ on the same coordinate system.

23. We let $y_1 = 2\cos x$ (which is a cosine curve with an amplitude of 2) and $y_2 = -\sin 2x$ (which is a sine curve with a period of π and reflected about the *x*-axis). Then we graph y_1, y_2, and $y = y_1 + y_2$ on the same coordinate system.

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25. We let $y_1 = \sin x$ and $y_2 = \sin\frac{x}{2}$ (which is a sine curve with a period of 4π).

Then we graph $y_1, y_2,$ and $y = y_1 + y_2$ on the same coordinate system.

27. We let $y_1 = \cos x$ and $y_2 = \cos 2x$ (which is a cosine curve with a period of π).

Then we graph $y_1, y_2,$ and $y = y_1 + y_2$ on the same coordinate system.

29. We let $y_1 = \sin x$ and $y_2 = \frac{1}{2}\cos 2x$ (which is a cosine curve with an amplitude of $\frac{1}{2}$ and a period of π). Then we graph $y_1, y_2,$ and $y = y_1 + y_2$ on the same coordinate system.

31. We let $y_1 = \sin x$ and $y_2 = -\cos x$ (which is a cosine curve reflected about the *x*-axis). Then we graph $y_1, y_2,$ and $y = y_1 + y_2$ on the same coordinate system.

33. $y = x \sin x$

x	0	$\pi/2$	π	$3\pi/2$	2π	$5\pi/2$	3π	$7\pi/2$	4π
$\sin x$	0	1	0	–1	0	1	0	–1	0
$x \sin x$	0	$\pi/2$	0	$-3\pi/2$	0	$5\pi/2$	0	$-7\pi/2$	0

For problems 35 through 43, see textbook answer section for graphs.

45. $v = \frac{s}{t}$

$= \frac{5\ ft}{20\ \text{sec}} = \frac{1}{4}\ ft/\text{sec}$

47. $v = r\omega$

$= 6\ m\left(3\frac{rad}{\text{sec}}\right)$

$= 18$ m/sec

$s = vt$

$= 18\ \frac{m}{\text{sec}} \cdot 10\ \text{sec}$

$= 180\ m$

49. $\omega = 5\frac{\text{rev}}{\text{min}} \cdot 2\pi\frac{\text{rad}}{rev}$

$= 10\ \pi$ rad/min

$v = r\omega$

$= (6\text{ in})\left(10\pi\ \frac{rad}{\text{min}}\right)$

$= 60\pi$ in/min

4.7 Inverse Trigonometric Functions

EVEN SOLUTIONS

2. Only functions whose graphs pass the horizontal line test will have an inverse that is also a function. We call this type of function a one-to-one function.

4. Because all six trigonometric functions are not one-to-one, we must restrict the domain in order to define an inverse function.

6. The notation $y = \arccos x$ means that *y* is the angle between 0 and π, inclusive, whose cosine equals *x*.

8. If $y = \tan^{-1} x$, then *y* will be an angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

10. If $y = \cos^{-1} x$, then *y* can terminate in quadrant I or II.

12. In order for $\sin^{-1}(\sin x) = x$, *x* must be a value between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

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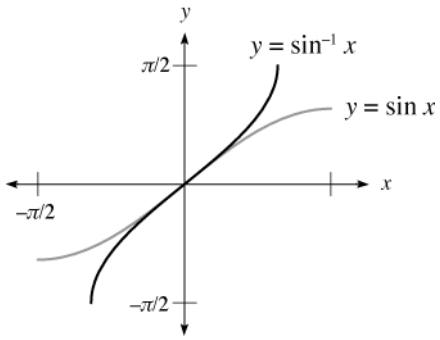
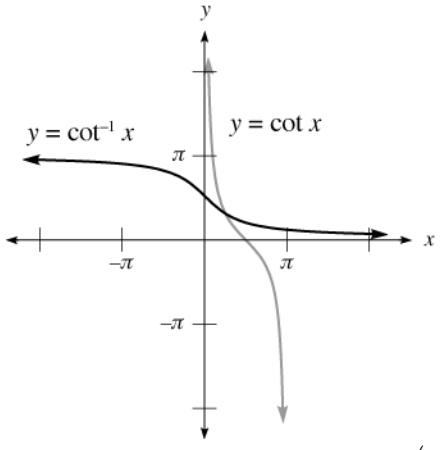
14. Sketching the graph of $y = \sin x$ between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ then reflecting across the line $y = x$ results in the graph of $y = \sin^{-1} x$:



16. Sketching the graph of $y = \cot x$ between 0 and π then reflecting across the line $y = x$ results in the graph of $y = \cot^{-1} x$:



18. Let $x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$, so $\cos x = \frac{\sqrt{3}}{2}$ and $0 \le x \le \pi$. Thus $x = \frac{\pi}{6}$, so $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$.

20. Let $x = \sin^{-1}(0)$, so $\sin x = 0$ and $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. Thus $x = 0$, so $\sin^{-1}(0) = 0$.

22. Let $x = \tan^{-1}(0)$, so $\tan x = 0$ and $-\frac{\pi}{2} < x < \frac{\pi}{2}$. Thus $x = 0$, so $\tan^{-1}(0) = 0$.

24. Let $x = \arccos(1)$, so $\cos x = 1$ and $0 \le x \le \pi$. Thus $x = 0$, so $\arccos(1) = 0$.

26. Let $x = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$, so $\sin x = -\frac{\sqrt{2}}{2}$ and $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. Thus $x = -\frac{\pi}{4}$, so $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$.

28. Let $x = \arctan\left(\frac{\sqrt{3}}{3}\right)$, so $\tan x = \frac{\sqrt{3}}{3}$ and $-\frac{\pi}{2} < x < \frac{\pi}{2}$. Thus $x = \frac{\pi}{6}$, so $\arctan\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$.

30. Let $x = \arcsin\left(-\frac{\sqrt{3}}{2}\right)$, so $\sin x = -\frac{\sqrt{3}}{2}$ and $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. Thus $x = -\frac{\pi}{3}$, so $\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$.

32. Let $x = \tan^{-1}\left(-\sqrt{3}\right)$, so $\tan x = -\sqrt{3}$ and $-\frac{\pi}{2} < x < \frac{\pi}{2}$. Thus $x = -\frac{\pi}{3}$, so $\tan^{-1}\left(-\sqrt{3}\right) = -\frac{\pi}{3}$.

34. Let $x = \cos^{-1}\left(-\frac{1}{2}\right)$, so $\cos x = -\frac{1}{2}$ and $0 \le x \le \pi$. Thus $x = \frac{2\pi}{3}$, so $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$.

36. Let $x = \arcsin(-1)$, so $\sin x = -1$ and $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. Thus $x = -\frac{\pi}{2}$, so $\arcsin(-1) = -\frac{\pi}{2}$.

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38. Using a calculator: $\sin^{-1}(-0.7021) \approx -44.6°$

40. Using a calculator: $\arccos(0.2967) \approx 72.7°$

42. Using a calculator: $\tan^{-1}(3.7990) \approx 75.3°$

44. Using a calculator: $\sin^{-1}(-0.6446) \approx -40.1°$

46. Using a calculator: $\arctan(-0.2679) \approx -15.0°$

48. Using a calculator: $\cos^{-1}(-0.9397) \approx 160.0°$

50. **a**. The value is $\sin^{-1}\left(-\frac{1}{2}\right) = -30°$.

b. The value is $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60°$.

c. The value is $\arcsin\left(-\frac{\sqrt{2}}{2}\right) = -45°$.

52. Let $\theta = \cos^{-1}\frac{x}{2}$, so $0 \le \theta \le \pi$. Then $\sin\theta \ge 0$, and so $|2\sin\theta| = 2\sin\theta$.

54. Let $\theta = \tan^{-1}\frac{x}{5}$, so $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. Then $\sec\theta > 0$, and so $|5\sec\theta| = 5\sec\theta$.

56. Since they are inverse functions: $\cos\left(\cos^{-1}\frac{5}{13}\right) = \frac{5}{13}$

58. Since they are inverse functions: $\sin\left(\sin^{-1}\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}$

60. Since they are inverse functions: $\tan\left(\tan^{-1}\frac{7}{24}\right) = \frac{7}{24}$

62. Evaluating: $\sin^{-1}(\sin 300°) = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -60°$ or $-\frac{\pi}{3}$

64. Evaluating: $\sin^{-1}\left(\sin\frac{\pi}{6}\right) = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

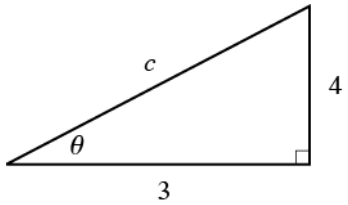
66. Evaluating: $\cos^{-1}(\cos 45°) = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = 45°$ or $\frac{\pi}{4}$

68. Evaluating: $\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

70. Evaluating: $\tan^{-1}(\tan 30°) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$ or $\frac{\pi}{6}$

72. Evaluating: $\tan^{-1}\left(\tan\frac{4\pi}{3}\right) = \tan^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}$

74. Let $\theta = \tan^{-1}\left(\frac{4}{3}\right)$, so $\tan\theta = \frac{4}{3}$. Drawing the triangle:



Using the Pythagorean Theorem: $c = \sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$

Therefore: $\csc\left(\tan^{-1}\frac{4}{3}\right) = \csc\theta = \frac{1}{\sin\theta} = \frac{1}{4/5} = \frac{5}{4}$

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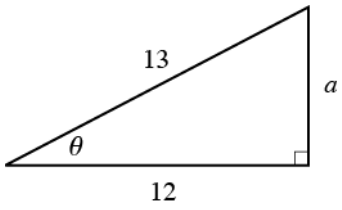
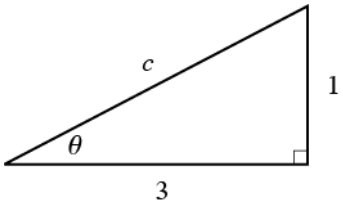
76. Let $\theta = \cos^{-1}\frac{12}{13}$, so $\cos\theta = \frac{12}{13}$. Drawing the triangle:



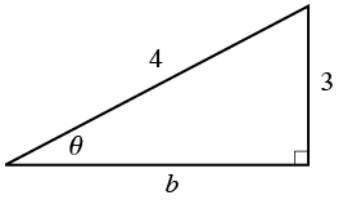
Using the Pythagorean Theorem: $a = \sqrt{13^2 - 12^2} = \sqrt{169 - 144} = \sqrt{25} = 5$. Therefore: $\tan\left(\cos^{-1}\frac{12}{13}\right) = \tan\theta = \frac{5}{12}$

78. Let $\theta = \tan^{-1}\frac{1}{3}$, so $\tan\theta = \frac{1}{3}$. Drawing the triangle:



Using the Pythagorean Theorem: $c = \sqrt{1^2 + 3^2} = \sqrt{1 + 9} = \sqrt{10}$. Therefore: $\cos\left(\tan^{-1}\frac{1}{3}\right) = \cos\theta = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$

80. Let $\theta = \sin^{-1}\frac{3}{4}$, so $\sin\theta = \frac{3}{4}$. Drawing the triangle:

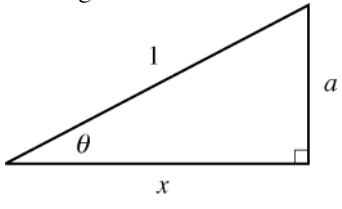


Using the Pythagorean Theorem: $b = \sqrt{4^2 - 3^2} = \sqrt{16 - 9} = \sqrt{7}$. Therefore: $\cos\left(\sin^{-1}\frac{3}{4}\right) = \cos\theta = \frac{\sqrt{7}}{4}$

82. $\cos^{-1}(\cos x) = x$ as long as $0 \le x \le \pi$.

84. $\sin\left(\sin^{-1}x\right) = x$ as long as $-1 \le x \le 1$.

86. Let $\theta = \cos^{-1}x$, so $\cos\theta = x$. Drawing the triangle:



Using the Pythagorean Theorem: $a = \sqrt{1 - x^2}$. Therefore: $\tan\left(\cos^{-1}x\right) = \tan\theta = \frac{\sqrt{1 - x^2}}{x}$

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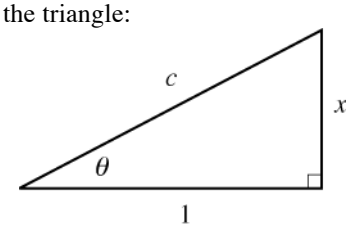
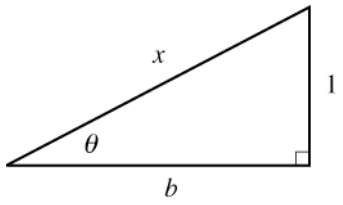
88. Let $\theta = \tan^{-1} x$, so $\tan\theta = x$. Drawing the triangle:



Using the Pythagorean Theorem: $c = \sqrt{x^2+1}$. Therefore: $\cos\left(\tan^{-1} x\right) = \cos\theta = \dfrac{1}{\sqrt{x^2+1}}$

90. Let $\theta = \sin^{-1}\dfrac{1}{x}$, so $\sin\theta = \dfrac{1}{x}$. Drawing the triangle:



Using the Pythagorean Theorem: $b = \sqrt{x^2-1}$. Therefore: $\tan\left(\sin^{-1}\dfrac{1}{x}\right) = \tan\theta = \dfrac{1}{\sqrt{x^2-1}}$

92. From Problem 90: $\csc\left(\sin^{-1}\dfrac{1}{x}\right) = \csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{1/x} = x$

94. Calculating the radian value for *d*:

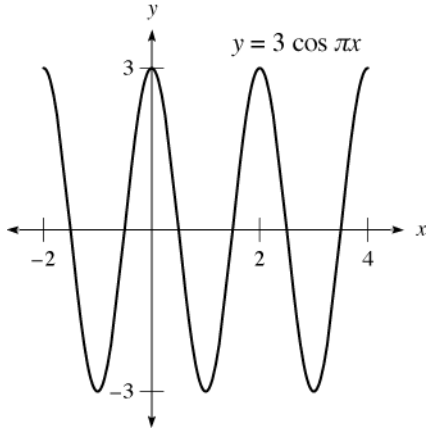
$$d = \cos^{-1}\left[\sin\left(21\frac{53.896}{60}\right)\sin\left(19\frac{43.219}{60}\right) + \cos\left(21\frac{53.896}{60}\right)\cos\left(19\frac{43.219}{60}\right)\cos\left(4\frac{33.406}{60}\right)\right]$$

$$\approx \cos^{-1}(0.9965)$$

$$\approx 0.0837$$

Multiply by *r* to obtain the great circle distance: $s = rd = (3960)(0.0837) \approx 331$ miles

96. The amplitude is 3 and the period is $\dfrac{2\pi}{\pi} = 2$:



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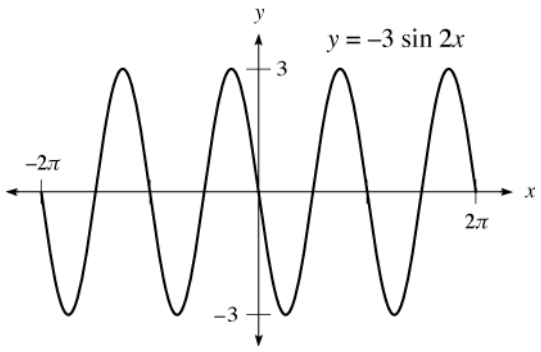
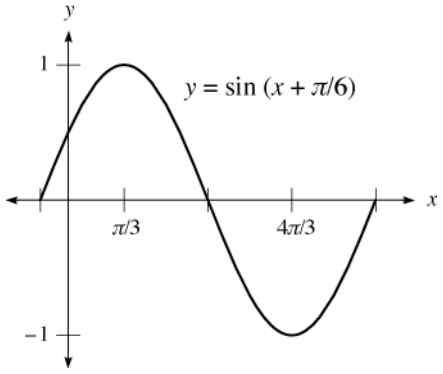
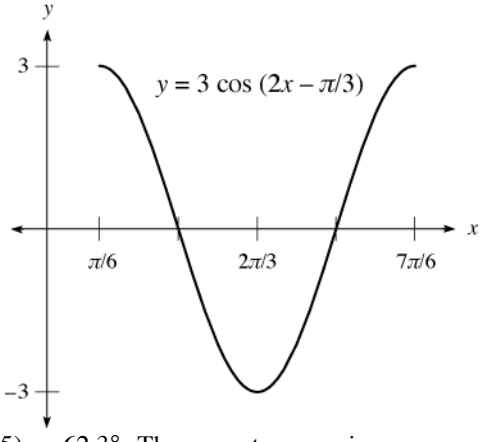
98. The amplitude is 3 and the period is $\frac{2\pi}{2} = \pi$. Note this curve is a reflection of $y = 3\sin 2x$ across the x-axis:



100. The amplitude is 1, the period is 2π, and the horizontal translation is $-\frac{\pi}{6}$:



102. The amplitude is 3, the period is $\frac{2\pi}{2} = \pi$, and the horizontal translation is $\frac{\pi/3}{2} = \frac{\pi}{6}$:



104. Using a calculator: $\arcsin(-0.8855) \approx -62.3°$. The correct answer is a.

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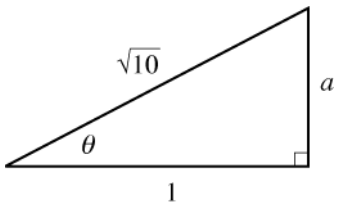
106. Let $\theta = \cos^{-1}\frac{1}{\sqrt{10}}$, so $\cos\theta = \frac{1}{\sqrt{10}}$. Drawing the triangle:



Using the Pythagorean Theorem: $a = \sqrt{\left(\sqrt{10}\right)^2 - 1^2} = \sqrt{10-1} = \sqrt{9} = 3$. Therefore: $\tan\left(\cos^{-1}\frac{1}{\sqrt{10}}\right) = \tan\theta = 3$

The correct answer is d.

ODD SOLUTIONS

1. interchange or reverse

3. reflection, $y = x$

5. $y = \sin^{-1} x$, $y = \arcsin x$

7. angle, $-\frac{\pi}{2}$, $\frac{\pi}{2}$, sine

9. I, IV

11. I, IV

For problems 13 and 15, see textbook answer section for graphs.

17. The angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose sine is $\frac{\sqrt{3}}{2}$ is $\frac{\pi}{3}$.

19. The angle between 0 and π whose cosine is -1 is π.

21. The angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose tangent is 1 is $\frac{\pi}{4}$.

23. The angle between 0 and π whose cosine is $-\frac{\sqrt{2}}{2}$ is $\frac{3\pi}{4}$.

25. The angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose sine is $-\frac{1}{2}$ is $-\frac{\pi}{6}$.

27. The angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose tangent is $\sqrt{3}$ is $\frac{\pi}{3}$.

29. The angle between 0 and π whose cosine is 0 is $\frac{\pi}{2}$.

31. The angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose tangent is $-\frac{\sqrt{3}}{3}$ is $-\frac{\pi}{6}$.

33. The angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose sine is 1 is $\frac{\pi}{2}$.

35. The angle between 0 and π whose cosine is $\frac{1}{2}$ is $\frac{\pi}{3}$.

37. Scientific Calculator: 0.1702 [*INV*] [*SIN*]

Graphing Calculator: [*2nd*] [*SIN*] [(] 0.1702 [)] [*ENTER*]

Answer: 9.8°

39. Scientific Calculator: 0.8425 [*INV*] [*COS*]

Graphing Calculator: [*2nd*] [*COS*] [(] 0.8425 [)] [*ENTER*]

Answer: 32.6°

41. Scientific Calculator: 0.3799 [*INV*] [*TAN*]

Graphing Calculator: [*2nd*] [*TAN*] [(] 0.3799 [)] [*ENTER*]

Answer: 20.8°

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43. Scientific Calculator: 0.4664 [+/−] [INV] [COS]

Graphing Calculator: [2nd] [COS] [(] [−] 0.4664 [)] [ENTER]

Answer: 117.8°

45. Scientific Calculator: 2.748 [+/−] [INV] [TAN]

Graphing Calculator: [2nd] [TAN] [(] [−] 2.748 [)] [ENTER]

Answer: -70.0°

47. Scientific Calculator: 0.7660 [+/−] [INV] [SIN]

Graphing Calculator: [2nd] [SIN] [(] [−] 0.7660 [)] [ENTER]

Answer: -50.0°

49. Using a TI-84 Plus,: graph $y_1 = \cos^{-1}(x)$ in degree mode. Set the window to x between -1 and 1 with Xscl = 0.5 and y between 0 and 180 with Yscl = 30.

a. Then push [2nd] [TRACE] [1:VALUE]. At the cursor, enter 1 [÷] 2 [ENTER]. The answer is $y = 60^\circ$.

b. Then push [2nd] [TRACE] [1:VALUE]. At the cursor, enter [(−)] [√] [(] 3 [)] [÷] 2 [ENTER]. The answer is $y = 150^\circ$.

c. Then push [2nd] [TRACE] [1:VALUE]. At the cursor, enter [√] [(] 2 [)] [÷] 2 [ENTER]. The answer is $y = 45^\circ$.

51. Follow the directions in problem 49, except graph $y_1 = \tan^{-1}(x)$ with the window set to x between -10 and 10.

a. Then push [2nd] [TRACE] [1:VALUE]. At the cursor, enter [(−)] 1 [ENTER]. The answer is $y = -45^\circ$.

b. Then push [2nd] [TRACE] [1:VALUE]. At the cursor, enter [√] [(] 3 [)] [ENTER]. The answer is 60°.

c. Then push [2nd] [TRACE] [1:VALUE]. At the cursor, enter [(−)] 1 [÷] [√] [(] 3 [)] [ENTER]. The answer is -30°.

53. If $\theta = \sin^{-1}\frac{x}{4}$, then $\sin\theta = \frac{x}{4}$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$. For any value in this interval, $\cos\theta$ will be positive. Therefore $|\cos\theta| = \cos\theta$ and we can simplify: $|4\cos\theta| = 4\cos\theta$.

55. Let $\theta = \sin^{-1}\frac{3}{5}$, then $\sin\theta = \frac{3}{5}$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$. We want to find $\sin\theta$ which is equal to $\frac{3}{5}$.

57. Let $\theta = \cos^{-1}\frac{1}{2}$, then $\cos\theta = \frac{1}{2}$ and $0 \le \theta \le \pi$. We want to find $\cos\theta$ which is equal to $\frac{1}{2}$.

59. Let $\theta = \tan^{-1}\frac{1}{2}$, then $\tan\theta = \frac{1}{2}$ and $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. We want to find $\tan\theta$ which is equal to $\frac{1}{2}$.

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61. First we want to find $\sin 225^\circ$: $\sin 225^\circ = -\sin 45^\circ = -\dfrac{\sqrt{2}}{2}$

Next we want to find $\sin^{-1}\left(-\dfrac{\sqrt{2}}{2}\right)$:

Let $\theta = \sin^{-1}\left(-\dfrac{\sqrt{2}}{2}\right)$

Then $\sin\theta = -\dfrac{\sqrt{2}}{2}$

$\sin\hat{\theta} = \dfrac{\sqrt{2}}{2}$

$\hat{\theta} = 45^\circ$ and $-90^\circ \le \theta \le 90^\circ$

Therefore, $\theta = -45^\circ$.

63. First we want to find $\sin\dfrac{\pi}{3}$: $\sin\dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}$

Next we want to find $\sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$:

Let $\theta = \sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$

Then $\sin\theta = \dfrac{\sqrt{3}}{2}$ and $-\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2}$

Therefore, $\theta = \dfrac{\pi}{3}$.

65. First we want to find $\cos 120^\circ$: $\cos 120^\circ = -\cos 60^\circ = -\dfrac{1}{2}$

Next we want to find $\cos^{-1}\left(-\dfrac{1}{2}\right)$:

Let $\theta = \cos^{-1}\left(-\dfrac{1}{2}\right)$

Then $\cos\theta = -\dfrac{1}{2}$ $\cos\hat{\theta} = \dfrac{1}{2}$

$\hat{\theta} = 60^\circ$ and $0^\circ \le \theta \le 180^\circ$

Therefore, $\theta = 180^\circ - 60^\circ = 120^\circ$.

67. First we want to find $\cos\dfrac{7\pi}{4}$: $\cos\dfrac{7\pi}{4} = \cos\dfrac{\pi}{4}$

$= \dfrac{\sqrt{2}}{2}$

Next we want to find $\cos^{-1}\left(\dfrac{\sqrt{2}}{2}\right)$:

Let $\theta = \cos^{-1}\left(\dfrac{\sqrt{2}}{2}\right)$

Then $\cos\theta = \dfrac{\sqrt{2}}{2}$ and $0 \le \theta \le \pi$

Therefore, $\theta = \dfrac{\pi}{4}$.

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69. First we want to find $\tan 45^\circ$: $\tan 45^\circ = 1$

Next we want to find $\tan^{-1} 1$:

Let $\theta = \tan^{-1} 1$

Then $\tan\theta = 1$ and $-90^\circ < \theta < 90^\circ$

Therefore, $\theta = 45^\circ$.

71. First we want to find $\tan \dfrac{5\pi}{6}$: $\tan \dfrac{5\pi}{6} = -\tan \dfrac{\pi}{6} = -\dfrac{\sqrt{3}}{3}$

Next we want to find $\tan^{-1}\left(-\dfrac{\sqrt{3}}{3}\right)$:

Let $\theta = \tan^{-1}\left(-\dfrac{\sqrt{3}}{3}\right)$

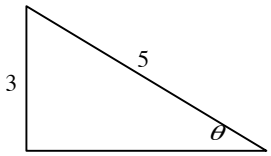
Then $\tan\theta = -\dfrac{\sqrt{3}}{3}$

$\tan\hat{\theta} = \dfrac{\sqrt{3}}{3}$

$\hat{\theta} = \dfrac{\pi}{6}$ and $-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$ Therefore, $\theta = -\dfrac{\pi}{6}$.

73. Let $\theta = \sin^{-1}\dfrac{3}{5}$, then $\sin\theta = \dfrac{3}{5}$ and $-\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2}$.

Next, we draw a triangle and find the adjacent side:



adjacent side $= \sqrt{5^2 - 3^2}$

$= \sqrt{25-9} = \sqrt{16} = 4$

Then, we find $\tan\theta = \dfrac{3}{4}$.

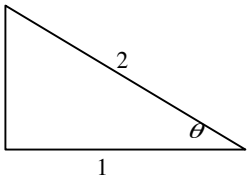
75. Let $\theta = \cos^{-1}\dfrac{1}{\sqrt{5}}$, then $\cos\theta = \dfrac{1}{\sqrt{5}}$ and $0 \le \theta \le \pi$.

We are looking for $\sec\theta$.

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{1/\sqrt{5}} = \sqrt{5}$

77. Let $\theta = \cos^{-1}\dfrac{1}{2}$, then $\cos\theta = \dfrac{1}{2}$ and $0 \le \theta \le \pi$.

Next, we draw a triangle and find the opposite side:



Opposite side $= \sqrt{2^2 - 1^2} = \sqrt{4-1} = \sqrt{3}$

Then we find $\sin\theta = \dfrac{\sqrt{3}}{2}$

79. Let $\theta = \tan^{-1}\dfrac{1}{2}$, then $\tan\theta = \dfrac{1}{2}$ and $-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$.

We want to find $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{1/2} = 2$

81. Since $y = \sin x$ and $y = \sin^{-1} x$ (for $-\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$) are inverse functions, one function will "undo" the action performed by the other. Therefore, $\sin^{-1}(\sin x) = x$.

83. Let $\theta = \cos^{-1} x$, then $\cos\theta = x$ and $0 \le \theta \le \pi$. We want to find $\cos\theta$ which is equal to x.

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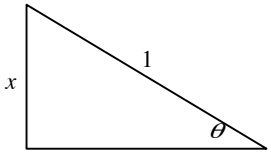
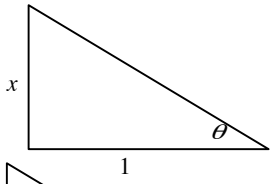
85. Let $\theta = \sin^{-1} x$, then $\sin\theta = x$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$. We draw a triangle and find the adjacent side: Adjacent side $= \sqrt{1^2 - x^2} = \sqrt{1 - x^2}$

From the figure, we find $\cos\theta = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$.



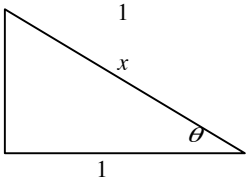
87. Let $\theta = \tan^{-1} x$, then $\tan\theta = x$ and $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. We draw a triangle and find the hypotenuse: hypotenuse $= \sqrt{x^2 + 1^2} = \sqrt{x^2 + 1}$

From the figure, we find $\sin\theta = \frac{x}{\sqrt{x^2+1}}$.



89. Let $\theta = \cos^{-1}\frac{1}{x}$, then $\cos\theta = \frac{1}{x}$ and $0 \le \theta \le \pi$. We draw a triangle and find the opposite side: Opposite side $= \sqrt{x^2 - 1^2} = \sqrt{x^2 - 1}$

From the figure, we find $\sin\theta = \frac{\sqrt{x^2-1}}{x}$.



91. Let $\theta = \cos^{-1}\frac{1}{x}$, then $\cos\theta = \frac{1}{x}$ and $0 \le \theta \le \pi$.

We want to find $\sec\theta = \frac{1}{\cos\theta} = \frac{1}{1/x} = x$

93. We compute the following on a calculator:

$d = \cos^{-1}[\sin(32^\circ 22.108')\sin(13^\circ 04.809') + \cos(32^\circ 22.108')\cos(13^\circ 04.809')\cos(64^\circ 41.178' - 59^\circ 29.263')]$

$= 1{,}370$ miles

95. The graph is a sine curve with amplitude of 2 and period of $\frac{2\pi}{\pi}$ *or* 2**.**

97. The graph is a cosine curve with amplitude of 3 and period of $\frac{2\pi}{1/2}$ *or* 4π**.**

The graph is reflected about the *x*-axis.

99. The graph is a standard sine curve with a horizontal shift of $\frac{\pi}{4}$.

101. Amplitude $= 3$ Period $= \frac{2\pi}{2} = \pi$ Horizontal shift $= \frac{\pi/3}{2} = \frac{\pi}{6}$

One Cycle: $0 \le 2x - \frac{\pi}{3} \le 2\pi$ Spacing $= \frac{1}{4}(\pi) = \frac{\pi}{4}$

$\frac{\pi}{3} \le 2x \le \frac{7\pi}{3}$

$\frac{\pi}{6} \le x \le \frac{7\pi}{6}$

The 5 points we use on the *x*-axis are $\frac{\pi}{6}, \frac{5\pi}{12}, \frac{2\pi}{3}, \frac{11\pi}{12}, \frac{7\pi}{6}$.

The 2 points we use on the *y*-axis are -3 and 3.

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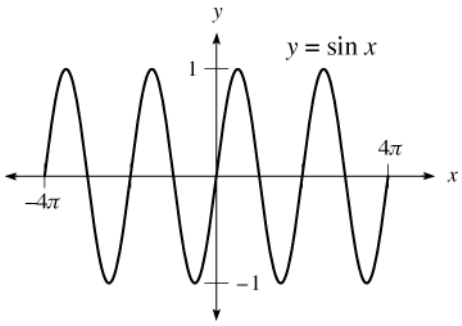
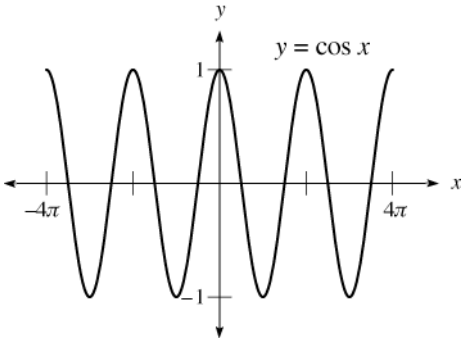
Chapter 4 Test

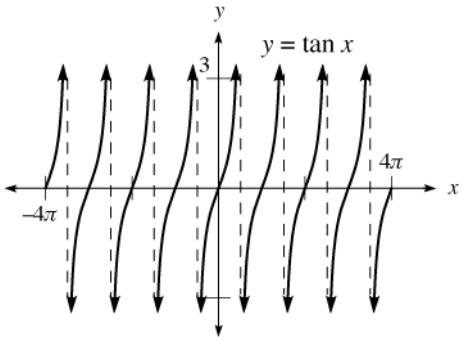
1. Sketching the graph:



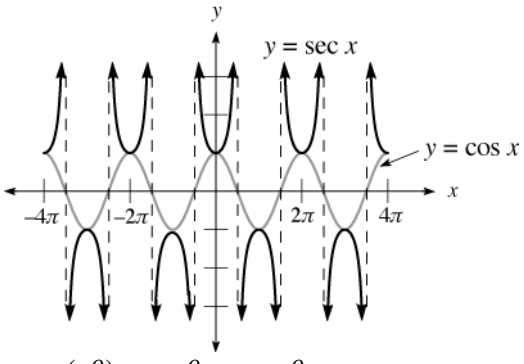
2. Sketching the graph:



3. Sketching the graph:



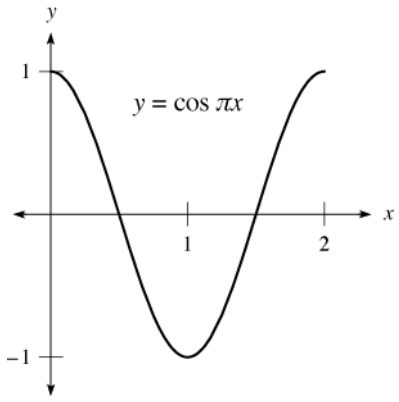
4. Sketching the graph:



5. Since sine is an odd function and cosine is an even function: $\cot(-\theta) = \dfrac{\cos(-\theta)}{\sin(-\theta)} = \dfrac{\cos\theta}{-\sin\theta} = -\dfrac{\cos\theta}{\sin\theta} = -\cot\theta$

6. Working from the left side: $\sin(-\theta)\sec(-\theta)\cot(-\theta) = -\sin\theta \cdot \dfrac{1}{\cos(-\theta)} \cdot \dfrac{\cos(-\theta)}{\sin(-\theta)} = -\sin\theta \cdot \dfrac{1}{\cos\theta} \cdot \dfrac{\cos\theta}{-\sin\theta} = 1$

7. The amplitude is 1 and the period is $\dfrac{2\pi}{\pi} = 2$:



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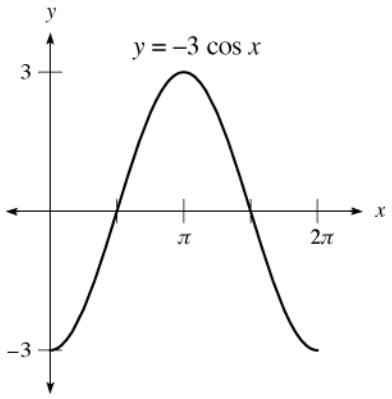
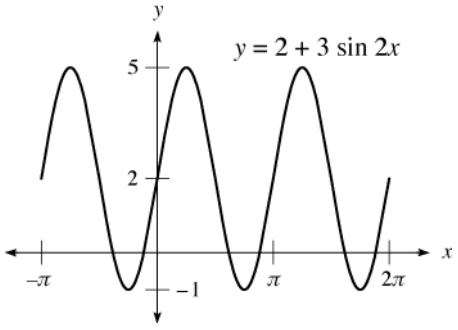
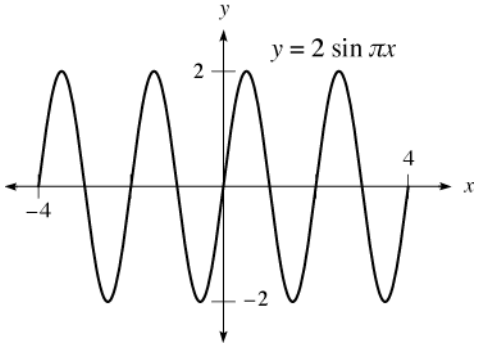
8. The amplitude is 3 and the period is 2π. Note this curve is a reflection of $y = 3\cos x$ across the x-axis:



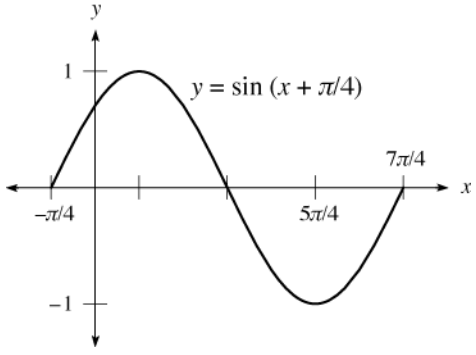
9. The amplitude is 3 and the period is $\frac{2\pi}{2} = \pi$. Note this curve is a vertical translation of $y = 3\sin 2x$ by 2 units:



10. The amplitude is 2 and the period is $\frac{2\pi}{\pi} = 2$:



11. The amplitude is 1, the period is 2π, and the horizontal shift is $-\frac{\pi}{4}$:



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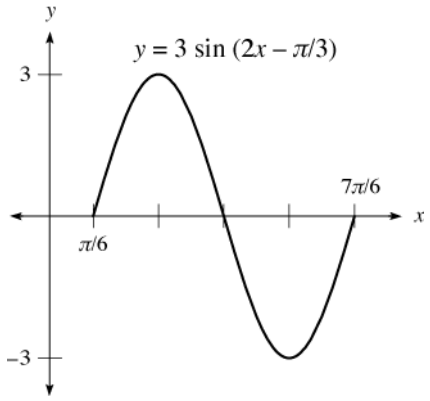
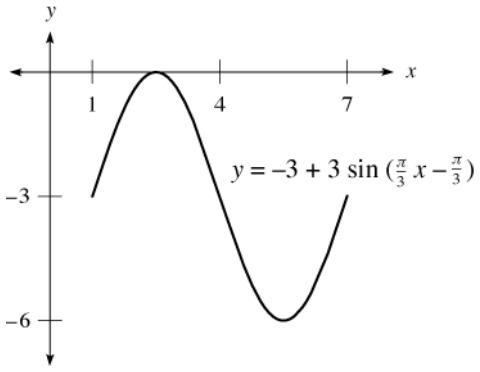
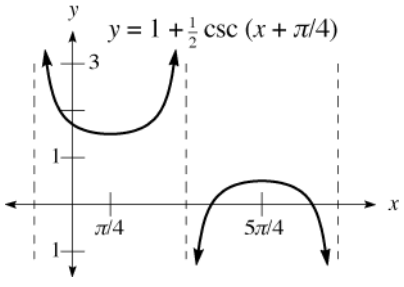
12. The amplitude is 3, the period is $\frac{2\pi}{2} = \pi$, and the horizontal shift is $\frac{\pi/3}{2} = \frac{\pi}{6}$:



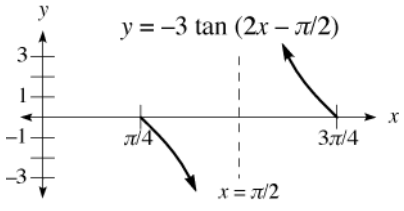
13. The amplitude is 3, the period is $\frac{2\pi}{\pi/3} = 6$, and the horizontal shift is $\frac{\pi/3}{\pi/3} = 1$. Note the curve is a vertical translation of $y = 3\sin\left(\frac{\pi}{3}x - \frac{\pi}{3}\right)$ by –3 units:



14. The period is 2π and the horizontal shift is $-\frac{\pi}{4}$:



15. The period is $\frac{\pi}{2}$ and the horizontal shift is $\frac{\pi/2}{2} = \frac{\pi}{4}$:



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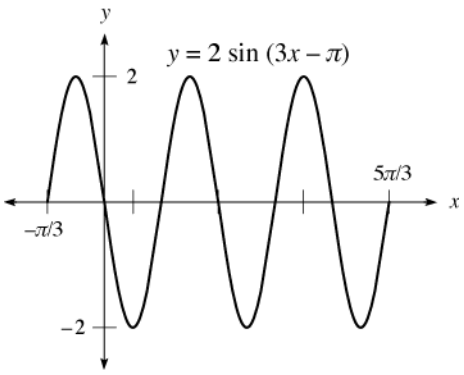
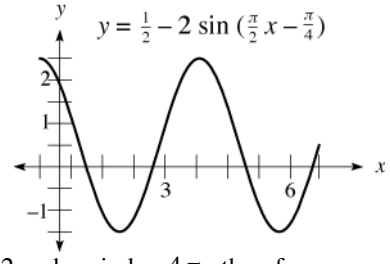
16. The amplitude is 2, the period is $\frac{2\pi}{3}$, and the horizontal shift is $\frac{\pi}{3}$:



17. The amplitude is 2, the period is $\frac{2\pi}{\pi/2} = 4$, and the horizontal shift is $\frac{\pi/4}{\pi/2} = \frac{1}{2}$. There is a vertical translation of $\frac{1}{2}$, and the curve is reflected across the x-axis:



18. This is a sine curve with amplitude = 2 and period = 4π, therefore:

$$\frac{2\pi}{B} = 4\pi$$
$$2\pi = 4\pi B$$
$$B = \frac{1}{2}$$

The horizontal shift is $-\pi$, therefore:

$$-\frac{C}{B} = -\pi$$
$$\frac{-C}{1/2} = -\pi$$
$$C = \frac{\pi}{2}$$

Thus the equation of the curve is $y = 2\sin\left(\frac{1}{2}x + \frac{\pi}{2}\right)$.

19. This is a sine curve with amplitude = $\frac{1}{2}(1-0) = \frac{1}{2}$ and a vertical translation of $\frac{1}{2}$ unit. The period is 4, therefore:

$$\frac{2\pi}{B} = 4$$
$$2\pi = 4B$$
$$B = \frac{\pi}{2}$$

There is no phase shift, thus the equation of the curve is $y = \frac{1}{2} + \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)$.

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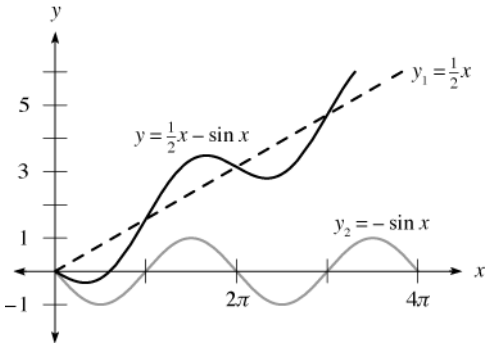
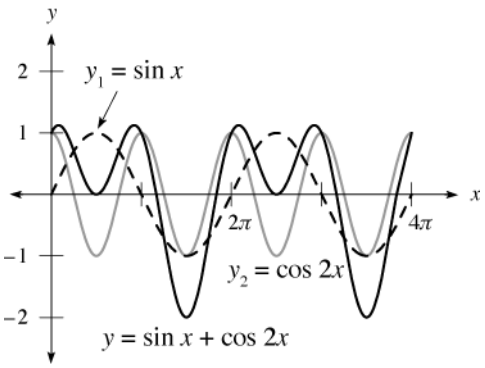
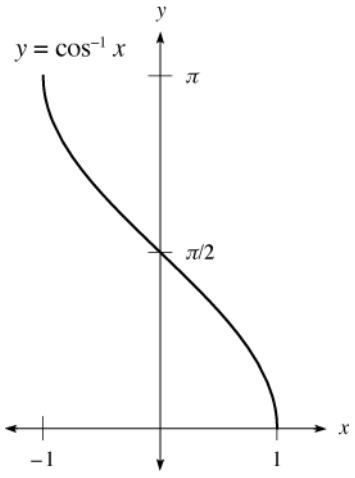
20. Graphing $y_1 = \frac{1}{2}x$, $y_2 = -\sin x$, and $y = \frac{1}{2}x - \sin x$:



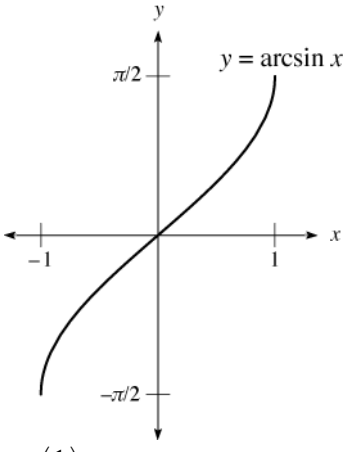
21. Graphing $y_1 = \sin x$, $y_2 = \cos 2x$, $y = \sin x + \cos 2x$:



22. Graphing $y = \cos^{-1} x$:



23. Graphing $y = \arcsin x$:



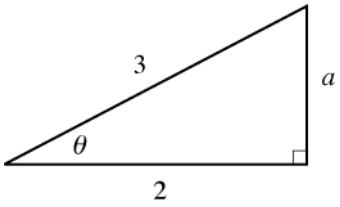
24. Let $x = \sin^{-1}\left(\frac{1}{2}\right)$, so $\sin x = \frac{1}{2}$ and $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. Thus $x = \frac{\pi}{6}$, so $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$.

25. Let $x = \arccos\left(-\frac{\sqrt{2}}{2}\right)$, so $\cos x = -\frac{\sqrt{2}}{2}$ and $0 \le x \le \pi$. Thus $x = \frac{3\pi}{4}$, so $\arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$.

26. Using a calculator: $\arcsin(0.9345) \approx 69.1°$

27. Using a calculator: $\arctan(-0.3028) \approx -16.8°$

28. Let $\theta = \cos^{-1}\frac{2}{3}$, so $\cos\theta = \frac{2}{3}$. Drawing the triangle:



Using the Pythagorean Theorem: $a = \sqrt{3^2 - 2^2} = \sqrt{9-4} = \sqrt{5}$. Therefore: $\sin\left(\cos^{-1}\frac{2}{3}\right) = \sin\theta = \frac{\sqrt{5}}{3}$

29. Evaluating: $\tan^{-1}\left(\tan\frac{7\pi}{6}\right) = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$

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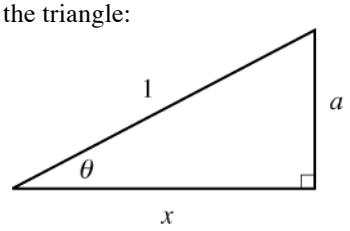
30. Let $\theta = \cos^{-1} x$, so $\cos\theta = x$. Drawing the triangle:



Using the Pythagorean Theorem: $a = \sqrt{1 - x^2}$. Therefore: $\tan\left(\cos^{-1} x\right) = \tan\theta = \dfrac{\sqrt{1 - x^2}}{x}$

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